

Contents

| | |
|---|-------------|
| List of Figures | xiii |
| List of Tables | xv |
| Preface | xvii |
| 1 Introduction | 3 |
| 1.1 Organization of the Book | 4 |
| 1.2 Useful Background | 6 |
| 1.2.1 Mathematics Background | 6 |
| 1.2.2 Probability and Statistics Background | 6 |
| 1.2.3 Finance Theory Background | 7 |
| 1.3 Notation | 8 |
| 1.4 Prices, Returns, and Compounding | 9 |
| 1.4.1 Definitions and Conventions | 9 |
| 1.4.2 The Marginal, Conditional, and Joint Distribution of Returns | 13 |
| 1.5 Market Efficiency | 20 |
| 1.5.1 Efficient Markets and the Law of Iterated Expectations | 22 |
| 1.5.2 Is Market Efficiency Testable? | 24 |
| 2 The Predictability of Asset Returns | 27 |
| 2.1 The Random Walk Hypotheses | 28 |
| 2.1.1 The Random Walk 1: IID Increments | 31 |
| 2.1.2 The Random Walk 2: Independent Increments | 32 |
| 2.1.3 The Random Walk 3: Uncorrelated Increments | 33 |
| 2.2 Tests of Random Walk 1: IID Increments | 33 |
| 2.2.1 Traditional Statistical Tests | 33 |
| 2.2.2 Sequences and Reversals, and Runs | 34 |

| | | |
|----------|---|------------|
| 2.3 | Tests of Random Walk 2: Independent Increments | 41 |
| 2.3.1 | Filter Rules | 42 |
| 2.3.2 | Technical Analysis | 43 |
| 2.4 | Tests of Random Walk 3: Uncorrelated Increments | 44 |
| 2.4.1 | Autocorrelation Coefficients | 44 |
| 2.4.2 | Portmanteau Statistics | 47 |
| 2.4.3 | Variance Ratios | 48 |
| 2.5 | Long-Horizon Returns | 55 |
| 2.5.1 | Problems with Long-Horizon Inferences | 57 |
| 2.6 | Tests For Long-Range Dependence | 59 |
| 2.6.1 | Examples of Long-Range Dependence | 59 |
| 2.6.2 | The Hurst-Mandelbrot Rescaled Range Statistic . . | 62 |
| 2.7 | Unit Root Tests | 64 |
| 2.8 | Recent Empirical Evidence | 65 |
| 2.8.1 | Autocorrelations | 66 |
| 2.8.2 | Variance Ratios | 68 |
| 2.8.3 | Cross-Autocorrelations and Lead-Lag Relations . . | 74 |
| 2.8.4 | Tests Using Long-Horizon Returns | 78 |
| 2.9 | Conclusion | 80 |
| 3 | Market Microstructure | 83 |
| 3.1 | Nonsynchronous Trading | 84 |
| 3.1.1 | A Model of Nonsynchronous Trading | 85 |
| 3.1.2 | Extensions and Generalizations | 98 |
| 3.2 | The Bid-Ask Spread | 99 |
| 3.2.1 | Bid-Ask Bounce | 101 |
| 3.2.2 | Components of the Bid-Ask Spread | 103 |
| 3.3 | Modeling Transactions Data | 107 |
| 3.3.1 | Motivation | 108 |
| 3.3.2 | Rounding and Barrier Models | 114 |
| 3.3.3 | The Ordered Probit Model | 122 |
| 3.4 | Recent Empirical Findings | 128 |
| 3.4.1 | Nonsynchronous Trading | 128 |
| 3.4.2 | Estimating the Effective Bid-Ask Spread | 134 |
| 3.4.3 | Transactions Data | 136 |
| 3.5 | Conclusion | 144 |
| 4 | Event-Study Analysis | 149 |
| 4.1 | Outline of an Event Study | 150 |
| 4.2 | An Example of an Event Study | 152 |
| 4.3 | Models for Measuring Normal Performance | 153 |
| 4.3.1 | Constant-Mean-Return Model | 154 |
| 4.3.2 | Market Model | 155 |

| | | |
|----------|---|------------|
| 4.3.3 | Other Statistical Models | 155 |
| 4.3.4 | Economic Models | 156 |
| 4.4 | Measuring and Analyzing Abnormal Returns | 157 |
| 4.4.1 | Estimation of the Market Model | 158 |
| 4.4.2 | Statistical Properties of Abnormal Returns | 159 |
| 4.4.3 | Aggregation of Abnormal Returns | 160 |
| 4.4.4 | Sensitivity to Normal Return Model | 162 |
| 4.4.5 | CARs for the Earnings-Announcement Example | 163 |
| 4.4.6 | Inferences with Clustering | 166 |
| 4.5 | Modifying the Null Hypothesis | 167 |
| 4.6 | Analysis of Power | 168 |
| 4.7 | Nonparametric Tests | 172 |
| 4.8 | Cross-Sectional Models | 173 |
| 4.9 | Further Issues | 175 |
| 4.9.1 | Role of the Sampling Interval | 175 |
| 4.9.2 | Inferences with Event-Date Uncertainty | 176 |
| 4.9.3 | Possible Biases | 177 |
| 4.10 | Conclusion | 178 |
| 5 | The Capital Asset Pricing Model | 181 |
| 5.1 | Review of the CAPM | 181 |
| 5.2 | Results from Efficient-Set Mathematics | 184 |
| 5.3 | Statistical Framework for Estimation and Testing | 188 |
| 5.3.1 | Sharpe-Lintner Version | 189 |
| 5.3.2 | Black Version | 196 |
| 5.4 | Size of Tests | 203 |
| 5.5 | Power of Tests | 204 |
| 5.6 | Nonnormal and Non-IID Returns | 208 |
| 5.7 | Implementation of Tests | 211 |
| 5.7.1 | Summary of Empirical Evidence | 211 |
| 5.7.2 | Illustrative Implementation | 212 |
| 5.7.3 | Unobservability of the Market Portfolio | 213 |
| 5.8 | Cross-Sectional Regressions | 215 |
| 5.9 | Conclusion | 217 |
| 6 | Multifactor Pricing Models | 219 |
| 6.1 | Theoretical Background | 219 |
| 6.2 | Estimation and Testing | 222 |
| 6.2.1 | Portfolios as Factors with a Riskfree Asset | 223 |
| 6.2.2 | Portfolios as Factors without a Riskfree Asset | 224 |
| 6.2.3 | Macroeconomic Variables as Factors | 226 |
| 6.2.4 | Factor Portfolios Spanning the Mean-Variance Frontier | 228 |

| | | |
|----------|---|------------|
| 6.3 | Estimation of Risk Premia and Expected Returns | 231 |
| 6.4 | Selection of Factors | 233 |
| 6.4.1 | Statistical Approaches | 233 |
| 6.4.2 | Number of Factors | 238 |
| 6.4.3 | Theoretical Approaches | 239 |
| 6.5 | Empirical Results | 240 |
| 6.6 | Interpreting Deviations from Exact Factor Pricing | 242 |
| 6.6.1 | Exact Factor Pricing Models, Mean-Variance Analysis, and the Optimal Orthogonal Portfolio | 243 |
| 6.6.2 | Squared Sharpe Ratios | 245 |
| 6.6.3 | Implications for Separating Alternative Theories | 246 |
| 6.7 | Conclusion | 251 |
| 7 | Present-Value Relations | 253 |
| 7.1 | The Relation between Prices, Dividends, and Returns | 254 |
| 7.1.1 | The Linear Present-Value Relation with Constant Expected Returns | 255 |
| 7.1.2 | Rational Bubbles | 258 |
| 7.1.3 | An Approximate Present-Value Relation with Time-Varying Expected Returns | 260 |
| 7.1.4 | Prices and Returns in a Simple Example | 264 |
| 7.2 | Present-Value Relations and US Stock Price Behavior | 267 |
| 7.2.1 | Long-Horizon Regressions | 267 |
| 7.2.2 | Volatility Tests | 275 |
| 7.2.3 | Vector Autoregressive Methods | 279 |
| 7.3 | Conclusion | 286 |
| 8 | Intertemporal Equilibrium Models | 291 |
| 8.1 | The Stochastic Discount Factor | 293 |
| 8.1.1 | Volatility Bounds | 296 |
| 8.2 | Consumption-Based Asset Pricing with Power Utility | 304 |
| 8.2.1 | Power Utility in a Lognormal Model | 306 |
| 8.2.2 | Power Utility and Generalized Method of Moments | 314 |
| 8.3 | Market Frictions | 314 |
| 8.3.1 | Market Frictions and Hansen-Jagannathan Bounds | 315 |
| 8.3.2 | Market Frictions and Aggregate Consumption Data | 316 |
| 8.4 | More General Utility Functions | 326 |
| 8.4.1 | Habit Formation | 326 |
| 8.4.2 | Psychological Models of Preferences | 332 |
| 8.5 | Conclusion | 334 |

| | | |
|-----------|---|------------|
| 9 | Derivative Pricing Models | 339 |
| 9.1 | Brownian Motion | 341 |
| 9.1.1 | Constructing Brownian Motion | 341 |
| 9.1.2 | Stochastic Differential Equations | 346 |
| 9.2 | A Brief Review of Derivative Pricing Methods | 349 |
| 9.2.1 | The Black-Scholes and Merton Approach | 350 |
| 9.2.2 | The Martingale Approach | 354 |
| 9.3 | Implementing Parametric Option Pricing Models | 355 |
| 9.3.1 | Parameter Estimation of Asset Price Dynamics | 356 |
| 9.3.2 | Estimating σ in the Black-Scholes Model | 361 |
| 9.3.3 | Quantifying the Precision of Option Price Estimators | 367 |
| 9.3.4 | The Effects of Asset Return Predictability | 369 |
| 9.3.5 | Implied Volatility Estimators | 377 |
| 9.3.6 | Stochastic Volatility Models | 379 |
| 9.4 | Pricing Path-Dependent Derivatives Via Monte Carlo Simulation | 382 |
| 9.4.1 | Discrete Versus Continuous Time | 383 |
| 9.4.2 | How Many Simulations to Perform | 384 |
| 9.4.3 | Comparisons with a Closed-Form Solution | 384 |
| 9.4.4 | Computational Efficiency | 386 |
| 9.4.5 | Extensions and Limitations | 390 |
| 9.5 | Conclusion | 391 |
| 10 | Fixed-Income Securities | 395 |
| 10.1 | Basic Concepts | 396 |
| 10.1.1 | Discount Bonds | 397 |
| 10.1.2 | Coupon Bonds | 401 |
| 10.1.3 | Estimating the Zero-Coupon Term Structure | 409 |
| 10.2 | Interpreting the Term Structure of Interest Rates | 413 |
| 10.2.1 | The Expectations Hypothesis | 413 |
| 10.2.2 | Yield Spreads and Interest Rate Forecasts | 418 |
| 10.3 | Conclusion | 423 |
| 11 | Term-Structure Models | 427 |
| 11.1 | Affine-Yield Models | 428 |
| 11.1.1 | A Homoskedastic Single-Factor Model | 429 |
| 11.1.2 | A Square-Root Single-Factor Model | 435 |
| 11.1.3 | A Two-Factor Model | 438 |
| 11.1.4 | Beyond Affine-Yield Models | 441 |
| 11.2 | Fitting Term-Structure Models to the Data | 442 |
| 11.2.1 | Real Bonds, Nominal Bonds, and Inflation | 442 |
| 11.2.2 | Empirical Evidence on Affine-Yield Models | 445 |

| | | |
|-----------|---|------------|
| 11.3 | Pricing Fixed-Income Derivative Securities | 455 |
| 11.3.1 | Fitting the Current Term Structure Exactly | 456 |
| 11.3.2 | Forwards and Futures | 458 |
| 11.3.3 | Option Pricing in a Term-Structure Model | 461 |
| 11.4 | Conclusion | 464 |
| 12 | Nonlinearities in Financial Data | 467 |
| 12.1 | Nonlinear Structure in Univariate Time Series | 468 |
| 12.1.1 | Some Parametric Models | 470 |
| 12.1.2 | Univariate Tests for Nonlinear Structure | 475 |
| 12.2 | Models of Changing Volatility | 479 |
| 12.2.1 | Univariate Models | 481 |
| 12.2.2 | Multivariate Models | 490 |
| 12.2.3 | Links between First and Second Moments | 494 |
| 12.3 | Nonparametric Estimation | 498 |
| 12.3.1 | Kernel Regression | 500 |
| 12.3.2 | Optimal Bandwidth Selection | 502 |
| 12.3.3 | Average Derivative Estimators | 504 |
| 12.3.4 | Application: Estimating State-Price Densities | 507 |
| 12.4 | Artificial Neural Networks | 512 |
| 12.4.1 | Multilayer Perceptrons | 512 |
| 12.4.2 | Radial Basis Functions | 516 |
| 12.4.3 | Projection Pursuit Regression | 518 |
| 12.4.4 | Limitations of Learning Networks | 518 |
| 12.4.5 | Application: Learning the Black-Scholes Formula | 519 |
| 12.5 | Overfitting and Data-Snooping | 523 |
| 12.6 | Conclusion | 524 |
| | Appendix | 527 |
| A.1 | Linear Instrumental Variables | 527 |
| A.2 | Generalized Method of Moments | 532 |
| A.3 | Serially Correlated and Heteroskedastic Errors | 534 |
| A.4 | GMM and Maximum Likelihood | 536 |
| | References | 541 |
| | Author Index | 587 |
| | Subject Index | 597 |

1

Introduction

FINANCIAL ECONOMICS is a highly empirical discipline, perhaps the most empirical among the branches of economics and even among the social sciences in general. This should come as no surprise, for financial markets are not mere figments of theoretical abstraction; they thrive in practice and play a crucial role in the stability and growth of the global economy. Therefore, although some aspects of the academic finance literature may seem abstract at first, there is a practical relevance demanded of financial models that is often waived for the models of other comparable disciplines.¹

Despite the empirical nature of financial economics, like the other social sciences it is almost entirely nonexperimental. Therefore, the primary method of inference for the financial economist is model-based statistical inference—financial econometrics. While econometrics is also essential in other branches of economics, what distinguishes financial economics is the central role that uncertainty plays in both financial theory and its empirical implementation. The starting point for every financial model is the uncertainty facing investors, and the substance of every financial model involves the impact of uncertainty on the behavior of investors and, ultimately, on market prices. Indeed, in the absence of uncertainty, the problems of financial economics reduce to exercises in basic microeconomics. The very existence of financial economics as a discipline is predicated on uncertainty.

This has important consequences for financial econometrics. The random fluctuations that require the use of statistical theory to estimate and test financial models are intimately related to the uncertainty on which those models are based. For example, the martingale model for asset prices has very specific implications for the behavior of test statistics such as the autocorrelation coefficient of price increments (see Chapter 2). This close connection between theory and empirical analysis is unparalleled in the

¹Bernstein (1992) provides a highly readable account of the interplay between theory and practice in the development of modern financial economics.

social sciences, although it has been the hallmark of the natural sciences for quite some time. It is one of the most rewarding aspects of financial econometrics, so much so that we felt impelled to write this graduate-level textbook as a means of introducing others to this exciting field.

Section 1.1 explains which topics we cover in this book, and how we have organized the material. We also suggest some ways in which the book might be used in a one-semester course on financial econometrics or empirical finance.

In Section 1.2, we describe the kinds of background material that are most useful for financial econometrics and suggest references for those readers who wish to review or learn such material along the way. In our experience, students are often more highly motivated to pick up the necessary background *after* they see how it is to be applied, so we encourage readers with a serious interest in financial econometrics but with somewhat less preparation to take a crack at this material anyway.

In a book of this magnitude, notation becomes a nontrivial challenge of coordination; hence Section 1.3 describes what method there is in our notational madness. We urge readers to review this carefully to minimize the confusion that can arise when $\hat{\beta}$ is mistaken for β and X is incorrectly assumed to be the same as \mathbf{X} .

Section 1.4 extends our discussion of notation by presenting notational conventions for and definitions of some of the fundamental objects of our study: prices, returns, methods of compounding, and probability distributions. Although much of this material is well-known to finance students and investment professionals, we think a brief review will help many readers.

In Section 1.5, we turn our attention to quite a different subject: the Efficient Markets Hypothesis. Because so much attention has been lavished on this hypothesis, often at the expense of other more substantive issues, we wish to dispense with this issue first. Much of the debate involves theological tenets that are empirically undecidable and, therefore, beyond the purview of this text. But for completeness—no self-respecting finance text could omit market efficiency altogether—Section 1.5 briefly discusses the topic.

1.1 Organization of the Book

In organizing this book, we have followed two general principles. First, the early chapters concentrate exclusively on stock markets. Although many of the methods discussed can be applied equally well to other asset markets, the empirical literature on stock markets is particularly large and by focusing on these markets we are able to keep the discussion concrete. In later chapters, we cover derivative securities (Chapters 9 and 12) and fixed-income securi-

ties (Chapters 10 and 11). The last chapter of the book presents nonlinear methods, with applications to both stocks and derivatives.

Second, we start by presenting statistical models of asset returns, and then discuss more highly structured economic models. In Chapter 2, for example, we discuss methods for predicting stock returns from their own past history, without much attention to institutional detail; in Chapter 3 we show how the microstructure of stock markets affects the short-run behavior of returns. Similarly, in Chapter 4 we discuss simple statistical models of the cross-section of individual stock returns, and the application of these models to event studies; in Chapters 5 and 6 we show how the Capital Asset Pricing Model and multifactor models such as the Arbitrage Pricing Theory restrict the parameters of the statistical models. In Chapter 7 we discuss longer-run evidence on the predictability of stock returns from variables other than past stock returns; in Chapter 8 we explore dynamic equilibrium models which can generate persistent time-variation in expected returns. We use the same principle to divide a basic treatment of fixed-income securities in Chapter 10 from a discussion of equilibrium term-structure models in Chapter 11.

We have tried to make each chapter as self-contained as possible. While some chapters naturally go together (e.g., Chapters 5 and 6, and Chapters 10 and 11), there is certainly no need to read this book straight through from beginning to end. For classroom use, most teachers will find that there is too much material here to be covered in one semester. There are several ways to use the book in a one-semester course. For example one teacher might start by discussing short-run time-series behavior of stock prices using Chapters 2 and 3, then cover cross-sectional models in Chapters 4, 5, and 6, then discuss intertemporal equilibrium models using Chapter 8, and finally cover derivative securities and nonlinear methods as advanced topics using Chapters 9 and 12. Another teacher might first present the evidence on short- and long-run predictability of stock returns using Chapters 2 and 7, then discuss static and intertemporal equilibrium theory using Chapters 5, 6, and 8, and finally cover fixed-income securities using Chapters 10 and 11.

There are some important topics that we have not been able to include in this text. Most obviously, our focus is almost exclusively on US domestic asset markets. We say very little about asset markets in other countries, and we do not try to cover international topics such as exchange-rate behavior or the home-bias puzzle (the tendency for each country's investors to hold a disproportionate share of their own country's assets in their portfolios). We also omit such important econometric subjects as Bayesian analysis and frequency-domain methods of time-series analysis. In many cases our choice of topics has been influenced by the dual objectives of the book: to explain the methods of financial econometrics, and to review the empirical literature in finance. We have tended to concentrate on topics that

involve econometric issues, sometimes at the expense of other equally interesting material—including much recent work in behavioral finance—that is econometrically more straightforward.

1.2 Useful Background

The many rewards of financial econometrics come at a price. A solid background in mathematics, probability and statistics, and finance theory is necessary for the practicing financial econometrician, for precisely the reasons that make financial econometrics such an engaging endeavor. To assist readers in obtaining this background (since only the most focused and directed of students will have it already), we outline in this section the topics in mathematics, probability, statistics, and finance theory that have become indispensable to financial econometrics. We hope that this outline can serve as a self-study guide for the more enterprising readers and that it will be a partial substitute for including background material in this book.

1.2.1 Mathematics Background

The mathematics background most useful for financial econometrics is not unlike the background necessary for econometrics in general: multivariate calculus, linear algebra, and matrix analysis. References for each of these topics are Lang (1973), Strang (1976), and Magnus and Neudecker (1988), respectively. Key concepts include

- multiple integration
- multivariate constrained optimization
- matrix algebra
- basic rules of matrix differentiation.

In addition, option- and other derivative-pricing models, and continuous-time asset pricing models, require some passing familiarity with the *Itô* or *stochastic calculus*. A lucid and thorough treatment is provided by Merton (1990), who pioneered the application of stochastic calculus to financial economics. More mathematically inclined readers may also wish to consult Chung and Williams (1990).

1.2.2 Probability and Statistics Background

Basic probability theory is a prerequisite for any discipline in which uncertainty is involved. Although probability theory has varying degrees of mathematical sophistication, from coin-flipping calculations to measure-theoretic foundations, perhaps the most useful approach is one that emphasizes the

intuition and subtleties of elementary probabilistic reasoning. An amazingly durable classic that takes just this approach is Feller (1968). Brieman (1992) provides similar intuition but at a measure-theoretic level. Key concepts include

- definition of a random variable
- independence
- distribution and density functions
- conditional probability
- modes of convergence
- laws of large numbers
- central limit theorems.

Statistics is, of course, the primary engine which drives the inferences that financial econometricians draw from the data. As with probability theory, statistics can be taught at various levels of mathematical sophistication. Moreover, unlike the narrower (and some would say “purer”) focus of probability theory, statistics has increased its breadth as it has matured, giving birth to many well-defined subdisciplines such as multivariate analysis, nonparametrics, time-series analysis, order statistics, analysis of variance, decision theory, Bayesian statistics, etc. Each of these subdisciplines has been drawn upon by financial econometricians at one time or another, making it rather difficult to provide a single reference for all of these topics. Amazingly, such a reference does exist: Stuart and Ord’s (1987) three-volume *tour de force*. A more compact reference that contains most of the relevant material for our purposes is the elegant monograph by Silvey (1975). For topics in time-series analysis, Hamilton (1994) is an excellent comprehensive text. Key concepts include

- Neyman-Pearson hypothesis testing
- linear regression
- maximum likelihood
- basic time-series analysis (stationarity, autoregressive and ARMA processes, vector autoregressions, unit roots, etc.)
- elementary Bayesian inference.

For continuous-time financial models, an additional dose of stochastic processes is a must, at least at the level of Cox and Miller (1965) and Hoel, Port, and Stone (1972).

1.2.3 *Finance Theory Background*

Since the *raison d’être* of financial econometrics is the empirical implementation and evaluation of financial models, a solid background in finance theory is the most important of all. Several texts provide excellent coverage

of this material: Duffie (1992), Huang and Litzenberger (1988), Ingersoll (1987), and Merton (1990). Key concepts include

- risk aversion and expected-utility theory
- static mean-variance portfolio theory
- the Capital Asset Pricing Model (CAPM) and the Arbitrage Pricing Theory (APT)
- dynamic asset pricing models
- option pricing theory.

1.3 Notation

We have found that it is far from simple to devise a consistent notational scheme for a book of this scope. The difficulty comes from the fact that financial econometrics spans several very different strands of the finance literature, each replete with its own firmly established set of notational conventions. But the conventions in one literature often conflict with the conventions in another. Unavoidably, then, we must sacrifice either internal notational consistency across different chapters of this text or external consistency with the notation used in the professional literature. We have chosen the former as the lesser evil, but we do maintain the following conventions throughout the book:

- We use boldface for vectors and matrices, and regular face for scalars. Where possible, we use bold uppercase for matrices and bold lowercase for vectors. Thus \mathbf{x} is a vector while \mathbf{X} is a matrix.
- Where possible, we use uppercase letters for the levels of variables and lowercase letters for the natural logarithms (logs) of the same variables. Thus if P is an asset price, p is the log asset price.
- Our standard notation for an innovation is the Greek letter ϵ . Where we need to define several different innovations, we use the alternative Greek letters η , ξ , and ζ .
- Where possible, we use Greek letters to denote parameters or parameter vectors.
- We use the Greek letter ι to denote a vector of ones.
- We use hats to denote sample estimates, so if β is a parameter, $\hat{\beta}$ is an estimate of β .
- When we use subscripts, we always use uppercase letters for the upper limits of the subscripts. Where possible, we use the same letters for upper limits as for the subscripts themselves. Thus subscript t runs from 1 to T , subscript k runs from 1 to K , and so on. An exception is that we will let subscript i (usually denoting an asset) run from 1 to N because this notation is so common. We use t and τ for time subscripts;

i for asset subscripts; k , m , and n for lead and lag subscripts; and j as a generic subscript.

- We use the timing convention that a variable is dated t if it is known by the end of period t . Thus R_t denotes a return on an asset held from the end of period $t-1$ to the end of period t .
- In writing variance-covariance matrices, we use Ω for the variance-covariance matrix of asset returns, Σ for the variance-covariance matrix of residuals from a time-series or cross-sectional model, and \mathbf{V} for the variance-covariance matrix of parameter estimators.
- We use script letters sparingly. \mathcal{N} denotes the normal distribution, and \mathcal{L} denotes a log likelihood function.
- We use $\Pr(\cdot)$ to denote the probability of an event.

The professional literature uses many specialized terms. Inevitably we also use these frequently, and we italicize them when they first appear in the book.

1.4 Prices, Returns, and Compounding

Virtually every aspect of financial economics involves *returns*, and there are at least two reasons for focusing our attention on returns rather than on prices. First, for the average investor, financial markets may be considered close to perfectly competitive, so that the size of the investment does not affect price changes. Therefore, since the investment “technology” is constant-returns-to-scale, the return is a complete and scale-free summary of the investment opportunity.

Second, for theoretical and empirical reasons that will become apparent below, returns have more attractive statistical properties than prices, such as stationarity and ergodicity. In particular, dynamic general-equilibrium models often yield nonstationary prices, but stationary returns (see, for example, Chapter 8 and Lucas [1978]).

1.4.1 Definitions and Conventions

Denote by P_t the price of an asset at date t and assume for now that this asset pays no dividends. The *simple net return*, R_t , on the asset between dates $t-1$ and t is defined as

$$R_t = \frac{P_t}{P_{t-1}} - 1. \quad (1.4.1)$$

The *simple gross return* on the asset is just one plus the net return, $1 + R_t$.

From this definition it is apparent that the asset’s gross return over the most recent k periods from date $t-k$ to date t , written $1 + R_t(k)$, is simply

equal to the product of the k single-period returns from $t - k + 1$ to t , i.e.,

$$\begin{aligned} 1 + R_t(k) &\equiv (1 + R_t) \cdot (1 + R_{t-1}) \cdots (1 + R_{t-k+1}) \\ &= \frac{P_t}{P_{t-1}} \cdot \frac{P_{t-1}}{P_{t-2}} \cdot \frac{P_{t-2}}{P_{t-3}} \cdots \frac{P_{t-k+1}}{P_{t-k}} = \frac{P_t}{P_{t-k}}, \end{aligned} \quad (1.4.2)$$

and its net return over the most recent k periods, written $R_t(k)$, is simply equal to its k -period gross return minus one. These multiperiod returns are called *compound* returns.

Although returns are scale-free, it should be emphasized that they are *not* unitless, but are always defined with respect to some time interval, e.g., one “period.” In fact, R_t is more properly called a *rate* of return, which is more cumbersome terminology but more accurate in referring to R_t as a rate or, in economic jargon, a *flow* variable. Therefore, a return of 20% is not a complete description of the investment opportunity without specification of the return horizon. In the academic literature, the return horizon is generally given explicitly, often as part of the data description, e.g., “The CRSP *monthly* returns file was used.”

However, among practitioners and in the financial press, a return-horizon of one year is usually assumed implicitly; hence, unless stated otherwise, a return of 20% is generally taken to mean an *annual* return of 20%. Moreover, multiyear returns are often *annualized* to make investments with different horizons comparable, thus:

$$\text{Annualized}[R_t(k)] = \left[\prod_{j=0}^{k-1} (1 + R_{t-j}) \right]^{1/k} - 1. \quad (1.4.3)$$

Since single-period returns are generally small in magnitude, the following approximation based on a first-order Taylor expansion is often used to annualize multiyear returns:

$$\text{Annualized}[R_t(k)] \approx \frac{1}{k} \sum_{j=0}^{k-1} R_{t-j}. \quad (1.4.4)$$

Whether such an approximation is adequate depends on the particular application at hand; it may suffice for a quick and coarse comparison of investment performance across many assets, but for finer calculations in which the volatility of returns plays an important role, i.e., when the higher-order terms in the Taylor expansion are not negligible, the approximation (1.4.4) may break down. The only advantage of such an approximation is convenience—it is easier to calculate an arithmetic rather than a geometric average—however, this advantage has diminished considerably with the advent of cheap and convenient computing power.

Continuous Compounding

The difficulty of manipulating geometric averages such as (1.4.3) motivates another approach to compound returns, one which is not approximate and also has important implications for modeling asset returns; this is the notion of continuous compounding. The *continuously compounded return* or *log return* r_t of an asset is defined to be the natural logarithm of its gross return $(1 + R_t)$:

$$r_t \equiv \log(1 + R_t) = \log \frac{P_t}{P_{t-1}} = p_t - p_{t-1}, \quad (1.4.5)$$

where $p_t \equiv \log P_t$. When we wish to emphasize the distinction between R_t and r_t , we shall refer to R_t as a *simple* return. Our notation here deviates slightly from our convention that lowercase letters denote the logs of uppercase letters, since here we have $r_t = \log(1 + R_t)$ rather than $\log(R_t)$; we do this to maintain consistency with standard conventions.

The advantages of continuously compounded returns become clear when we consider multiperiod returns, since

$$\begin{aligned} r_t(k) &= \log(1 + R_t(k)) = \log((1 + R_t) \cdot (1 + R_{t-1}) \cdots (1 + R_{t-k+1})) \\ &= \log(1 + R_t) + \log(1 + R_{t-1}) + \cdots + \log(1 + R_{t-k+1}) \\ &= r_t + r_{t-1} + \cdots + r_{t-k+1}, \end{aligned} \quad (1.4.6)$$

and hence the continuously compounded multiperiod return is simply the sum of continuously compounded single-period returns. Compounding, a multiplicative operation, is converted to an additive operation by taking logarithms. However, the simplification is not merely in reducing multiplication to addition (since we argued above that with modern calculators and computers, this is trivial), but more in the modeling of the statistical behavior of asset returns over time—it is far easier to derive the time-series properties of additive processes than of multiplicative processes, as we shall see in Chapter 2.

Continuously compounded returns do have one disadvantage. The simple return on a portfolio of assets is a weighted average of the simple returns on the assets themselves, where the weight on each asset is the share of the portfolio's value invested in that asset. If portfolio p places weight w_{ip} in asset i , then the return on the portfolio at time t , R_{pt} , is related to the returns on individual assets, R_{it} , $i = 1 \dots N$, by $R_{pt} = \sum_{i=1}^N w_{ip} R_{it}$. Unfortunately continuously compounded returns do not share this convenient property. Since the log of a sum is not the same as the sum of logs, r_{pt} does not equal $\sum_{i=1}^N w_{ip} r_{it}$.

In empirical applications this problem is usually minor. When returns are measured over short intervals of time, and are therefore close to zero, the continuously compounded return on a portfolio is close to the weighted

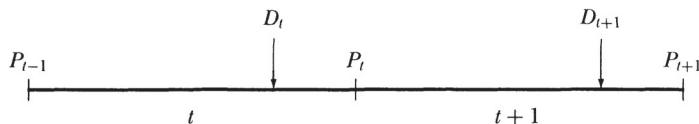


Figure 1.1. Dividend Payment Timing Convention

average of the continuously compounded returns on the individual assets: $r_{pt} \approx \sum_{i=1}^N w_{ip} r_{it}$.² We use this approximation in Chapter 3. Nonetheless it is common to use simple returns when a cross-section of assets is being studied, as in Chapters 4–6, and continuously compounded returns when the temporal behavior of returns is the focus of interest, as in Chapters 2 and 7.

Dividend Payments

For assets which make periodic dividend payments, we must modify our definitions of returns and compounding. Denote by D_t the asset’s dividend payment at date t and assume, purely as a matter of convention, that this dividend is paid just before the date- t price P_t is recorded; hence P_t is taken to be the *ex-dividend* price at date t . Alternatively, one might describe P_t as an end-of-period asset price, as shown in Figure 1.1. Then the net simple return at date t may be defined as

$$R_t = \frac{P_t + D_t}{P_{t-1}} - 1. \quad (1.4.7)$$

Multiperiod and continuously compounded returns may be obtained in the same way as in the no-dividends case. Note that the continuously compounded return on a dividend-paying asset, $r_t = \log(P_t + D_t) - \log(P_{t-1})$, is a nonlinear function of log prices and log dividends. When the ratio of prices to dividends is not too variable, however, this function can be approximated by a linear function of log prices and dividends, as discussed in detail in Chapter 7.

Excess Returns

It is often convenient to work with an asset’s excess return, defined as the difference between the asset’s return and the return on some reference asset. The reference asset is often assumed to be riskless and in practice is usually a short-term Treasury bill return. Working with simple returns, the

²In the limit where time is continuous, Ito’s Lemma, discussed in Section 9.1.2 of Chapter 9, can be used to relate simple and continuously compounded returns.

simple excess return on asset i is

$$Z_{it} = R_{it} - R_{0t}, \quad (1.4.8)$$

where R_{0t} is the reference return. Alternatively one can define a log excess return as

$$z_{it} = r_{it} - r_{0t}. \quad (1.4.9)$$

The excess return can also be thought of as the payoff on an *arbitrage portfolio* that goes long in asset i and short in the reference asset, with no net investment at the initial date. Since the initial net investment is zero, the return on the arbitrage portfolio is undefined but its dollar payoff is proportional to the excess return as defined above.

1.4.2 The Marginal, Conditional, and Joint Distribution of Returns

Having defined asset returns carefully, we can now begin to study their behavior across assets and over time. Perhaps the most important characteristic of asset returns is their randomness. The return of IBM stock over the next month is unknown today, and it is largely the explicit modeling of the sources and nature of this uncertainty that distinguishes financial economics from other social sciences. Although other branches of economics and sociology do have models of stochastic phenomena, in none of them does uncertainty play so central a role as in the pricing of financial assets—without uncertainty, much of the financial economics literature, both theoretical and empirical, would be superfluous. Therefore, we must articulate at the very start the types of uncertainty that asset returns might exhibit.

The Joint Distribution

Consider a collection of N assets at date t , each with return R_{it} at date t , where $t = 1, \dots, T$. Perhaps the most general model of the collection of returns $\{R_{it}\}$ is its joint distribution function:

$$G(R_{11}, \dots, R_{N1}; R_{12}, \dots, R_{N2}; \dots; R_{1T}, \dots, R_{NT}; \mathbf{x} \mid \boldsymbol{\theta}), \quad (1.4.10)$$

where \mathbf{x} is a vector of *state variables*, variables that summarize the economic environment in which asset returns are determined, and $\boldsymbol{\theta}$ is a vector of fixed parameters that uniquely determines G . For notational convenience, we shall suppress the dependence of G on the parameters $\boldsymbol{\theta}$ unless it is needed.

The probability law G governs the stochastic behavior of asset returns and \mathbf{x} , and represents the sum total of all knowable information about them. We may then view financial econometrics as the statistical inference of $\boldsymbol{\theta}$, given G and realizations of $\{R_{it}\}$. Of course, (1.4.10) is far too general to

be of any use for statistical inference, and we shall have to place further restrictions on G in the coming sections and chapters. However, (1.4.10) does serve as a convenient way to organize the many models of asset returns to be developed here and in later chapters. For example, Chapters 2 through 6 deal exclusively with the joint distribution of $\{R_{it}\}$, leaving additional state variables \mathbf{x} to be considered in Chapters 7 and 8. We write this joint distribution as G_R .

Many asset pricing models, such as the Capital Asset Pricing Model (CAPM) of Sharpe (1964), Lintner (1965a, b), and Mossin (1966) considered in Chapter 5, describe the joint distribution of the cross section of returns $\{R_{1t}, \dots, R_{Nt}\}$ at a single date t . To reduce (1.4.10) to this essentially static structure, we shall have to assert that returns are statistically independent through time and that the joint distribution of the cross-section of returns is identical across time. Although such assumptions seem extreme, they yield a rich set of implications for pricing financial assets. The CAPM, for example, delivers an explicit formula for the trade-off between risk and expected return, the celebrated security market line.

The Conditional Distribution

In Chapter 2, we place another set of restrictions on G_R which will allow us to focus on the *dynamics* of individual asset returns while abstracting from cross-sectional relations between the assets. In particular, consider the joint distribution F of $\{R_{i1}, \dots, R_{iT}\}$ for a given asset i , and observe that we may always rewrite F as the following product:

$$F(R_{i1}, \dots, R_{iT}) = F_{i1}(R_{i1}) \cdot F_{i2}(R_{i2} \mid R_{i1}) \cdot F_{i3}(R_{i3} \mid R_{i2}, R_{i1}) \\ \cdots F_{iT}(R_{iT} \mid R_{iT-1}, \dots, R_{i1}). \quad (1.4.11)$$

From (1.4.11), the temporal dependencies implicit in $\{R_{it}\}$ are apparent. Issues of predictability in asset returns involve aspects of their *conditional* distributions and, in particular, how the conditional distributions evolve through time.

By placing further restrictions on the conditional distributions $F_{it}(\cdot)$, we shall be able to estimate the parameters θ implicit in (1.4.11) and examine the predictability of asset returns explicitly. For example, one version of the random-walk hypothesis is obtained by the restriction that the conditional distribution of return R_{it} is equal to its marginal distribution, i.e., $F_{it}(R_{it} \mid \cdot) = F_{it}(R_{it})$. If this is the case, then returns are temporally independent and therefore unpredictable using past returns. Weaker versions of the random walk are obtained by imposing weaker restrictions on $F_{it}(R_{it} \mid \cdot)$.

The Unconditional Distribution

In cases where an asset return's conditional distribution differs from its marginal or unconditional distribution, it is clearly the conditional distribu-

tion that is relevant for issues involving predictability. However, the properties of the unconditional distribution of returns may still be of some interest, especially in cases where we expect predictability to be minimal.

One of the most common models for asset returns is the temporally independently and identically distributed (IID) normal model, in which returns are assumed to be independent over time (although perhaps cross-sectionally correlated), identically distributed over time, and normally distributed. The original formulation of the CAPM employed this assumption of normality, although returns were only implicitly assumed to be temporally IID (since it was a static “two-period” model). More recently, models of asymmetric information such as Grossman (1989) and Grossman and Stiglitz (1980) also use normality.

While the temporally IID normal model may be tractable, it suffers from at least two important drawbacks. First, most financial assets exhibit limited liability, so that the largest loss an investor can realize is his total investment and no more. This implies that the smallest net return achievable is -1 or -100% . But since the normal distribution’s support is the entire real line, this lower bound of -1 is clearly violated by normality. Of course, it may be argued that by choosing the mean and variance appropriately, the probability of realizations below -1 can be made arbitrarily small; however it will never be zero, as limited liability requires.

Second, if single-period returns are assumed to be normal, then multi-period returns cannot also be normal since they are the *products* of the single-period returns. Now the *sums* of normal single-period returns are indeed normal, but the sum of single-period simple returns does not have any economically meaningful interpretation. However, as we saw in Section 1.4.1, the sum of single-period continuously compounded returns does have a meaningful interpretation as a multiperiod continuously compounded return.

The Lognormal Distribution

A sensible alternative is to assume that continuously compounded single-period returns r_{it} are IID normal, which implies that single-period gross simple returns are distributed as IID *lognormal* variates, since $r_{it} \equiv \log(1 + R_{it})$. We may express the lognormal model then as

$$r_{it} \sim \mathcal{N}(\mu_i, \sigma_i^2). \quad (1.4.12)$$

Under the lognormal model, if the mean and variance of r_{it} are μ_i and σ_i^2 , respectively, then the mean and variance of simple returns are given by

$$E[R_{it}] = e^{\mu_i + \frac{\sigma_i^2}{2}} - 1 \quad (1.4.13)$$

$$\text{Var}[R_{it}] = e^{2\mu_i + \sigma_i^2} [e^{\sigma_i^2} - 1]. \quad (1.4.14)$$

Alternatively, if we assume that the mean and variance of simple returns R_{it} are m_i and s_i^2 , respectively, then under the lognormal model the mean and variance of r_{it} are given by

$$E[r_{it}] = \log \frac{m_i + 1}{\sqrt{1 + \left(\frac{s_i}{m_i + 1}\right)^2}} \quad (1.4.15)$$

$$\text{Var}[r_{it}] = \log \left[1 + \left(\frac{s_i}{m_i + 1}\right)^2 \right]. \quad (1.4.16)$$

The lognormal model has the added advantage of not violating limited liability, since limited liability yields a lower bound of zero on $(1 + R_{it})$, which is satisfied by $(1 + R_{it}) = e^{r_{it}}$ when r_{it} is assumed to be normal.

The lognormal model has a long and illustrious history, beginning with the dissertation of the French mathematician Louis Bachelier (1900), which contained the mathematics of Brownian motion and heat conduction, five years prior to Einstein's (1905) famous paper. For other reasons that will become apparent in later chapters (see, especially, Chapter 9), the lognormal model has become the workhorse of the financial asset pricing literature.

But as attractive as the lognormal model is, it is not consistent with all the properties of historical stock returns. At short horizons, historical returns show weak evidence of skewness and strong evidence of excess kurtosis. The *skewness*, or normalized third moment, of a random variable ϵ with mean μ and variance σ^2 is defined by

$$S[\epsilon] \equiv E \left[\frac{(\epsilon - \mu)^3}{\sigma^3} \right]. \quad (1.4.17)$$

The *kurtosis*, or normalized fourth moment, of ϵ is defined by

$$K[\epsilon] \equiv E \left[\frac{(\epsilon - \mu)^4}{\sigma^4} \right]. \quad (1.4.18)$$

The normal distribution has skewness equal to zero, as do all other symmetric distributions. The normal distribution has kurtosis equal to 3, but *fat-tailed* distributions with extra probability mass in the tail areas have higher or even infinite kurtosis.

Skewness and kurtosis can be estimated in a sample of data by constructing the obvious sample averages: the sample mean

$$\hat{\mu} \equiv \frac{1}{T} \sum_{t=1}^T \epsilon_t, \quad (1.4.19)$$

the sample variance

$$\hat{\sigma}^2 \equiv \frac{1}{T} \sum_{t=1}^T (\epsilon_t - \hat{\mu})^2, \quad (1.4.20)$$

the sample skewness

$$\hat{S} \equiv \frac{1}{T\hat{\sigma}^3} \sum_{t=1}^T (\epsilon_t - \hat{\mu})^3, \quad (1.4.21)$$

and the sample kurtosis

$$\hat{K} \equiv \frac{1}{T\hat{\sigma}^4} \sum_{t=1}^T (\epsilon_t - \hat{\mu})^4. \quad (1.4.22)$$

In large samples of normally distributed data, the estimators \hat{S} and \hat{K} are normally distributed with means 0 and 3 and variances $6/T$ and $24/T$, respectively (see Stuart and Ord [1987, Vol. 1]). Since 3 is the kurtosis of the normal distribution, sample *excess kurtosis* is defined to be sample kurtosis less 3. Sample estimates of skewness for daily US stock returns tend to be negative for stock indexes but close to zero or positive for individual stocks. Sample estimates of excess kurtosis for daily US stock returns are large and positive for both indexes and individual stocks, indicating that returns have more mass in the tail areas than would be predicted by a normal distribution.

Stable Distributions

Early studies of stock market returns attempted to capture this excess kurtosis by modeling the distribution of continuously compounded returns as a member of the *stable* class (also called the *stable Pareto-Lévy* or *stable Paretian*), of which the normal is a special case.³ The stable distributions are a natural generalization of the normal in that, as their name suggests, they are stable under addition, i.e., a sum of stable random variables is also a stable random variable. However, nonnormal stable distributions have more probability mass in the tail areas than the normal. In fact, the nonnormal stable distributions are so fat-tailed that their variance and all higher moments are infinite. Sample estimates of variance or kurtosis for random variables with

³The French probabilist Paul Lévy (1924) was perhaps the first to initiate a general investigation of stable distributions and provided a complete characterization of them through their log-characteristic functions (see below). Lévy (1925) also showed that the tail probabilities of stable distributions approximate those of the Pareto distribution, hence the term “stable Pareto-Lévy” or “stable Paretian” distribution. For applications to financial asset returns, see Blattberg and Gonedes (1974); Fama (1965); Fama and Roll (1971); Fielitz (1976); Fielitz and Rozell (1983); Granger and Morgenstern (1970); Hagerman (1978); Hsu, Miller, and Wichern (1974); Mandelbrot (1963); Mandelbrot and Taylor (1967); Officer (1972); Samuelson (1967, 1976); Simkowitz and Beedles (1980); and Tucker (1992).

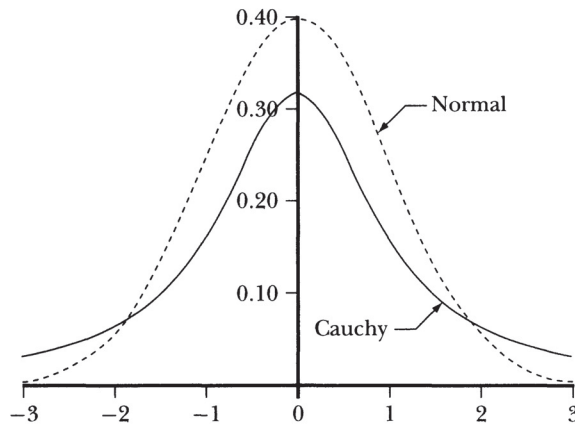


Figure 1.2. Comparison of Stable and Normal Density Functions

these distributions will not converge as the sample size increases, but will tend to increase indefinitely.

Closed-form expressions for the density functions of stable random variables are available for only three special cases: the normal, the Cauchy, and the Bernoulli cases.⁴ Figure 1.2 illustrates the Cauchy distribution, with density function

$$f(x) = \frac{1}{\pi} \frac{\gamma}{\gamma^2 + (x - \delta)^2} . \quad (1.4.23)$$

In Figure 1.2, (1.4.23) is graphed with parameters $\delta = 0$ and $\gamma = 1$, and it is apparent from the comparison with the normal density function (dashed lines) that the Cauchy has fatter tails than the normal.

Although stable distributions were popular in the 1960's and early 1970's, they are less commonly used today. They have fallen out of favor partly because they make theoretical modelling so difficult; standard finance theory

⁴However, Lévy (1925) derived the following explicit expression for the logarithm of the characteristic function $\varphi(t)$ of any stable random variable X : $\log \varphi(t) \equiv \log E[e^{itX}] = i\delta t - \gamma|t|^\alpha [1 - i\beta \operatorname{sgn}(t) \tan(\alpha\pi/2)]$, where $(\alpha, \beta, \delta, \gamma)$ are the four parameters that characterize each stable distribution. $\delta \in (-\infty, \infty)$ is said to be the *location* parameter, $\beta \in (-\infty, \infty)$ is the *skewness index*, $\gamma \in (0, \infty)$ is the *scale* parameter, and $\alpha \in (0, 2]$ is the *exponent*. When $\alpha = 2$, the stable distribution reduces to a normal. As α decreases from 2 to 0, the tail areas of the stable distribution become increasingly “fatter” than the normal. When $\alpha \in (1, 2)$, the stable distribution has a finite mean given by δ , but when $\alpha \in (0, 1]$, even the mean is infinite. The parameter β measures the symmetry of the stable distribution; when $\beta = 0$ the distribution is symmetric, and when $\beta > 0$ (or $\beta < 0$) the distribution is skewed to the right (or left). When $\beta = 0$ and $\alpha = 1$ we have the Cauchy distribution, and when $\alpha = 1/2$, $\beta = 1$, $\delta = 0$, and $\gamma = 1$ we have the Bernoulli distribution.

almost always requires finite second moments of returns, and often finite higher moments as well. Stable distributions also have some counterfactual implications. First, they imply that sample estimates of the variance and higher moments of returns will tend to increase as the sample size increases, whereas in practice these estimates seem to converge. Second, they imply that long-horizon returns will be just as non-normal as short-horizon returns (since long-horizon returns are sums of short-horizon returns, and these distributions are stable under addition). In practice the evidence for non-normality is much weaker for long-horizon returns than for short-horizon returns.

Recent research tends instead to model returns as drawn from a fat-tailed distribution with finite higher moments, such as the t distribution, or as drawn from a mixture of distributions. For example the return might be conditionally normal, conditional on a variance parameter which is itself random; then the unconditional distribution of returns is a mixture of normal distributions, some with small conditional variances that concentrate mass around the mean and others with large conditional variances that put mass in the tails of the distribution. The result is a fat-tailed unconditional distribution with a finite variance and finite higher moments. Since all moments are finite, the Central Limit Theorem applies and long-horizon returns will tend to be closer to the normal distribution than short-horizon returns. It is natural to model the conditional variance as a time-series process, and we discuss this in detail in Chapter 12.

An Empirical Illustration

Table 1.1 contains some sample statistics for individual and aggregate stock returns from the Center for Research in Securities Prices (CRSP) for 1962 to 1994 which illustrate some of the issues discussed in the previous sections. Sample moments, calculated in the straightforward way described in (1.4.19)–(1.4.22), are reported for value- and equal-weighted indexes of stocks listed on the New York Stock Exchange (NYSE) and American Stock Exchange (AMEX), and for ten individual stocks. The individual stocks were selected from market-capitalization deciles using 1979 end-of-year market capitalizations for all stocks in the CRSP NYSE/AMEX universe, where International Business Machines is the largest decile's representative and Continental Materials Corp. is the smallest decile's representative.

Panel A reports statistics for daily returns. The daily index returns have extremely high sample excess kurtosis, 34.9 and 26.0 respectively, a clear sign of fat tails. Although the excess kurtosis estimates for daily individual stock returns are generally less than those for the indexes, they are still large, ranging from 3.35 to 59.4. Since there are 8179 observations, the standard error for the kurtosis estimate under the null hypothesis of normality is $\sqrt{24/8179} = 0.054$, so these estimates of excess kurtosis are overwhelmingly

statistically significant. The skewness estimates are negative for the daily index returns, -1.33 and -0.93 respectively, but generally positive for the individual stock returns, ranging from -0.18 to 2.25 . Many of the skewness estimates are also statistically significant as the standard error under the null hypothesis of normality is $\sqrt{6/8179} = 0.027$.

Panel B reports sample statistics for monthly returns. These are considerably less leptokurtic than daily returns—the value- and equal-weighted CRSP monthly index returns have excess kurtosis of only 2.42 and 4.14 , respectively, an order of magnitude smaller than the excess kurtosis of daily returns. As there are only 390 observations the standard error for the kurtosis estimate is also much larger, 0.248 . This is one piece of evidence that has led researchers to use fat-tailed distributions with finite higher moments, for which the Central Limit Theorem applies and drives longer-horizon returns towards normality.

1.5 Market Efficiency

The origins of the Efficient Markets Hypothesis (EMH) can be traced back at least as far as the pioneering theoretical contribution of Bachelier (1900) and the empirical research of Cowles (1933). The modern literature in economics begins with Samuelson (1965), whose contribution is neatly summarized by the title of his article: “Proof that Properly Anticipated Prices Fluctuate Randomly”.⁵ In an informationally efficient market—not to be confused with an allocationally or Pareto-efficient market—price changes must be unforecastable if they are properly anticipated, i.e., if they fully incorporate the expectations and information of all market participants.

Fama (1970) summarizes this idea in his classic survey by writing: “A market in which prices always ‘fully reflect’ available information is called ‘efficient’.” Fama’s use of quotation marks around the words “fully reflect” indicates that these words are a form of shorthand and need to be explained more fully. More recently, Malkiel (1992) has offered the following more explicit definition:

A capital market is said to be efficient if it fully and correctly reflects all relevant information in determining security prices. Formally, the market is said to be efficient with respect to some information set . . . if security prices would be unaffected by revealing that information to all participants. Moreover, efficiency with respect to an information set

⁵Bernstein (1992) discusses the contributions of Bachelier, Cowles, Samuelson, and many other early authors. The articles reprinted in Lo (1996) include some of the most important papers in this literature.

Table 1.1. Stock market returns, 1962 to 1994.

| Security | Mean | Standard Deviation | Skewness | Excess Kurtosis | Minimum | Maximum |
|---------------------------------|-------|--------------------|----------|-----------------|---------|---------|
| Panel A: Daily Returns | | | | | | |
| Value-Weighted Index | 0.044 | 0.82 | -1.33 | 34.92 | -18.10 | 8.87 |
| Equal-Weighted Index | 0.073 | 0.76 | -0.93 | 26.03 | -14.19 | 9.83 |
| International Business Machines | 0.039 | 1.42 | -0.18 | 12.48 | -22.96 | 11.72 |
| General Signal Corp. | 0.054 | 1.66 | 0.01 | 3.35 | -13.46 | 9.43 |
| Wrigley Co. | 0.072 | 1.45 | -0.00 | 11.03 | -18.67 | 11.89 |
| Interlake Corp. | 0.043 | 2.16 | 0.72 | 12.35 | -17.24 | 23.08 |
| Raytech Corp. | 0.050 | 3.39 | 2.25 | 59.40 | -57.90 | 75.00 |
| Ampco-Pittsburgh Corp. | 0.053 | 2.41 | 0.66 | 5.02 | -19.05 | 19.18 |
| Energen Corp. | 0.054 | 1.41 | 0.27 | 5.91 | -12.82 | 11.11 |
| General Host Corp. | 0.070 | 2.79 | 0.74 | 6.18 | -23.53 | 22.92 |
| Garan Inc. | 0.079 | 2.35 | 0.72 | 7.13 | -16.67 | 19.07 |
| Continental Materials Corp. | 0.143 | 5.24 | 0.93 | 6.49 | -26.92 | 50.00 |
| Panel B: Monthly Returns | | | | | | |
| Value-Weighted Index | 0.96 | 4.33 | -0.29 | 2.42 | -21.81 | 16.51 |
| Equal-Weighted Index | 1.25 | 5.77 | 0.07 | 4.14 | -26.80 | 33.17 |
| International Business Machines | 0.81 | 6.18 | -0.14 | 0.83 | -26.19 | 18.95 |
| General Signal Corp. | 1.17 | 8.19 | -0.02 | 1.87 | -36.77 | 29.73 |
| Wrigley Co. | 1.51 | 6.68 | 0.30 | 1.31 | -20.26 | 29.72 |
| Interlake Corp. | 0.86 | 9.38 | 0.67 | 4.09 | -30.28 | 54.84 |
| Raytech Corp. | 0.83 | 14.88 | 2.73 | 22.70 | -45.65 | 142.11 |
| Ampco-Pittsburgh Corp. | 1.06 | 10.64 | 0.77 | 2.04 | -36.08 | 46.94 |
| Energen Corp. | 1.10 | 5.75 | 1.47 | 12.47 | -24.61 | 48.36 |
| General Host Corp. | 1.33 | 11.67 | 0.35 | 1.11 | -38.05 | 42.86 |
| Garan Inc. | 1.64 | 11.30 | 0.76 | 2.30 | -35.48 | 51.60 |
| Continental Materials Corp. | 1.64 | 17.76 | 1.13 | 3.33 | -58.09 | 84.78 |

Summary statistics for daily and monthly returns (in percent) of CRSP equal- and value-weighted stock indexes and ten individual securities continuously listed over the entire sample period from July 3, 1962 to December 30, 1994. Individual securities are selected to represent stocks in each size decile. Statistics are defined in (1.4.19)–(1.4.22).

... implies that it is impossible to make economic profits by trading on the basis of [that information set].

Malkiel's first sentence repeats Fama's definition. His second and third sentences expand the definition in two alternative ways. The second sentence suggests that market efficiency can be tested by revealing information to

market participants and measuring the reaction of security prices. If prices do not move when information is revealed, then the market is efficient with respect to that information. Although this is clear conceptually, it is hard to carry out such a test in practice (except perhaps in a laboratory).

Malkiel's third sentence suggests an alternative way to judge the efficiency of a market, by measuring the profits that can be made by trading on information. This idea is the foundation of almost all the empirical work on market efficiency. It has been used in two main ways. First, many researchers have tried to measure the profits earned by market professionals such as mutual fund managers. If these managers achieve superior returns (after adjustment for risk) then the market is not efficient with respect to the information possessed by the managers. This approach has the advantage that it concentrates on real trading by real market participants, but it has the disadvantage that one cannot directly observe the information used by the managers in their trading strategies (see Fama [1970, 1991] for a thorough review of this literature).

As an alternative, one can ask whether hypothetical trading based on an explicitly specified information set would earn superior returns. To implement this approach, one must first choose an information set. The classic taxonomy of information sets, due to Roberts (1967), distinguishes among

Weak-form Efficiency: The information set includes only the history of prices or returns themselves.

Semistrong-Form Efficiency: The information set includes all information known to all market participants (*publicly available* information).

Strong-Form Efficiency: The information set includes all information known to any market participant (*private* information).

The next step is to specify a model of "normal" returns. Here the classic assumption is that the normal returns on a security are constant over time, but in recent years there has been increased interest in equilibrium models with time-varying normal security returns.

Finally, abnormal security returns are computed as the difference between the return on a security and its normal return, and forecasts of the abnormal returns are constructed using the chosen information set. If the abnormal security return is unforecastable, and in this sense "random," then the hypothesis of market efficiency is not rejected.

1.5.1 Efficient Markets and the Law of Iterated Expectations

The idea that efficient security returns should be random has often caused confusion. Many people seem to think that an efficient security price should

be smooth rather than random. Black (1971) has attacked this idea rather effectively:

A perfect market for a stock is one in which there are no profits to be made by people who have no special information about the company, and in which it is difficult even for people who do have special information to make profits, because the price adjusts so rapidly as the information becomes available. . . . Thus we would like to see *randomness* in the prices of successive transactions, rather than great continuity. . . . Randomness means that a series of small upward movements (or small downward movements) is very unlikely. If the price is going to move up, it should move up all at once, rather than in a series of small steps. . . . Large price movements are desirable, so long as they are not consistently followed by price movements in the opposite direction.

Underlying this confusion may be a belief that returns cannot be random if security prices are determined by discounting future cash flows. Smith (1968), for example, writes: “I suspect that even if the random walkers announced a perfect mathematic proof of randomness, I would go on believing that in the long run future earnings influence present value.”

In fact, the discounted present-value model of a security price is entirely consistent with randomness in security returns. The key to understanding this is the so-called *Law of Iterated Expectations*. To state this result we define information sets I_t and J_t , where $I_t \subset J_t$ so all the information in I_t is also in J_t but J_t is superior because it contains some extra information. We consider expectations of a random variable X conditional on these information sets, written $E[X | I_t]$ or $E[X | J_t]$. The Law of Iterated Expectations says that $E[X | I_t] = E[E[X | J_t] | I_t]$. In words, if one has limited information I_t , the best forecast one can make of a random variable X is the forecast of the forecast one would make of X if one had superior information J_t . This can be rewritten as $E[X - E[X | J_t] | I_t] = 0$, which has an intuitive interpretation: One cannot use limited information I_t to predict the forecast error one would make if one had superior information J_t .

Samuelson (1965) was the first to show the relevance of the Law of Iterated Expectations for security market analysis; LeRoy (1989) gives a lucid review of the argument. We discuss the point in detail in Chapter 7, but a brief summary may be helpful here. Suppose that a security price at time t , P_t , can be written as the rational expectation of some “fundamental value” V^* , conditional on information I_t available at time t . Then we have

$$P_t = E[V^* | I_t] = E_t V^*. \quad (1.5.1)$$

The same equation holds one period ahead, so

$$P_{t+1} = E[V^* | I_{t+1}] = E_{t+1} V^*. \quad (1.5.2)$$

But then the expectation of the change in the price over the next period is

$$E_t[P_{t+1} - P_t] = E_t[E_{t+1}[V^*] - E_t[V^*]] = 0, \quad (1.5.3)$$

because $I_t \subset I_{t+1}$, so $E_t[E_{t+1}[V^*]] = E_t[V^*]$ by the Law of Iterated Expectations. Thus realized changes in prices are unforecastable given information in the set I_t .

1.5.2 Is Market Efficiency Testable?

Although the empirical methodology summarized here is well-established, there are some serious difficulties in interpreting its results. First, any test of efficiency must assume an equilibrium model that defines normal security returns. If efficiency is rejected, this could be because the market is truly inefficient or because an incorrect equilibrium model has been assumed. This *joint hypothesis* problem means that market efficiency as such can never be rejected.

Second, perfect efficiency is an unrealistic benchmark that is unlikely to hold in practice. Even in theory, as Grossman and Stiglitz (1980) have shown, abnormal returns will exist if there are costs of gathering and processing information. These returns are necessary to compensate investors for their information-gathering and information-processing expenses, and are no longer abnormal when these expenses are properly accounted for. In a large and liquid market, information costs are likely to justify only small abnormal returns, but it is difficult to say how small, even if such costs could be measured precisely.

The notion of *relative* efficiency—the efficiency of one market measured against another, e.g., the New York Stock Exchange vs. the Paris Bourse, futures markets vs. spot markets, or auction vs. dealer markets—may be a more useful concept than the all-or-nothing view taken by much of the traditional market-efficiency literature. The advantages of relative efficiency over absolute efficiency are easy to see by way of an analogy. Physical systems are often given an efficiency rating based on the relative proportion of energy or fuel converted to useful work. Therefore, a piston engine may be rated at 60% efficiency, meaning that on average 60% of the energy contained in the engine's fuel is used to turn the crankshaft, with the remaining 40% lost to other forms of work such as heat, light, or noise.

Few engineers would ever consider performing a statistical test to determine whether or not a given engine is perfectly efficient—such an engine exists only in the idealized frictionless world of the imagination. But measuring relative efficiency—relative to the frictionless ideal—is commonplace. Indeed, we have come to expect such measurements for many household products: air conditioners, hot water heaters, refrigerators, etc. Similarly,

market efficiency is an idealization that is economically unrealizable, but that serves as a useful benchmark for measuring relative efficiency.

For these reasons, in this book we do not take a stand on market efficiency itself, but focus instead on the statistical methods that can be used to test the joint hypothesis of market efficiency and market equilibrium. Although many of the techniques covered in these pages are central to the market-efficiency debate—tests of variance bounds, Euler equations, the CAPM and the APT—we feel that they can be more profitably applied to measuring efficiency rather than to testing it. And if some markets turn out to be particularly inefficient, the diligent reader of this text will be well-prepared to take advantage of the opportunity.

Author Index

- Ait-Sahalia, 340, 370, 392, 451, 507,
510–512
Abel, 260, 327, 328
Acharya, 175
Adams, 411
Admati, 99
Affleck-Graves, 107, 208
Ainslie, 334
Aitchison, 123
Aiyagari, 316
Aldous, 41
Alexander, G., 155
Alexander, S., 42, 65
Allen, 44
Amihud, 103, 104, 107, 316
Amin, 379, 381
Ammer, 421, 445
Andersen, 472
Anderson, 192
Andrews, 535
Arnold, 357
Arrow, 507
Aschauer, 326
Ashford, 123
Ashley, 150
Asquith, 174, 179
Atchison, 85

Bachelier, 16, 20, 32, 341
Backus, 429, 435, 454, 455, 457, 465
Bagehot, 103, 107
Bailey, 155

Bakay, 150
Balduzzi, 423
Ball, C., 107–109, 117, 122, 123, 177,
379, 392
Ball, R., 150
Banz, 211, 509
Barberis, 333
Barclay, 176
Barker, 150
Barr, 443, 445
Barron, A., 512
Barron, R., 512
Barsky, 283
Barton, 41
Basu, 211
Beckers, 379
Beedles, 17
Benartzi, 333
Bernard, 167
Bernstein, 3, 20, 339
Bertola, 423
Bertsimas, 99, 107, 365, 366
Bick, 507
Bierwag, 406
Billingsley, 341, 344
Black, 23, 182, 211, 339, 350, 351,
354, 356, 367, 379, 442, 455, 457,
462, 464, 485, 497, 510, 523
Blanchard, 259
Blattberg, 17, 208, 379
Bliss, 418, 422
Blume, L., 44

- Blume, M., 42, 43, 65, 100, 178, 211, 323, 324
Boehmer, 167
Boldrin, 474
Bollerslev, 381, 481, 483, 488, 489, 491–494, 496
Bonomo, 334
Boudoukh, 79, 134
Box, 47, 140
Brainard, 317
Braun, 494
Breedon, 221, 298, 306, 317, 508, 509
Breen, 251
Brennan, 108, 381, 441, 449
Brenner, 452, 455
Brickley, 179
Brieman, 7
Brock, 44, 470, 474, 478, 479
Brodsky, 472
Bronfman, 136
Broomhead, 516
Brown, D., 44
Brown, P., 150
Brown, R., 430, 441, 443, 452, 455
Brown, S., 150, 154, 157, 171, 177, 311, 443, 452
Burnside, 304
Butler, 85

Campbell, C., 173
Campbell, J., 48, 49, 214, 221, 239, 255, 257, 258, 261–264, 268, 273, 274, 278, 281, 283, 285, 286, 311, 313–315, 317, 318, 320, 321, 323–327, 330, 332, 372, 395, 408, 415, 419, 421, 422, 435, 443, 445, 448, 494, 497
Carleton, 409
Carlstein, 472
Cecchetti, 304, 310
Chamberlain, 92, 229, 238, 504
Chan, K., 110, 449
Chan, L., 107, 410
Chen, N., 239, 240, 424
Cho, 117, 121–123
Choi, 102
Chou, 381, 481

Christie, A., 379, 497
Christie, W., 107, 110
Chung, C., 504
Chung, K., 6
Cochrane, 48, 49, 52, 274, 302, 327, 330, 332
Cohen, 84, 85, 88, 104, 107
Collins, 167
Cone, 110
Connor, 221, 237–241
Constantinides, 316, 318, 326, 327, 330, 442, 507
Cooper, 409
Cootner, 65
Copeland, 103, 107, 108
Corrado, 172, 173
Cowles, 20, 35–37, 65
Cox, D., 7, 140
Cox, J., 31, 318, 340, 341, 349, 355, 379, 380, 382, 414, 429, 435, 436, 440, 441, 443, 444, 449, 456, 458, 463, 508, 509
Crack, 113
Craig, 474
Curcio, 107, 109
Cutler, 317

Dahm, 123
Dann, 180
Darken, 516
David, 41
Davis, D., 84
Davis, M., 316
Day, 474
Deaton, 504
DeBondt, 212
Debreu, 507
Dechert, 470, 478, 479
De Long, 283
DeLong, 317, 333
Demsetz, 103, 107
Dent, 167
Derman, 442, 455, 457, 464, 507
Dhrymes, 220
Diaconis, 41
Diamond, 260
Diba, 259

- Dickey, 65
Dimson, 85
Ding, 486
Dolley, 149
Domowitz, 54
Donaldson, 283
Duffie, 8, 318, 380, 441
Dufour, 55
Dunn, 327
Dupire, 507
Durlauf, 276–278
Dybvig, 220, 221, 351, 433, 440, 443, 444, 452, 456, 464
- Easley, 44, 99, 103, 107, 140
Eckbo, B., 179
Eckbo, E., 175
Edwards, 43
Eichenbaum, 326
Eikeboom, 102
Einstein, 16, 32
Engle, 54, 257, 381, 469, 481, 482, 485, 486, 489, 492, 494, 496, 531
Epstein, 305, 315, 319, 320, 334
Estrella, 424
Eytan, 92
- Fabozzi, 396, 405, 406
Fama, 20, 22, 41–43, 54, 65, 78, 79, 150, 157, 208, 211, 212, 215, 221, 240, 241, 248, 249, 266–268, 274, 379, 418, 421, 422, 424
Fang, 387, 388
Faust, 48, 49, 52
Feller, 7
Ferguson, 44
Ferson, 327, 448, 534
Fielitz, 17
Fischel, 110
Fisher, 85, 150
Fishman, 387
Flavin, 279
Foerster, 534
Fong, 412
Foresi, 423, 457
Foster, 381, 485
Frankel, 84
- Frees, 117, 121–123
French, 72, 78, 79, 157, 211, 212, 240, 241, 248, 249, 266–268, 274, 366, 421, 497
Friedman, 518
Friend, 211, 220, 323, 324
Froot, 259, 288, 333, 424
Fuller, 45, 46, 65
Furbush, 110
- Gabr, 472
Galai, 103, 107
Gallant, A., 522, 535
Galli, 84
Garber, 258
Garcia, 334
Garman, 380, 481, 507
Gatto, 385, 391
George, 107, 135
Gertler, 316
Gibbons, 193, 196, 199, 206, 245, 246, 298, 317, 445, 446, 448, 455
Gilles, 275
Giovannini, 84, 320
Girosi, 512, 516, 517, 522
Gleick, 473
Glosten, 101–103, 106, 107, 135, 486, 488, 497
Godek, 110
Goetzmann, 311
Goldberger, 504
Goldenberg, 379
Goldman, 385, 391
Goldstein, 102
Gonedes, 17, 208, 379
Gonzalez-Rivera, 489
Goodhart, 107, 109
Gordon, 256
Gottlieb, 121–123
Graham, 114
Grandmont, 474
Granger, 17, 59, 60, 65, 91, 257, 470, 472, 486
Granito, 406
Grassberger, 476–478
Gray, 452, 455
Gregory, 435

- Grinblatt, 221
Grossman, H., 259
Grossman, S., 15, 24, 110, 305, 306,
308, 317, 318, 335
Gultekin, B., 220
Gultekin, M., 220
Gultekin, N., 369
Gurland, 123
- Hagerman, 17
Hakansson, 507
Hald, 30
Hall, A., 527
Hall, R., 276, 278, 305, 311
Hamilton, 7, 257, 360, 372, 451, 472,
473, 481, 527, 535, 536
Hammersley, 386–388
Hampel, 523
Handscorn, 387
Hansen, B., 488
Hansen, L., 269, 270, 292, 295, 296,
301, 302, 304, 306, 309, 311, 314,
315, 326, 332, 359–361, 392, 448,
527, 531, 536, 540
Härdle, 501, 502
Hardouvelis, 424
Harjes, 452, 455
Harpaz, 92
Harris, J., 107, 110
Harris, L., 103, 107–109, 121–123,
135
Harrison, 31, 355, 380, 508
Harvey, A., 490, 493
Harvey, C., 217, 314, 494–497
Hasbrouck, 107
Hausman, 50, 51, 109, 122–124, 128,
136–138, 143
Hawawini, 85
He, H., 315, 316, 507
Heath, 455, 457, 458, 464
Heaton, 304, 316, 318, 327, 332, 360
Hegde, 107
Hentschel, 485, 488, 497
Hertz, 512
Heston, 379, 446, 448
Hicks, 418
Ho, 103, 104, 107, 455–457, 464
Hodrick, 267–270, 274, 285, 286,
448, 536
Hoel, 7, 346
Hofmann, 379
Hogarth, 332
Holden, 474
Holt, 84
Hornik, 522
Hosking, 59, 60
Hsieh, 474, 475, 479
Hsu, 17
Huang, C., 8, 380, 391, 461, 508
Huang, R., 107, 110, 135
Huberman, 221, 231
Hull, 340, 349, 378–381, 459, 464,
510
Hurst, 59, 62
Hutchinson, 340, 392, 510, 512, 519,
522
- Ingersoll, 8, 210, 295, 318, 351, 380,
382, 406, 414, 429, 433, 435, 436,
440, 441, 443, 444, 449, 456, 458,
463
Irish, 504
Itô, 348
- Jöreskog, 234
Jackwerth, 370, 507
Jacquier, 490
Jagannathan, 214, 292, 296, 301,
304, 309, 315, 486, 488, 496, 497
Jain, 177
James, 180
Jamshidian, 463
Jarrell, 178, 179
Jarrow, 455, 457, 458, 464
Jegadeesh, 212, 266, 274
Jennings, 44
Jensen, 150, 179, 211
Jerison, 348
Jobson, 196, 223, 224
Johnson, 379
Jones, H., 35–37, 65
Joyeux, 59, 60
- Kagel, 84

- Kahneman, 333
Kalay, 121–123
Kalos, 387
Kamstra, 283
Kan, 441
Kandel, E., 110
Kandel, S., 200, 215, 217, 221, 231,
286, 309, 310
Kane, 168, 396
Kani, 507
Karasinski, 442
Karolyi, 449
Kaufman, 406
Kaul, 107, 135
Keim, 100, 107, 267, 421
Kendall, 65, 273
Kennan, 475
Kim, M., 79, 274
Kim, S., 490
Kindleberger, 258
Klass, 481
Kleidon, 110, 277, 278
Knuth, 114
Kocherlakota, 302, 310
Kogan, 365, 366
Kohlase, 474
Koo, 321
Korajczyk, 237–241, 251
Korkie, 196, 223, 224
Kothari, 212, 251
Kreps, 31, 319, 332, 355, 380, 508
Krogh, 512
Kroner, 381, 452, 455, 481, 491
Kwon, 397, 415, 420, 451, 453
Kyle, 99, 317

Laibson, 334
Lakonishok, 44, 107, 248, 249
Lam, 304, 310
Lanen, 175
Lang, 6
Leamer, 472, 523
LeBaron, 44, 475, 479
Ledoit, 113
Lee, C., 136
Lee, M., 452, 455
Lee, S., 455–457, 464, 488

Lee, T., 123
Lehmann, 236, 238, 240, 241, 246,
284
Leland, 507
LeRoy, 23, 256, 275, 276, 279, 280
Leroy, 31, 102, 508
Lévy, 17
Lilien, 494
Lintner, 14, 156, 181
Litzenberger, 8, 176, 216, 298, 317,
412, 461, 508, 509
Ljung, G., 47
Lo, 20, 47–49, 52–55, 63, 64, 74, 76,
78, 79, 84, 85, 89, 93–95, 99, 107,
109, 122–124, 128, 130, 133,
136–138, 143, 212, 248, 249, 251,
340, 359, 365, 366, 370, 371, 374,
375, 377, 379, 392, 472, 504, 507,
510–512, 519, 522–524
Loewenstein, 334
Longstaff, 392, 429, 437–439, 442,
444, 449, 455, 507
Lorenz, 473
Lowe, 516
Lucas, D., 316, 318
Lucas, R., Jr., 9, 31, 102, 508
Lumsdaine, 488
Luttmer, 304, 315, 316
Lutz, 413, 418

Macaulay, 403, 420
MacBeth, 211, 215, 369
Mackay, 258
MacKinlay, A. C., 47–49, 52–55, 74,
76, 78, 79, 85, 89, 93–95, 99,
109, 122–124, 128, 130, 133,
136–138, 143, 193, 210, 212, 245,
248–251, 366, 379, 472, 504, 510,
523, 524
Maddala, 123
Madhavan, 107
Magee, 43
Magnus, 6
Maier, 84, 85, 88, 104, 107
Majluf, 179
Maksimovic, 175
Malatesta, 167, 175

- Malgrange, 474
Malkiel, 20
Mandelbrot, 17, 54, 59, 62, 63, 65, 379
Mankiw, L., 260
Mankiw, N. G., 48, 49, 278, 289, 313, 317, 318, 320, 326, 337, 422, 423
Manne, 178
Mark, 274, 304, 310
Markowitz, 181
Marsh, 121, 123–125, 143, 277, 278, 289
Marx, 110
Mason, A., 410
Mason, S., 507
Mauldon, 388
Mayers, 214
McCallum, 425
McCullagh, 123
McCulloch, J., 397, 410, 411, 414, 415, 417, 420, 451, 453
McCulloch, R., 217
McCulloch, W., 512
McDonald, 208
McQueen, 149
Mech, 134
Mehra, 293, 302, 303, 307, 332, 334
Mei, 237
Melino, 308, 317, 396, 489, 490
Mendelson, 103, 104, 107, 316
Merton, 6, 8, 184, 219, 221, 277, 278, 289, 315, 318, 322, 339, 340, 346, 349–351, 354, 365, 392, 481, 507, 510
Merville, 369
Miccheli, 517
Mikkelson, 179
Milgrom, 101–103, 107
Miller, H., 7
Miller, M., 110, 509
Miller, R., 17, 107
Milne, 463
Miron, 423
Mishkin, 424
Mitchell, 149, 179
Modest, 92, 236, 238, 240, 241, 315, 316
Modigliani, 418
Monahan, 535
Monro, 515
Mood, 39, 41
Moody, 516
Morganstern, 17, 65
Morrison, 235, 238
Morse, 175
Morton, 455, 457, 458, 464
Mossin, 14
Muirhead, 192, 193
Muller, 472
Mullins, 174, 179
Musumeci, 167
Muth, 56
Myers, J., 150
Myers, S., 179
Naik, 452, 455
Neftci, 44
Nelson, C., 79, 274, 313, 413
Nelson, D., 381, 481, 484–486, 488, 489, 494
Nelson, W., 317
Netter, 149, 179
Neudecker, 6
Newey, 55, 130, 269, 270, 535, 536
Ng, 379, 381, 485, 486, 491
Niederhoffer, 107, 109
Nimalendran, 107, 135
Norman, 316
O'Brien, 475
O'Hara, 44, 84, 99, 103, 107, 140
Obstfeld, 259, 288
Officer, 17, 481
Ogaki, 360, 527
Ord, 7, 17
Osborne, 65, 107, 109
Pagan, 485
Palmer, 512
Papell, 474
Parkinson, 481
Partch, 179
Paskov, 388
Patashnik, 114

- Pau, 44
Pearson, 438, 441
Pennacchi, 445
Perold, 507
Perron, 366, 372, 472
Pesaran, 475
Pfleiderer, 99
Phillips, P., 277
Phillips, S., 100
Pierce, 47
Pitts, 512
Platen, 379
Poggio, 340, 392, 510, 512, 516, 517, 519, 522
Polson, 490
Port, 7, 346
Porter, 275, 276, 280
Porteus, 319
Poterba, 48, 49, 78, 79, 266, 317
Potter, 475
Poulsen, 167, 178
Powell, 516, 517
Prabhala, 175
Prelec, 334
Prescott, 293, 302, 303, 307, 332, 334
Priestley, 470, 471
Procaccia, 476–478
- Radner, 508
Rady, 507
Ramaswamy, 216, 381, 445, 446, 455
Ready, 136
Reddington, 405
Reder, 332
Reinsch, 522
Richard, 295
Richardson, 47–49, 58, 79, 134, 210, 274, 422
Rietz, 310, 311
Ritter, 156
Robbins, 515
Roberds, 423
Roberts, 22, 30, 65
Robins, 494
Robinson, P., 471
Rogalski, 369
Roley, 149
- Rolfo, 412
Roll, 17, 72, 101–103, 106, 128, 134, 135, 143, 145, 150, 184, 213, 216, 239, 240, 243, 366
Roma, 379
Romano, 47
Romer, 278, 289
Rosenfeld, 121, 123–125, 143
Ross, D., 110
Ross, S., 31, 156, 193, 196, 206, 216, 219, 220, 239, 240, 245, 246, 311, 318, 341, 355, 380, 382, 414, 429, 433, 435, 436, 440, 441, 443, 444, 449, 456, 458, 463, 508, 509
Rossi, 490
Roth, 84
Rothschild, 92, 238, 491
Roy, 55
Rozell, 17
Ruback, 179
Rubin, 234
Rubinstein, 340, 341, 349, 351, 370, 381, 461, 507, 509
Rudebusch, 423
Ruiz, 490, 493
Runkle, 423, 486, 488, 497
Ruud, 504
- Salandro, 102
Samuelson, 17, 20, 23, 30, 256, 321
Sanders, 449
Sayers, 474
Schaefer, 412, 430, 441, 443, 452, 455
Scheinkman, 359–361, 392, 470, 475, 478, 479
Schipper, 167, 180
Schoenholtz, 408, 421, 422
Scholes, 85, 88, 177, 211, 339, 350, 351, 354, 356, 367, 462, 510
Schultz, 107
Schuss, 346
Schwartz, E., 429, 438, 439, 441, 442, 444, 449, 455
Schwartz, R., 84, 85, 88, 104, 107
Schweizer, 379
Schwert, G., 149, 485, 497
Sclove, 472

- Scott, 276, 379
SEC, 84, 108
Sentana, 497
Shanken, 85, 193, 196, 199, 200, 206,
212, 215–217, 220, 222, 226, 233,
245, 246, 251
Shanno, 379
Shapiro, J., 107
Shapiro, M., 278, 289, 317, 320, 422
Sharpe, 14, 155, 156, 181
Shastri, 102
Shea, 412
Shephard, 490, 493
Shiller, 255, 257, 258, 261–263, 265,
267, 275, 276, 278, 281, 283, 305,
306, 308, 317, 318, 366, 395–397,
408, 419, 421, 422, 443, 445
Shimko, 370, 577
Shleifer, 248, 249, 317, 333
Sias, 134
Siegel, A., 413
Siegel, J., 311
Siegmund, 472
Silvey, 7, 123, 358
Simkowitz, 17
Simonds, 85
Sims, 91
Singer, 348
Singleton, 306, 311, 314, 326, 327,
332, 418, 429
Skelton, 406
Sloan, 212, 251
Smidt, 107
Smith, A., 23
Smith, C., 100
Smith, J., 110
Smith, T., 47, 79
Sofianos, 107
Sosin, 385, 391
Stambaugh, 100, 178, 214, 215, 217,
267, 273, 286, 309, 310, 421, 422,
446, 448, 497
Starks, 134
Startz, 313, 326
Steigerwald, 279
Stewart, 474
Stiglitz, 15, 24
Stinchcombe, 522
Stock, 48, 49, 58, 79, 274, 422
Stoker, 504, 505
Stoll, 103–105, 107, 110, 135
Stone, 7, 346
Strang, 6
Stroock, 348
Stuart, 7, 17
Stuetzle, 518
Subba Rao, 472
Suits, 410
Summers, 48, 49, 78, 79, 260, 265,
266, 317, 333
Sun, 429, 435, 438, 441
Sundaresan, M., 92
Sundaresan, S., 326, 327, 330, 396
Sunier, 494
Sutch, 418
Svensson, 413
Taqqu, 63
Taylor, H., 17
Taylor, M., 44
Taylor, S., 485
Teräsvirta, 470
Thaler, 212, 333
Thayer, 234
Thisted, 123
Thombs, 47
Thompson, J., 474
Thompson, R., 167, 175, 180
Tiniç, 103, 107, 369
Tirole, 259, 260
Titman, 212, 221
Tjøstheim, 470
Toevs, 406
Tong, 470, 472
Torous, 107–109, 177, 392
Toy, 442, 455, 457, 464
Traub, 388
Treyner, 44
Tsay, 476
Tschoegl, 107–109
Tucker, 17
Tufano, 507
Turnbull, 463, 489, 490
Tversky, 333

- Unal, 168
- van Deventer, 411
- Vasicek, 412, 429, 432, 434, 441, 449
- Vayanos, 316
- Vishny, 248, 249, 333
- Volterra, 471
- Wahba, 523
- Waldmann, 317, 333
- Wallis, J., 59, 63
- Wang, J., 92, 99, 371, 373–375, 377
- Wang, Y., 387, 388
- Wang, Z., 214, 496
- Warner, 150, 154, 171, 177
- Wasley, 173
- Watson, 259
- Weil, P., 305, 310, 315, 319, 320
- Weil, R., 406
- Weinstein, 157
- West, 55, 130, 258, 269, 270, 275, 278, 280, 289, 535, 536
- Whaley, 107
- Wheatley, 317
- Whitcomb, 84, 85, 88, 104, 107
- White, A., 378–381, 464
- White, H., 54, 174, 489, 512, 515, 522, 527, 536, 539
- Whitelaw, 134
- Whiteman, 423
- Whitlock, 387
- Wichern, 17
- Wiener, 348
- Wiggins, 378–381, 489, 490
- Wilcox, 316
- Williams, J., 85, 88, 175, 177
- Williams, R., 6
- Willig, 110
- Woodford, 474, 475
- Wooldridge, 488, 489, 491, 494, 497
- Working, 65
- Yaari, 334
- Zeckhauser, 260
- Zehna, 367
- Zeldes, 317
- Zhou, G., 217
- Zhou, Z., 335
- Zin, 305, 315, 319, 320, 334, 435, 454, 455, 457, 465

Subject Index

- absolute value GARCH model, 485
- activation function, 513
- affine-yield models of the term structure, 428, 441, 445
- aggregate consumption
 - aggregation, 305
 - Consumption Capital Asset Pricing Model, 316
- American option, 349
- amplitude-dependent exponential autoregression (EXPAR) models, 470
- antipersistence, 60
- antithetic variates method, 388
- arbitrage opportunities, 339
 - state price vector, 295
 - bond excess returns, 414
 - Merton's approach to option pricing, 351
- arbitrage portfolios, 351
- Arbitrage Pricing Theory (APT), 8, 85, 92, 219. *See also* Capital Asset Pricing Model, multifactor models
- exact factor pricing, 221
- factor risk exposure, 221
- pervasive factors, 221
- riskfree return, 220
- well-diversified market portfolio, 221
- ARCH models, 469, 482. *See also* GARCH models
 - option pricing, 381
- arithmetic Brownian motion, 32, 344. *See also* Brownian motion
- Arrow-Debreu securities, 507
- artificial neural network, 512. *See also* learning networks
- Asian options, 382
- ask price, 83
- asymptotic distribution
 - GMM estimator, 533
 - IV estimator, 529
 - ML estimator, 350, 538
- asymptotic order, 343
- asymptotically efficient estimator, 358, 530
- autocorrelation coefficients, 44, 66, 145
- autocorrelation matrices, 75, 76, 131. *See also* cross-autocorrelation
- autocovariance coefficients, 45
- autocovariance matrices, 74
- Autoregressive Conditionally Heteroskedastic models, 469, 482. *See also* GARCH models
- average derivative estimators, 505
- average rate options, 382, 386

- backpropagation, 515
- bandwidth, 500
 - optimal bandwidth selection, 502
- barrier models, 121
- barrier options, 391

- Bayesian inference, 7
- BDS test, 479
- BEKK model, 491
- benchmark portfolio, 298
- Berkeley Options Database, 107
- Bernoulli distribution, 18
- beta, 155, 182, 496
- bias
 - finite-sample bias in long-horizon regressions, 273
- bid price, 83
- bid-ask bounce, 101, 134
- bid-ask spread, 99, 146, 147
 - adverse-selection cost component, 103
 - estimating the effective bid-ask spread, 134
 - inventory cost component, 103
 - order-processing cost component, 103
- bilinear model, 471
- binary threshold model, 512
- binomial tree for the short-term interest rate, 442
- birth and death options, 391
- Black-Scholes and Merton option pricing model, 339, 350. *See also* option pricing models
 - estimator for α , 364
 - adjusting the Black-Scholes formula for predictability, 375
 - assumptions, 350
 - Black-Scholes formula, 352, 371, 373, 519
 - CAPM, 351
 - deterministic volatility, 379
 - estimator for σ^2 , 361, 374, 375
 - stochastic volatility, 380
 - implied volatility, 377
 - option sensitivities, 354
- borrowing constraints, 315
- Box-Cox transformation, 140
- Box-Pierce Q -statistic, 67
- Brock-Dechert-Scheinman test, 478
- Brownian motion, 344
 - arithmetic, 32, 344
 - estimator for α , 364
 - estimator for σ^2 , 361
 - geometric, 347
 - properties, 344
- bubbles, 258
- bullish vertical spread, 509
- Butterfly Effect, 473
- call option, 349
- callable bond, 395
- Capital Asset Pricing Model (CAPM), 14, 181. *See also* Arbitrage Pricing Theory, Intertemporal Capital Asset Pricing Model, data-snooping biases, mean-variance efficient-set mathematics, multifactor models, sample selection biases
 - anomalies, 211
 - applications, 183
 - Black version, 182, 196
 - book-market effect conditional, 496
 - cross-sectional regression tests, 215. *See also* errors-in-variables
 - heteroskedasticity, 208
 - intertemporal equilibrium models, 323
 - January effect, 100
 - non-normality, 208
 - nonsynchronous trading, 85
 - option pricing, 351
 - power of tests, 204
 - price-earnings-ratio effect, 211
 - Sharpe-Lintner version, 182, 189
 - size effect, 211, 496
 - size of tests, 203
 - temporal dependence, 208
 - unobservability of the market portfolio, 213, 216.
- CAPM. *See* Capital Asset Pricing Model
- catching up with the Joneses, 327, 328. *See also* habit formation models
- Cauchy distribution, 18
- CCAPM. *See* Consumption Capital Asset Pricing Model

- ceiling function, 114
 - chaos theory, 473. *See also*
 - deterministic nonlinear dynamical systems
 - clientele effects, 412
 - closeness indicator, 477
 - Cobb-Douglas utility, 326
 - coefficient functions, 356
 - cointegration, 257
 - the term structure of interest rates, 419
 - complete asset markets, 295
 - compound options, 391
 - conditional volatility models. *See*
 - ARCH models, GARCH models
 - connection strength, 513
 - consistent and uniformly asymptotically normal (CUAN) estimators, 358
 - constant-correlation model, 492
 - constant-expected-return hypothesis, 255
 - and vector autoregressive methods, 281
 - and volatility tests, 276
 - Consumption Capital Asset Pricing Model (CCAPM), 304
 - aggregate consumption and, 316
 - Epstein-Zin-Weil recursive utility model, 319. *See also*
 - Epstein-Zin-Weil model
 - instrumental variables (IV) regression, 311
 - investor heterogeneity, 317
 - power utility, 305. *See also*
 - lognormal asset pricing models
 - substituting consumption out of the model, 320
- consumption growth, 311, 434
- consumption of stockholders and nonstockholders, 317
- continuous-record asymptotics, 364
- contrarian investment strategies, 76
- control variate method, 387
- convexity, 406
- correlation coefficient, 44
- correlation dimension, 478
- correlation integral, 477
- cost of capital estimation, 183
- coupon bonds, 396, 401
 - convexity, 406
 - coupon rate, 401
 - duration, 403
 - effective duration, 406
 - forward rates, 408
 - immunization, 405
 - loglinear model, 406
 - Macaulay's duration, 403
 - modified duration, 405
 - price, 401, 409
 - yield to maturity, 401
- covariance stationarity, 484
- Cowles-Jones ratio, 35. *See also*
 - Random Walk 1 model
- Cox, Ingersoll, and Ross model, 436
- Cox-Ross option pricing technique, 390. *See also* risk-neutral option-pricing method
- cross-autocorrelation, 74, 75, 84, 129.
 - See also* autocorrelation matrices
- cross-sectional models, 173
- cross-sectional restrictions on the term structure, 452
- cross-validation, 502
- crude Monte Carlo, 386
- curse of dimensionality, 504
- data-snooping, 212, 240, 246, 249, 251, 523
- default risk, 406
- degrees of freedom, 523
- Delta of an option, 353, 512
- delta method, 51, 540
- delta-hedging, 512, 522
- derivative securities, 339, 455. *See also* fixed-income derivative securities, option pricing
 - forward contract, 458
 - futures contract, 459
- deterministic nonlinear dynamical systems, 473
 - logistic map, 525
 - sensitivity to initial conditions, 473
 - tent map, 474, 476, 525

- testing. *See* testing for
 - deterministic nonlinear dynamical processes
- difference-stationary process, 65, 372. *See also* unit root process
- diffusion function, 356
- discount bonds, 396, 397
 - estimating the zero-coupon term structure, 409
 - forward rate, 399
 - holding-period return, 398
 - immunization, 405
 - term structure of interest rates, 397
 - yield curve, 397
 - yield spread, 397
 - yield to maturity, 397
- discount function, 410. *See* spline estimation
- discounted value
 - of future dividends, 256
 - of the stock price, 255
- discrete-time models
 - of option pricing, 381
 - of stochastic volatility, 489
- discretization, 383, 385
- distribution. *See* asymptotic distribution, returns
- dividend-price ratio, 264, 268
- dividend-ratio model, 263
- dividends, 12, 254
- double bottoms. *See* technical analysis
- down and out options, 391
- drift, 31, 356
- dual-currency options, 391
- dual-equity options, 391
- durable goods, 326, 332
- duration, 403. *See also* coupon bonds
- duration of nontrading, 87
- dynamic hedging strategy, 521
- dynamic trading strategies, 352, 391
- Dynkin operator, 360

- effective duration, 406. *See also* coupon bonds
- effective spread, 102

- efficiency. *See* asymptotic efficiency
- Efficient Markets Hypothesis (EMH), 20
 - Semistrong-Form Efficiency, 22, 30
 - Strong-Form Efficiency, 22, 30
 - Weak-form Efficiency, 22, 30
- EGARCH model, 486, 488
- EH. *See* expectations hypothesis
- elasticity, 405
- elasticity of intertemporal substitution, 305
 - hyperbolic discounting and, 334
 - separating risk aversion from intertemporal substitution, 319
 - the riskless interest rate and, 309
- embedding dimension, 476
- EMH. *See* Efficient Markets Hypothesis
- Epstein-Zin-Weil recursive utility model, 319
 - consumption-wealth ratio, 321
 - cross-sectional asset pricing formula, 322
 - equity premium puzzle, 323
 - factor asset pricing model, 324
 - substituting consumption out of the model, 320
- equity premium puzzle
 - catching up with the Joneses model, 328
 - Hansen-Jagannathan volatility bound, 302
 - lognormal asset pricing model with Epstein-Zin-Weil utility, 323
 - lognormal asset pricing model with power utility, 307
- equity repurchases, 256, 287
- equivalent martingale measure, 355, 383, 508
- errors-in-variables, 216
- Euler equation, 293, 508. *See also* stochastic discount factor
 - Cobb-Douglas utility model, 326
 - ratio models of habit formation, 328
- European option, 349

- event-study analysis, 149. *See also*
 - nonparametric tests
 - abnormal return, 150, 151
 - Arbitrage Pricing Theory, 156
 - Capital Asset Pricing Model, 156
 - clustering, 166
 - constant-mean-return model, 151, 154
 - cross-sectional models, 173
 - cumulative abnormal return, 160
 - earnings-announcement example, 152
 - estimation window, 152
 - event window, 151
 - event-date uncertainty, 176
 - factor model, 155
 - generalized method of moments, 154, 174
 - inference with changing variances, 167
 - law and economics, 149
 - legal liability, 149, 179
 - market model, 151, 155, 158
 - market-adjusted-return model, 156
 - nonsynchronous trading, 177
 - normal return, 151
 - post-event window, 157
 - sampling interval, 175
 - skewness of returns, 172
 - standardized cumulative abnormal return, 160
 - test power, 168
- exact factor pricing, 221
 - interpreting deviations, 242
 - mean-variance efficient set mathematics, 243
 - nonrisk-based alternatives, 248
 - optimal orthogonal portfolio, 243, 245, 248
 - risk-based alternative, 247
 - Sharpe ratio, 245, 247, 248, 252
 - tangency portfolio, 245, 247
- excess kurtosis 17, 488, 512. *See also*
 - kurtosis, returns
- excess returns, 12, 182, 268, 291
- exercise price, 349
- exotic securities, 391
- expansion of the states, 357
- EXPAR models, 470
- expectations hypothesis (EH), 413, 418, 419. *See also* pure
 - expectations hypothesis, term structure of interest rates
 - empirical evidence, 418
 - log expectations hypothesis, 432, 437
 - preferred habitats, 418
 - yield spreads, 418
- expected discounted value. *See* discounted value
- exponential GARCH model, 486, 488
- exponential spline, 412
- face value, 396
- factor analysis, 234
- factor model, 155. *See also* multifactor models
- fair game. *See* martingale
- fat tail, 16, 480. *See also* kurtosis
- finite-dimensional distributions (FDDs), 344, 364
- Fisher information matrix. *See* information matrix
- fixed-income derivative securities, 455
 - Black-Scholes formula, 462
 - Heath-Jarrow-Morton model, 457
 - Ho-Lee model, 456
 - homoskedastic single-factor model, 463
 - option pricing, 461
 - term structure of implied volatility, 463
- fixed-income securities, 395
- floor function, 114
- Fokker-Planck equation, 359
- foreign currency, 5, 382, 386, 390
- forward equation, 359
- forward rate, 399, 438, 440. *See also* term structure of interest rates
 - coupon-bearing term structure, 408

- forward-rate curve, 400, 412
- log forward rate, 400, 408
- pure expectations hypothesis, 414, 417
- yield to maturity, 400
- forward trading, 399
- fractionally differenced time series, 60
- fractionally integrated time series.
See fractionally differenced time series
- fundamental asset, 356
- fundamental value, 258, 288

- Gamma of an option, 353
- GARCH models, 483, 486, 487
 - absolute value GARCH model, 485
 - additional explanatory variables, 488
 - BEKK model, 491
 - conditional market model, 493
 - conditional nonnormality, 488
 - constant-correlation model, 492
 - estimation, 487, 489
 - excess kurtosis in standardized residuals, 488
 - GARCH(1,1) model, 483, 497
 - GARCH-M model, 494
 - IGARCH model, 484
 - interest rate volatility, 452
 - multivariate, 490
 - persistence, 483
 - QGARCH model, 497
 - single-factor GARCH(1,1) model, 491
 - stationary distribution, 484
 - US stock returns, 488
 - VECH model, 491
- GARCH-in-mean model, 494
- GARCH-M model, 494
- Gaussian kernel, 501
- Generalized Autoregressive Conditionally Heteroskedastic models, 483. *See also* GARCH models
- Generalized Error Distribution, 489
- generalized inverse of a matrix, 244, 245
- Generalized Method of Moments (GMM), 174, 208, 222, 314, 359, 448, 449, 455, 489, 494, 532
 - asymptotic distribution, 533
 - asymptotic variance, 533
 - Newey-West estimator, 535
 - stochastic differential equation, 359
 - weighting matrix, 533
- geometric Brownian motion, 383.
See also Brownian motion
 - risk-neutralized process, 355, 370
- GMM. *See* Generalized Method of Moments
- Goldman-Sosin-Gatto option price formula, 385, 394
- Gordon growth model, 256
 - dynamic Gordon growth model, 263
- government spending in the utility function, 326
- Granger-causality, 91
- Greeks, 353

- habit formation, 327
 - Abel model, 327
 - Campbell-Cochrane model, 330
 - Constantinides model, 330
 - external-habit models, 327
 - internal-habit models, 327
 - difference models, 329
 - ratio models, 327
- Hamilton Markov-switching model, 472
- Hansen's test of overidentifying restrictions, 531
- Hansen-Jagannathan volatility bound, 296. *See also* stochastic discount factor
 - benchmark portfolio, 298
 - Equity Premium Puzzle, 302
 - geometric interpretation, 298
 - lognormal asset pricing model
 - with power utility and, 309
 - market frictions and, 315

- maximum correlation portfolio, 298
- mean-variance efficiency, 298
- nonnegativity constraints, 301
- Heath-Jarrow-Morton model, 457.
See also pricing fixed-income derivative securities
- Heaviside activation function, 513
- hedge portfolios, 322
- hedge ratio, 352, 353. *See also* delta-hedging
- heterogeneous investors, 318, 335
- heteroskedasticity- and
 - autocorrelation-consistent standard errors, 130, 174, 268, 534
- heteroskedasticity-consistent estimators, 54
- hidden layer, 514
- hidden units, 514
- historical volatility, 378
- Ho-Lee model, 456, 464. *See also* pricing fixed-income derivative securities
- holding-period return, 397
- homoskedastic single-factor term-structure model, 429, 452, 457
- Hotelling T^2 statistic, 232
- Hsieh test of nonlinearity, 475
- Hull and White stochastic volatility model, 380
- Hurst-Mandelbrot rescaled range statistic. *See* rescaled range statistic
- hyperbasis functions, 517
- hyperbolic discounting, 334

- idiosyncratic risk, 72, 92, 221, 318
- IGARCH model, 484
- IID. *See* independent and identical distribution
- immunization, 405
- implied volatility, 377
- importance sampling, 388
- income effect, 321. *See also* substitution effect
- income risk, 318
- incomplete markets, 296, 392
- independent and identical distribution (IID), 15, 33, 475
- indexed bonds, 395
- indirect slope estimator, 505
- infinitesimal generator, 360
- information matrix, 191, 358, 538
- information-matrix equality, 539
- input layer, 513
- instrumental variables (IV)
 - regression, 311, 313, 494, 527, 535
- instruments, 447, 528
- integrated GARCH model, 484
- interest rate. *See* coupon bonds, discount bonds, forward rate, interest-rate forecasts, short-term interest rate, term structure of interest rates, yield spread, riskless interest rate
- interest-rate forecasts, 418
- internal rate of return, 401
- interpolation problems, 516
- Intertemporal Capital Asset Pricing Model (ICAPM), 219, 221, 291.
See also Capital Asset Pricing Model, multifactor models
- intertemporal marginal rate of substitution, 294
- intertemporal substitution effect, 331. *See also* elasticity of intertemporal substitution
- investor heterogeneity and, 317
- irregularly sampled data, 363
- ISE estimator, 505
- isoelastic preferences. *See* power utility
- Itô process, 348. *See also* Brownian motion, stochastic differential equation
- Itô's Lemma, 348, 351
- IV regression. *See* instrumental variables regression

- January effect, 100
- Joseph Effect, 59

- kernel regression, 500
 - average derivative estimators, 505
 - convergence property, 501
 - curse of dimensionality, 504
 - optimal bandwidth selection, 502
 - universal approximation property, 515
 - weight function, 500
- Kronecker product, 532
- kurtosis, 16, 19, 81, 480, 488. *See also* returns

- labor income, 318
- Lagrangian function, 184
- latent-variable models, 446
- Law of Iterated Expectations, 24, 255
- lead-lag relations. *See* cross-autocorrelation
- learning networks, 512, 518
 - Black-Scholes formula and, 519
 - limitations, 518
 - multilayer perceptrons, 512
 - projection pursuit regression, 518
 - radial basis functions, 516
- legal liability, 149, 179
- leisure in the utility function, 326
- leverage hypothesis, 497
- likelihood function, 362, 537. *See also* maximum likelihood estimation
- likelihood ratio test, 193
- liquidity effects, 405
- local averaging, 500, 502
- local volatility, 375
- log dividend-price ratio, 264
- log forward rate, 400, 408. *See also* forward rate
- log holding-period return, 398
- log pure expectations hypothesis, 414
- log yield, 397, 399, 408. *See also* yield to maturity
- log-likelihood function, 190, 358, 487. *See also* maximum likelihood estimation
- logistic function, 513
- logistic map, 525

- loglinear approximation, 260, 320, 406
 - accuracy, 262
 - coupon bonds, 406
 - intertemporal budget constraint, 320
- lognormal distribution, 15
- lognormal model of asset pricing, 306
 - Cobb-Douglas utility, 326
 - Epstein-Zin-Weil recursive utility, 319
 - external-habit model, 328
 - power utility, 306
- long-horizon regressions, 267
 - R^2 statistics, 271
 - orthogonality tests, 279
 - variance ratio, 272
 - bias, 273
 - dividend-price ratio, 268
 - dynamic asset-allocation models, 287
 - finite-sample inference, 273
 - investment strategies, 287
- long-horizon returns, 55, 78. *See also* variance ratio
- long-range dependence, 59
- Longstaff-Schwartz model, 438
- lookback options, 391
- loss aversion, 333

- Macaulay's duration, 403
- marked to market, 459
- market efficiency, 20, 30. *See also* Efficient Markets Hypothesis
- market frictions, 314
 - aggregate consumption data, 316
 - Hansen-Jagannathan bounds, 315
- market microstructure, 83. *See also* barrier models, bid-ask spread, bid-ask bounce, nonsynchronous trading, price discreteness, rounding models
- market model, 155
 - conditional, 493
- market portfolio, 155, 181, 323, 495
- Markov chain, 38, 81, 145

- Markov process, 357
- Markov-switching models, 472
- martingale, 28, 256
- martingale convergence theorem, 484
- martingale pricing technique, 354
- maturity, 396
- maximum correlation portfolio, 298.
 - See also* Hansen-Jagannathan volatility bound
- maximum likelihood, 7
- maximum likelihood estimation, 358, 536
 - asymptotic distribution, 538
 - continuous-record asymptotics, 364
 - factor analysis, 234
 - GARCH models, 487
 - information matrix, 538
 - information-matrix equality, 539
 - irregularly sampled data, 363
 - option price, 367
 - quasi-maximum likelihood estimation, 539
 - stochastic differential equation, 357
 - White specification test, 539
- mean reversion, 89
- mean-variance efficient-set
 - mathematics, 184, 243, 298
 - global minimum-variance portfolio, 185, 217
 - Hansen-Jagannathan volatility bound, 298
 - minimum-variance frontier, 185
 - Sharpe ratio, 188
 - tangency portfolio, 188, 196, 218
 - zero-beta portfolio, 182, 185, 218
- m -histories, 112
- mixed distribution, 481
- mixture of normal distributions, 481
- ML estimation. *See* maximum likelihood estimation
- MLP. *See* multilayer perceptron
- modified duration, 405. *See also* duration
- moment conditions, 359
- Monte Carlo simulation methods, 340, 382, 386
 - antithetic variates method, 388
 - comparisons with closed-form solutions, 384
 - computational cost, 386
 - control variate method, 387
 - crude Monte Carlo, 386
 - discretization, 383, 385
 - efficiency, 386
 - importance sampling, 388
 - limitations, 390
 - number-theoretic method, 388
 - path-dependent option pricing, 382
 - stratified sampling, 388
 - variance-reduction techniques, 387
- mortgage-backed securities, 406
- multifactor models, 219, 324. *See also* Arbitrage Pricing Theory, exact factor pricing, Intertemporal Capital Asset Pricing Model, selection of factors
- Black version of the CAPM, 224, 229
- cross-sectional regression approach, 222, 233
- empirical studies, 240
- Epstein-Zin-Weil recursive utility model, 324
- estimation of expected returns, 231
- estimation of risk premia, 231
- factor portfolios spanning the mean-variance frontier, 228
- Generalized Method of Moments, 222
- Hotelling T^2 statistic, 232
- macroeconomic variables as factors, 226
- portfolios as factors, 223
- term-structure models, 440
- multilayer perceptron (MLP), 512
- multiplicative linear congruential generators (MLCG), 525
- multi-point moment conditions, 361

- multiquadrics, 517
- multivariate GARCH models, 490
- multivariate stochastic-volatility models, 493
- Nadaraya-Watson kernel estimator, 500
- network topology, 514
- networks, 512. *See also* learning networks
- neural networks, 512. *See also* learning networks
- Newey-West estimator, 535
- news impact curve, 485
- n -histories, 476
- no-arbitrage condition, 339
- noise traders, 317
- nominal bonds, 442
- nominal stochastic discount factor, 443. *See also* stochastic discount factor
- nonlinear ARMA models, 471
- nonlinear autoregressive models, 471
- nonlinear dynamical systems, 473. *See also* deterministic nonlinear dynamical systems
- nonlinear-in-mean time-series models, 469
- nonlinear-in-variance time-series models, 469
- nonlinear least squares estimation, 515, 518
- nonlinear moving-average models, 469
- nonlinear time-series analysis, 468
- nonlinearity testing. *See* testing for nonlinear structure
- nonparametric estimation, 498, 515
 - universal approximation property, 515
- nonparametric option pricing methods, 340, 392, 510
- nonparametric tests, 172
- nonperiodic cycles, 63
- nonseparability in utility, 326
- nonsynchronous trading, 84, 177
 - empirical findings, 128
 - nontrading process, 145
 - nontrading. *See* nonsynchronous trading
 - normal distribution, 15
 - number-theoretic method, 388
- offer. *See* ask price
- optimal orthogonal portfolio, 243, 245, 248. *See also* exact factor pricing
- option pricing, 349
 - adjusting the Black-Scholes formula for predictability, 375
 - Black-Scholes and Merton option pricing model, 350
 - Black-Scholes formula, 371, 373
 - discrete-time models, 381
 - estimator for σ^2 , 374
 - incomplete markets, 392
 - martingale approach, 354
 - maximum likelihood estimation, 367
 - nonparametric methods, 392
 - path-dependent options, 382
 - risk-neutral pricing method, 382
 - state-price densities, 509
- option sensitivities, 353
- ordered probit model, 122, 136
 - maximum likelihood estimation, 127, 141
- Ornstein-Uhlenbeck process, 360, 371, 434
- orthogonality condition, 528
- orthogonality tests, 276
- O-U process. *See* Ornstein-Uhlenbeck process
- output layer, 513
- overfitting, 498, 523
- par, 401
- parallel processing, 515
- parametric option-pricing model, 340, 356
- path-dependent derivatives, 340
- path-dependent options, 340, 382
- PEH. *See* pure expectations hypothesis

- perfect-foresight stock price, 275
 - observability in finite samples, 278
- perfectly hedged portfolio, 352. *See also* delta-hedging strategy
- performance evaluation, 183
- permanent shock, 65. *See also* unit root process
- persistence in expected stock returns, 265
- persistence in volatility, 483, 492
- peso problem, 310
- piecewise-linear models, 472
- plain vanilla options, 349
- Poincaré section, 475
- polynomial models, 471
- portfolio performance evaluation, 183
- power utility, 305, 434
- PPR. *See* projection pursuit regression
- precautionary savings, 310, 331
- predictability of stock returns, 27, 267
 - Black-Scholes formula, 371, 375
 - Campbell-Cochrane model, 332
 - dividend-price ratio, 268
 - rational bubbles, 260
 - time-varying risk-aversion, 332
- preferred habitats, 418
- price
 - clustering, 109, 145
 - discovery, 107
 - discreteness, 109, 143
 - ex-dividend, 12
 - impact, 107, 143
 - ticks, 108
- price of risk, 432, 495
- price-earnings-ratio effect, 211
- pricing kernel, 294
- principal components, 236
- principle of invariance, 367
- projection pursuit regression, 518
- prospect theory, 333
- psychological models of preferences, 332
- pure expectations hypothesis (PEH), 413. *See also* expectations hypothesis, term structure of interest rates
 - alternatives, 418
 - implications, 417
 - log pure expectations hypothesis, 414
 - preferred habitats, 418
- put option, 349
- QGARCH model, 497
- Q -statistic, 47
- quadratic form, 528
- quadratic GARCH model, 497
- quasi-maximum likelihood estimation, 489, 539
- R^2 statistic, 271
- radial basis functions (RBFs), 516
- rainbow options, 391
- random number generators, 525
- random walk, 27. *See also* long-horizon returns, long-range dependence, Random Walk 1 model, Random Walk 2 model, Random Walk 3 model, technical analysis, unit root processes, variance difference, variance ratio
 - continuous-time limit, 344
 - discrete-time random walk, 341
 - empirical evidence, 65
- Random Walk 1 model, 28, 31, 33
 - R -statistics, 34
 - canonical correlation, 34
 - Cowles-Jones ratio, 35
 - eigenvalues of the covariance matrices, 34
 - Kendall τ correlation test, 34
 - likelihood ratio statistic, 34
 - nonparametric tests, 34
 - runs test, 38
 - semiparametric tests, 34
 - sequences and reversals, 35
 - Spearman rank correlation test, 34
 - Spearman's footrule test, 34
- Random Walk 2 model, 28, 32, 41
 - filter rules, 42

- technical analysis, 43
- Random Walk 3 model, 28, 33, 44.
 - See also* Box-Pierce Q -statistic, variance difference, variance ratio
 - portmanteau statistics, 47
- rank test, 172
- rational bubbles, 258
- RBFs. *See* radial basis functions
- regularization, 516, 522
- replicating portfolio, 353, 380, 391
- replication in Monte Carlo simulation, 383
- representative agent models, 292, 305
- rescaled range statistic, 62
- returns, 254
 - annualized, 10
 - Bernoulli distribution, 18
 - Cauchy distribution, 18
 - compound, 10
 - conditional distribution, 14
 - continuously compounded, 11, 255
 - discount bond, 407
 - discreteness, 110
 - discreteness bias, 116
 - excess, 12
 - excess kurtosis 17, 488, 512
 - forecasting returns, 268
 - gross, 9
 - holding-period return, 398
 - independently and identically distributed (IID), 15
 - joint distribution, 13
 - kurtosis, 16, 19, 81, 480, 488
 - log, 11, 255
 - lognormal distribution, 15
 - net, 9
 - normal distribution, 15
 - simple, 11
 - skewness, 17, 81, 172, 498
 - stable distribution, 17
 - unconditional distribution, 15
 - unexpected stock returns, 264, 284
 - virtual, 85. *See also* nonsynchronous trading
 - riding the yield curve, 416
- risk aversion
 - difference habit-formation models, 329
 - Equity Premium Puzzle, 308, 323, 329
 - first-order, 334
 - loss aversion, 333
 - separating risk aversion from intertemporal substitution, 319
 - time-varying, 330, 335
- risk prices, 325
- risk-neutral option pricing method, 354, 370, 382, 509
- risk-neutral pricing density, 508
- risk-neutrality, 354
- risk-neutralized process, 355
- risk-return tradeoff, 14, 181
- risk-sharing, 318
- riskfree interest rate. *See* riskless interest rate
- riskfree rate puzzle, 310, 329
- riskless interest rate, 182, 306, 309, 319, 328, 331
- rolling standard deviation, 481
- rotational indeterminacy, 234
- rounding models, 114
- R/S statistic. *See* rescaled range statistic
- RW1 model. *See* Random Walk 1 model
- RW2 model. *See* Random Walk 2 model
- RW3 model. *See* Random Walk 3 model
- sample paths, 383
- sample selection biases, 212, 251
- sampling interval, 364
- scale-invariance, 305
- score vector, 537
- SDM. *See* state-dependent models
- security market line, 14
- selection of factors, 233

- cross-sectional generalized least squares (GLS), 235
 - data-snooping, 240, 246, 251
 - factor analysis, 234
 - principal components, 236
 - rotational indeterminacy, 234
 - strict factor structure, 234, 239
 - self-exciting threshold
 - autoregression (SETAR) models, 470
 - self-financing portfolio, 339, 351
 - sequences and reversals, 35
 - serial correlation, 44
 - SETAR models. *See* self-exciting threshold autoregression (SETAR) models
 - Sharpe ratio, 188, 245, 247, 300
 - short-term interest rate, 430, 449. *See also* discount bonds, riskless interest rate
 - GARCH effects on volatility, 452
 - regime-switching, 451
 - shortsales constraints, 315
 - sign test, 172
 - size effect, 211, 496
 - size-sorted portfolio, 70, 75, 129
 - skewness, 17, 81, 172, 498. *See also* returns
 - slope estimators, 505
 - small stocks, 211, 496
 - smoothing, 499, 517. *See also* kernel regression
 - solvency constraint, 315
 - spanning, 380, 391
 - SPD. *See* state-price density
 - specification tests
 - Hansen's test, 531
 - White test, 539
 - spline estimation, 410, 412, 517
 - exponential spline model, 412
 - tax-adjusted spline model, 412
 - spot rate, 414, 417
 - spread. *See* bid-ask spread, yield spread
 - spread-lock interest rate swaps, 391
 - square-root single-factor term-structure model, 435, 454
 - stable distribution, 17
 - standard Brownian motion, 344. *See also* Brownian motion
 - state prices, 295, 507
 - state-dependent models (SDM), 470
 - state-price density (SPD), 507
 - stationary time-series process, 484.
 - See also* unit root process
 - stochastic approximation, 515
 - stochastic differential equation, 346, 356
 - GMM estimation, 359
 - Itô's Lemma, 348
 - maximum likelihood estimation, 357
 - multiplication rules, 347
 - stochastic discount factor, 294, 427, 429. *See also* Euler equation
 - equity premium puzzle, 302
 - habit-formation difference models, 331
 - Hansen-Jagannathan volatility bound, 296
 - nominal, 443
 - nonnegativity, 295, 301
 - power utility, 309
 - state-price density, 508
 - uniqueness, 296
- stochastic trend, 65. *See also* unit root process
- stochastic-volatility models, 379, 489, 493
 - multivariate, 493
- stratified sampling, 388
- strict factor structure, 234, 239
- strike price, 349
- STRIPS, 396
- stroboscopic map, 475
- structural breaks, 472
- Student-*t* distribution, 210, 489
- substitution effect, 321, 331. *See also* elasticity of intertemporal substitution
- supershares, 507
- support and resistance levels, 43
- surplus consumption ratio, 330
- survivorship bias, 311. *See also*

- sample selection biases
- synthetic convertible bonds, 391

- tail thickness, 480
- tax clientele, 405
- tax-adjusted spline model of the term structure, 412
- technical analysis, 43. *See also* Random Walk 2 model
- temporary shock, 65. *See also* unit root process
- tent map, 474, 476, 525
- term premia, 418
- term structure of implied volatility, 463
- term structure of interest rates, 397. *See also* yield curve, term-structure models, expectations hypothesis, pure expectations hypothesis
- cointegration, 419
- forecasting interest rates, 418
- spline estimation, 410
- tax effects, 411
- vector autoregressive (VAR) methods, 422
- term-structure models, 427. *See also* fixed-income derivative securities
- affine-yield models, 428, 441, 445
- Cox, Ingersoll, and Ross model, 436
- cross-sectional restrictions, 452
- fixed-income derivative securities, 455
- Ho-Lee model, 456, 464. *See also* pricing fixed-income derivative securities
- homoskedastic single-factor model, 429, 452, 457
- latent-variable models, 446
- Longstaff-Schwartz model, 438
- square-root single-factor model, 435, 454
- stochastic discount factor, 427
- two-factor model, 438
- Vasicek model, 434
- testing for nonlinear structure

- Brock-Dechert-Scheinkman test, 478
- Hsieh test, 475
- Tsay test, 476
- Theta, 353
- threshold, 472
- threshold autoregression (TAR), 472
- time aggregation, 94, 129
- time inconsistency, 334
- time-nonseparability in the utility function, 327, 329. *See also* habit formation models
- trace operator, 74
- Trades and Quotes (TAQ) database, 107
- training a learning network, 515, 518
- training path, 519
- transactions costs, 315
- transactions data, 107, 136
- transition density function, 358
- Treasury securities, 395
- STRIPS, 396
- Treasury bills, 396
- Treasury notes and bonds, 396
- when-issued market, 399
- zero-coupon yield curve, 397
- trend-stationary process, 65, 372. *See also* unit root process
- trending Ornstein-Uhlenbeck process, 371
- Tsay test of nonlinearity, 476
- two-factor term-structure model, 438
- two-stage least squares (2SLS) estimation, 530
- variance-covariance matrix, 531
- lognormal asset pricing model with power utility, 312

- unit root process, 64, 257
- term structure of interest rates, 419
- volatility process, 484
- volatility tests, 277
- universal approximation property, 515

- VAR methods. *See* vector autoregressive methods

- variance, 15, 17
- variance-bounds tests, 277
- variance difference. *See* variance ratio
- variance inequality, 276
- variance ratio, 48, 68
 - long-horizon regressions, 272
 - Random Walk 1 model, 49
 - Random Walk 3 model, 53
- variance-bounds tests, 277
- variance-reduction techniques, 387
- Vasicek model, 434
- VECH model, 491
- vech operator, 490
- vector autoregressive (VAR)
 - methods
 - multiperiod forecasts, 280
 - present-value relations, 279
 - price volatility, 280
 - return volatility, 284
- Vega, 353
- volatility
 - deterministic, 379
 - historical, 378
 - implied, 377
 - stochastic, 378, 380, 489, 493
- volatility estimation. *See* ARCH models, GARCH models, stochastic-volatility models
- volatility feedback, 497
- volatility smiles, 512
- volatility tests, 275
 - finite-sample considerations, 278
 - Marsh and Merton model, 277
 - orthogonality tests, 276
 - unit roots, 277
 - variance-bounds tests, 277
- Volterra series, 471
- Wald test, 192, 281, 539
- weight function, 500
- weighting matrix
 - GMM estimation, 532, 534
 - IV estimation, 528, 530
- white noise, 346
- White specification test, 539
- Wiener process, 344. *See also* arithmetic Brownian motion
- wild card option, 459
- Wishart distribution, 192
- Wold Representation Theorem, 468
- yield curve, 397, 432, 438, 440. *See also* term structure of interest rates
- yield spread, 397, 418
- yield to maturity, 397, 401
- zero-beta asset, 294
- zero-coupon bonds, 396