Contents

CHAPTER ONE

Into the Forge

Pythagoras knew not to trust his ears. Sage and scientist, he understood that such organs of perception might well reveal some truths, but he was aware that they might always also lead him into error. In his *Fundamentals of Music*, Boethius relates that Pythagoras "put no credence in the ears," since, being bodily parts like all others, they are subject to incessant change.[1] Sometimes they vary on account of external and accidental circumstances; sometimes they begin to differ by necessity, as when they age. Pythagoras could hardly have expected more from acoustical devices made by men. Musical instruments were, for him, "sources of much variability and inconstancy."[2] More than once, he had studied the nature of strings. Their tones may change for reasons almost too numerous to enumerate. Depending on their matter, depending on their length and width, depending on the air about them and the force with which one plucks them, cords will inevitably produce different noises. Pythagoras had observed that other instruments are of a like nature. The sole certainty about them is that sooner or later, they must abandon "the state of their previous stability," causing their tones to change perceptibly. In awareness of the implications of these facts, Pythagoras had resolved to liberate himself and his investigations, as much as he could, from the troubling consequences of sensible things. He would study the laws of sound without needing to encounter them in bodily form, and he would acquire knowledge of the properties of audible things by the aid of reason alone.

The project was boldly conceived, but it was never to reach completion.

Just as Pythagoras decided on the course his inquiry must take, he was suddenly distracted. "As if impelled by a kind of divine will," the thinker found himself led from the place of his customary calculations out into the world. Enchanted, he wandered toward a forge. From within the smithy came the noise of many hammers. "Somehow," we learn, "they emitted a single consonance from differing sounds." Astonished, Pythagoras began to grasp what he had discovered. "He was in the presence of what he had long sought, and he approached the smiths' work as if spellbound."[3]

Wonder soon passed to reasoned reflection. The noise was not autonomous; it came from the activity of men, carried out with tools in one particular surrounding. Pythagoras sought to learn the cause of the coincidence of sounds, but this was no easy task. How to tell the smiths from the smithy, the hammerers from the hammers? First he tested a hypothesis. "He commanded the workers to exchange their tools among each other." Then the blacksmiths began their work anew. The startlingly single consonance remained. At least one indubitable conclusion could now be drawn: "The property of sounds did not rest in the muscles of men; instead, it followed the exchanged hammers." Yet the concord, properly defined, could not be said to move. Its place was stable; indisputably, it lay not in the workers, but in their tools. More exactly, the consonance lay in one of the hammers' many sensible properties. This was a property that may well have been incidental to their instrumentality: namely, that of possessing mass, insofar as it may be measured with precision. Pythagoras was quick to recognize this point: the "single consonance" resulted from the relations between the hammers' weights, which caused a set of pleasing sounds. For Boethius, a thinker of late antiquity, the relations of the various hammers' weights were naturally best expressed in the technical terms of Graeco-Latin arithmetic. He writes:

> There happened to be five hammers, and those which sounded together the consonance of the diapason were found to be double in weight. Pythagoras determined further that the same one, the one that was the double of the second, was the sesquitertian of another, with which it sounded a diatesseron. Then he found that this same, the duple of the above pair, formed the

> sesquialter ratio of still another, and that it joined with it in the consonance of the diapente. These two, to which the first double proved to be sesquitertian and sesquialter, were discovered in turn to hold the sesquioctave ratio between themselves.[4]

That summary, Boethius concedes, might also be more simply put. It suffices to imagine the following account: "So that what has been said might be clearer, let the weights of the four hammers be contained in the following numbers: twelve; nine; eight; six."[5]

Pythagoras immediately understood that the discovery was significant. Quickly, he returned home and, by new means, repeated the experiment. "First," Boethius narrates, "he attached corresponding weights to strings and discerned by ear their consonances; then, he applied the double and mean and fitted other ratios to lengths of pipes." Next, he "poured ladles of corresponding weights into glasses, and he struck these glasses — set in order according to various weights — with a rod of copper or iron." Finally, "he turned to strings, measuring their length and thickness, that he might test further."

He reached a set of findings that could be demonstrated most easily by means of a simple device: the monochord. This instrument consists of a single string, stretched over a sound box, fastened at both ends, and whose length is divided by a bridge that can be moved at will. When plucked or played, the cord will emit a single tone. It is obvious that as the length of the string is gradually diminished, its pitch will grow increasingly acute. But with the ring of the smithy still in his ears, Pythagoras had come to grasp far more. Regularities could be observed between adjustments in length and changes in sound; correlations could therefore be established between geometrical and sonorous phenomena. Three equivalences, in particular, were immediately remarkable. An open string will produce a tone. When it is halved, it will produce another, exactly one octave above the first. Then the cord will have emitted the interval known to the ancients as "diapason." When, instead, the string is divided into three sections, two of which are played, a new interval will be audible: the fifth, known to the Greeks and Romans as "diapente." When, finally, the string is divided into four sections, three of which are played, the cord will produce a tone higher than the open string by the distance of a fourth; thus the instrument will have sounded the "diatesseron."

Suddenly, the seemingly endless diversity of sounds acquired a new simplicity. Acoustical intervals were now expressible as arithmetical relations. The proof lay in the reduction of the sound of the octave to the relation of two to one (2:1); that of the fifth, to the relation of three to two (3:2); that of the fourth, to the relation of four to three (4:3). In short, the natural world could be transcribed—not, to be sure, by the letters of alphabets, which differ among themselves according to the variety of human idioms, but by those various collections of unity that the ancients understood to be "numbers." The consequences of this fact for the understanding of the physical world were great. In sensible things, one could find the intelligible; in the changing, the immutable. Through his analysis of sound, Pythagoras had reached the basis of his metaphysics. This was a doctrine Aristotle recorded in a series of lapidary propositions: "things are the same as numbers" (ἀριθμοὺς εἶναι αὐτα τὰ πράγματα);[6] "things are like numbers" (μιμήσει τὰ ὄντα φασὶν εἶναι τῶν ἀριθμῶν);[7] "things and numbers are composed of the same elements" (τὰ τῶν ἀριθμῶν στοιχεῖα τῶν ὄντων στοιχεῖα πάντων ὑπέλαβον εἶναι).[8] Such statements suggest differing, perhaps conflicting, positions.[9] But despite their variety, they attribute a single program to Pythagoras and his successors: to find, in the idea of number, a key to the understanding of the natural world.

That understanding could advance far, yet ultimately it was to falter. One may take the findings in the forge as an illustration of the project and its limits. Pythagoras developed an arithmetical doctrine founded on a series of four terms that corresponded to the four respective weights of the hammers in consonance: twelve; nine; eight; and six. Restricting himself to these integers, Pythagoras could thus express the numeric relations that, on a monochord, produce the octave (12:6), the fifth (9:6, or 12:8), and the fourth (8:6, or 12:9). But those relations also reflected simpler proportions. "Twelve to six" might be rewritten as "two to one"; "nine to six, or twelve to eight" could be rewritten as "three to two"; "eight to six, or twelve to nine" might be rewritten as "four to three." In short, all three fundamental acoustic intervals were expressible by the relations of the first four natural numbers; these terms alone sufficed for the analysis of the concord in the forge. Later, Pythagoras's followers would take a further step. The first four numbers became, for them, the units of a cosmological "fourfold" (τετρακτύς).[10] Their arithmetical summation would produce the unity of ten, furnishing the basis of all further calculation. Their geometrical arrangement would compose the "pebble figure" of the perfect triangle—that is, an equilateral triangle each of whose sides contains four points, followed by three, two, and one. Thinkers in the tradition would attribute diverse and far-reaching values to each of these arithmetical elements. Speusippus, for example, taught that the point is one; the line two; the triangle three; the pyramid four.[11] A source indicates that, among Pythagoreans, the principle was the object of a sacred oath: "No, I swear by he who transmitted to our soul the fourfold, / Which contains the source and the root of eternal nature."[12]

Yet the truth is that the set of four was lacking from the beginning. In the forge, the transcription was obstinately incomplete. One remained uncounted. Describing the instruments that produced the concord of sound, Boethius observes: "There happened to be five hammers." The relative weights of all but one could be perfectly notated with the aid of the first four numbers. But there was also a fifth. Boethius devotes no more than one sentence to the fate of this most unmusical of instruments: "The fifth hammer," he writes in passing, "which was discordant with all the others, was discarded" (*Quintus vero est reiectus, qui cunctis erat inconsonans*).[13] Sudden, certain, and apparently irrevocable, that "discarding" merits some reflection. What was the fifth hammer, such that Pythagoras chose so decidedly to reject it? Boethius suggests only the most minimal of answers, and it is not easily understood. He writes that the fifth hammer was "discordant with all the others." But that statement implies a question: What is this "all," if something—if even only one thing—sounds in utter dissonance with it?

The presence of the fifth hammer seems to belie the totality of the set of four. It could do so, however, in at least two ways, which suggest differing and indeed contradictory interpretations of the last instrument of percussion. Boethius's public may have concluded that the presence of the fifth hammer betrayed a fault to be ascribed not to Pythagoras, but to our own lowly world. Such an explanation might well be reconciled with the canons of ancient knowledge. It might be recalled that the classical thinkers, as a rule, sought to grasp principles of nature that would by definition be eternal, immutable, and necessary, but they also taught that particular things are by essence corruptible, changing, and hence uncertain. The early readers of Boethius's work may have understood the fifth hammer in such terms: for them, its discordant sound may have testified to the limits of the sublunary sphere, where natural science, even in its most developed forms, could predict no physical event with certainty. Only beyond the moon, in the noble regions that enclose this corruptible world, could mathematical principles and deductions find an exact realm of application. But today, of course, there is a more obvious solution to the problem of the noisy part. One may choose to point an accusing finger at the primitive, if ingenious theorist, inferring simply that something in the Pythagorean calculus was amiss. An error—whether of observation or outlook, measurement or method—could also have kept Pythagoras from finding a place for the tool in his system of proportions. One might well reason that had his analysis been correct, it would have admitted no remainder, for a scientific study surely tolerates no exceptions. Yet each solution, however imaginable, conceals an obscurity. What was the world of ancient knowledge, if it allowed—and perhaps demanded—a sound discordant with "all the others"? And what is the universe of modern science, if it, by contrast, cannot permit the noise of a single inconsonant part?

One can only speculate as to the reasons for the fifth instrument's discord. But this much can hardly be contested: although Pythagoras wished not to include the last hammer in his equivalences of noise and number, he nonetheless perceived it. Boethius leaves little doubt: immobile before the forge, "as if spellbound," the sage "overheard the beating of hammers somehow emit a single consonance from differing sounds." Thus the fifth tool beat, no less than one of five. Perhaps, in his momentary distraction, Pythagoras found himself drawn to that very instrument: the hammer with no number and no master, which somehow—yet impossibly—sounded both "in a single consonance" and in utter discordance "with all." One wonders whether the "kind of divine will" that caused the thinker to abandon his sheltered contemplations may not have had a part to play in this mysterious quintet. The spirit that deterred Pythagoras from his reasoned inquiry may have also been the one that remitted him to the sensible organs that he never meant to trust. Doubtless, the dispossession was transitory. But its consequences were to be lasting. Pythagoras might well return to his research; he might well reject all instruments. Dimly or distinctly, if only for a moment, he had nonetheless perceived a being without measure. It is difficult to imagine that he thought nothing of it. Yet it is certain that his followers, Boethius not least, later took pains to record it. Transcribed as an event of sound to which no certain quantity could be assigned, that resonance was to lure others into the forge discovered by Pythagoras. There, in fidelity and infidelity to their master, they would learn to perceive the harmonies of a music that no numbers may transcribe.

Index