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Introduction

Respecting the Text

IN *Wittgenstein* (1976 and 1987) I located what I took to be the key move in Wittgenstein's reflections on the possibility of a private language in *PI* 198. There Wittgenstein presents the following problem concerning rule-following:

PI 198. "But how can a rule shew me what I have to do at this point? Whatever I do is, on some interpretation, in accord with the rule."—That is not what we ought to say, but rather: any interpretation still hangs in the air along with what it interprets, and cannot give it any support. Interpretations by themselves do not determine meaning.

Three sections later, he refers to this problem as a paradox:

PI 201. This was our paradox: no course of action could be determined by a rule, because every course of action can be made out to accord with the rule. The answer was: if everything can be made out to accord with the rule, then it can also be made out to conflict with it.

And so there would be neither accord nor conflict here.

On the basis of this paradox, Wittgenstein then presents the following strong anti-privacy claim concerning rule-following:

PI 202. And hence also ‘obeying a rule’ is a practice. And to think one is obeying a rule is not to obey a rule. Hence it is not possible to obey a rule ‘privately’: otherwise thinking one was obeying a rule would be the same thing as obeying it.

From earlier sections of *Philosophical Investigations* we know that Wittgenstein holds:

(1) Language is a rule-governed activity.

So when, in *PI* 202, he explicitly tells us that

(2) It is not possible to obey a rule ‘privately,’

the thesis that a private language is impossible seems to follow at once.

This is not an implausible reading of the text, for it squares with much else that Wittgenstein says on the possibility of a private language. All the same, I now find this reading out of focus. It runs counter to Wittgenstein’s claim—his insistence—that he is not engaged in presenting and defending philosophical theses. I have in mind passages of the following kind:

PI 109. It was true to say that our considerations could not be scientific ones. It was not of any possible interest to us to find out empirically “that, contrary to our preconceived ideas, it is possible to think such-and-such”—whatever that may mean. . . . And we may not

advance any kind of theory. There must not be anything hypothetical in our considerations. We must do away with all *explanation*, and description alone must take its place. And this description gets its light, that is to say its purpose, from the philosophical problems. These are, of course, not empirical problems; they are solved, rather, by looking into the workings of our language, and that in such a way as to make us recognize those workings: *in despite of* an urge to misunderstand them. The problems are solved, not by giving new information, but by arranging what we have always known. Philosophy is a battle against the bewitchment of our intelligence by means of language.

PI 124. Philosophy may in no way interfere with the actual use of language; it can in the end only describe it.

For it cannot give it any foundation either.

It leaves everything as it is.

PI 126. Philosophy simply puts everything before us, and neither explains nor deduces anything.—Since everything lies open to view there is nothing to explain. For what is hidden, for example, is of no interest to us.

One might also give the name “philosophy” to what is possible *before* all new discoveries and inventions.

Wittgenstein’s methodological passages have, of course, been noticed and remarked on by most commentators. But they are not always accepted as fundamental guides to understanding Wittgenstein’s reflections on specific topics. After a brief (perhaps solemn) acknowledgment, they often drift out of sight. Crispin Wright, however, with admirable forthrightness, explicitly pushes these methodological pronouncements aside in these words:

[I]t is difficult to reconcile Wittgenstein's pronouncements about the kind of thing which he thinks he ought to be doing with what he actually seems to do. Not that his actual treatment of the particular issues seems flatly inconsistent with his general methodological ideas. Rather, we can put the would-be interpreter's difficulty like this: it is doubtful how anyone who read only a bowdlerized edition of the *Investigations*, from which all reference to philosophical method and the nature and place of philosophy had been removed, would be able to arrive at the conclusion that the author viewed those matters in just the way in which Wittgenstein professes to do. At the time I write this, the complaint is justified that the great volume of commentary on the *Investigations* has so far done very little to clarify either how we should interpret the general remarks on philosophy so as to have our understanding enhanced of Wittgenstein's treatment of specific questions, or conversely. (What are the 'well-known facts' arranged in the course of the Private Language discussion?) Wittgenstein's later views on philosophy constitute one of the so far least well understood aspects of his thought. (Wright 1980, p. 262)

Michael Dummett also rejects some of Wittgenstein's central constraints—in particular, his descriptivism—but does not suggest, as Wright seems to, that Wittgenstein's methodological restrictions are mere hand-waving.

We all stand, or should stand, in the shadow of Wittgenstein, in the same way that much earlier generations once stood in the shadow of Kant. Some things in his philosophy, however, I cannot see any reason for accepting: and one is the belief that philosophy, as such, must never criticize but only describe. This belief

was fundamental in the sense that it determined the whole manner in which, in his later writings, he discussed philosophical problems; not sharing it, I could not respect his work as I do if I regarded his arguments and insights as depending on the truth of that belief. (Dummett 1991, p. xi)

Perhaps an interesting philosophical position with a Wittgensteinian cast could emerge with Wittgenstein's methodological restrictions relaxed. That, however, is not my present concern.

As indicated in the preface, I will not get deeply involved with the secondary literature on Wittgenstein's philosophy. There are, however, some exceptions. For my purposes it is important to distinguish my reading of Wittgenstein's so-called skeptical paradox of rule-following found in *PI* 201 from one put forward some years later by Saul Kripke. The matter at issue is not one of priority. I do not claim to have published a Kripke-like interpretation of *PI* 201 four years before he did. My claim is that Kripke's understanding of the paradox in *PI* 201 is fundamentally wrong and I want to take precautions against having criticisms that have been leveled against his understanding of the paradox also leveled against mine. I will go into all this at the beginning of the next chapter. In chapter 2, I adopt a treatment of the notion of a *criterion* that is significantly different from one forcefully presented by P.M.S. Hacker in *Meaning and Mind* (1990). There are other references to the secondary literature—mostly acknowledgments. In general, however, I stay within the margins of Wittgenstein's text.

Returning to *PI* 109, it can be taken as a representative of the kind of passage that Crispin (the Expunger) Wright would delete. In some ways it may seem perplex-

ing. It begins by declaring that “it was true to say that our considerations could not be scientific ones.” This is a view that Wittgenstein expressed in the *Tractatus*, and perhaps he is alluding to it here. (See, for example, *TLP* 4.111.) He then makes the more specific point that he is not engaged in an empirical investigation intended to establish new facts—facts that may be contrary to “our preconceived ideas.” A few sentences later, he tells us that the philosophical problems that concern him “are, of course, not empirical problems; they are solved, rather, by looking into the workings of our language, and that in such a way as to make us recognize those workings.” This may seem strange, for isn’t “looking into the workings of our language” itself an empirical investigation? If so, in the same paragraph, Wittgenstein seems both to renounce and to recommend an empirical investigation of language. What is going on? The brief answer is this. Wittgenstein is not using the notion of an empirical investigation in a wide sense where the perceptual examination of any object—say, a spot on one’s tie—counts as an empirical investigation. He is making the narrower claim that his examination of the workings of language is not a scientific investigation intended to turn up *new*, perhaps exciting, facts about language; he claims to be dealing only with commonplace facts—matters open to anyone.

Why, then, is Wittgenstein concerned with the workings of language *at all*? How does the activity of describing the commonplace workings of language gain significance? His explicit answer is that such descriptions get their “light, that is to say [their] purpose, from . . . philosophical problems” (*PI* 109). I think this claim is of *first importance* for understanding Wittgenstein’s concern with language.

This remark distances Wittgenstein from an approach that has played a dominant role in philosophy for more than a century. Many philosophers have been attracted to the idea that producing a *theory* of language—or, more specifically, developing a theory of *meaning*—is the first task for philosophy. With such a theory in hand, one can then turn to the problems of philosophy, possessing the tools needed for their proper solution or, perhaps, their dissolution. On this approach, theory of meaning comes first; the treatment of other philosophical problems comes later. The logical positivists' attempt to formulate an empiricist criterion of cognitive meaning is one example of this approach. There have been many others. In contrast, I want to suggest that Wittgenstein's concern with philosophy is antecedent to and controls his reflections on language. In the absence of these antecedent philosophical perplexities, I do not think that Wittgenstein would have any philosophical interest (as opposed, say, to a literary interest or philological interest) in language at all. If this is correct, then it is at least misleading to refer to Wittgenstein as a philosopher of language.

Here a comparison between Wittgenstein's later philosophy and J. L. Austin's so-called ordinary language philosophy may be illuminating. Looking at their writings, one is immediately struck, not only by their differences in style, but also by the profound differences in temperament that these stylistic differences reflect. All the same, many of their most basic commitments are similar. Contrary to a long tradition of attributing our philosophical problems to the inaccuracy, vagueness, and crudeness of our everyday language, both Wittgenstein and Austin treat our ordinary or common use of language with respect. Austin and Wittgenstein agree in thinking that it is the philosopher's misunderstanding and misuse

(even abuse) of ordinary language—not ordinary language itself—that generates philosophical confusion.

Austin also shares Wittgenstein's appreciation of the rich diversity (the motley) of the uses of language. Both target the implicit tendency of philosophers to take descriptive language as paradigmatic for understanding the nature of language. In his classic paper "Other Minds," Austin speaks of those who commit the *descriptive fallacy*, as he labels it (Austin 1979, p. 103), which parallels Wittgenstein's attack on what he calls "a particular picture of human language," namely "individual words in language name objects—sentences are combinations of such names" (PI 1).

There are, however, important differences between Wittgenstein's and Austin's approaches. Austin thought of himself as a participant in a project involving the "joint labors of philosophers, grammarians, and numerous other students of language" that would yield, sometime in the twenty-first century, "a true and comprehensive science of language" (Austin 1979, p. 232). Wittgenstein would, I think, have little interest in the development of such "a true and comprehensive science of language," even if, contrary to what he believed, it could be successfully pulled off. Such a theory would be an empirical theory, and, as Wittgenstein explicitly states, the philosophical questions that concern him cannot be resolved by an appeal to empirical theories. I will assume that, when he says this, he means it.

The difference between Wittgenstein's and Austin's treatments of philosophical problems comes out in another way. Austin saw our ordinary language as a rich system of finely tuned fields of subtle contrasts. Problems arise, he thought, when philosophers come clomping in with heavy-footed categories that distort these fields,

yielding nonsense in the guise of sense. Philosophical problems are solved or dissolved by a meticulous “teasing out” of fine distinctions. Wittgenstein, in contrast, has little interest in drawing fine distinctions of the kind for which Austin was famous.

PI 106. Here it is difficult as it were to keep our heads up,—to see that we must stick to the subjects of our every-day thinking, and not go astray and imagine that we have to describe extreme subtleties, which in turn we are after all quite unable to describe with the means at our disposal. We feel as if we had to repair a torn spider’s web with our fingers.

The task of the philosopher, he tells us, is not to talk “about shades of meaning [where the only thing] in question [is] to find words to hit on the correct nuance” (*PI* 254).¹

¹Wittgenstein adds an important qualification to this claim that the philosopher need not engage in subtleties.

PI 254. [Nuances are] in question in philosophy only where we have to give a psychologically exact account of the temptation to use a particular kind of expression. What we ‘are tempted to say’ in such a case is, of course, not philosophy; but it is its raw material. Thus, for example, what a mathematician is inclined to say about the objectivity and reality of mathematical facts, is not a philosophy of mathematics, but something for philosophical treatment.

Sophistication and subtlety are not needed for recognizing plain facts of linguistic usage. Subtlety and a sense of nuance are, however, needed to unravel the tangled webs philosophers weave in creating complex cognitive illusions. To use a crude analogy, it can be easy to identify a paranoid’s basic false belief: He thinks he is being followed by CIA agents. Understanding the mental structures that robustly support this belief may be a subtle, complex problem indeed.

Rather than dwelling on fine distinctions—for example, noting the difference between doing something accidentally and doing it unwittingly—Wittgenstein is usually concerned with broader categories of language: the status of mathematical expressions, of first-person ascriptions of feeling or propositional attitudes, of proper names, general terms, and so on. I do not think Wittgenstein holds that serious philosophical problems arise from confounding terms that lie within a system of contrasting terms. The errors that concern Wittgenstein are not fine-grained errors; they are errors that arise from viewing a whole domain of discourse—a whole genre—in an improper or misleading way, for example, viewing first-person ascriptions of pain on the model of ascriptions of colors to objects, thinking that the only difference is one of subject matter. A parallel mistake occurs with respect to mathematical propositions.

Wittgenstein uses various strategies in dealing with such broad-scale confusions. One is to introduce simple language games of the kind found at the beginning of *Philosophical Investigations*, where generic differences in the uses of language are made transparent.

PI 5. It disperses the fog to study the phenomena of language in primitive kinds of application in which one can command a clear view of the aim and functioning of the words.

He also attempts to exhibit generic differences in uses of language by calling attention to the different ways in which people are trained to employ them. For example, learning how to recite an ordered series of sounds (numerals-to-be) is a standard starting point for learning arithmetic, but nothing similar is used to teach a child

how to ascribe colors to objects. If assertions are under discussion, then various kinds of assertion can be distinguished by examining their modes of verification. For Wittgenstein, all the things he is pointing to are commonplace—things no one doubts, but that we often ignore or consider trite—when engaged in the peculiarly detached business of doing philosophy.

PI 127. The work of the philosopher consists in assembling reminders for a particular purpose.

Another aspect of Wittgenstein's approach, one that seems particularly obnoxious to many philosophers, is what I will call his *defactoism*. I will explain this notion in the next chapter, but the sense of what I have in mind is captured in passages of the following kind:

PI 217. If I have exhausted the justifications I have reached bedrock, and my spade is turned. Then I am inclined to say: "This is simply what I do."

OC 344. My *life* consists in my being content to accept many things.

Passages of this kind are sometimes taken as signs of Wittgenstein's conservatism, pessimism, faintheartedness, or unwillingness to confront serious philosophical issues. Criticisms of this kind treat Wittgenstein's defactoism as an expression of defeat, whereas, instead, defactoism presents a fundamental challenge to the legitimacy of the philosophical enterprise as it is commonly pursued. Defactoism is a central feature of Wittgenstein's later philosophy. Treating it as a basic and unifying theme of his later philosophy is central to this textual study. Chapter 1 is dedicated to this topic.

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