Contents

1

The Ball

Footballs often have black and white panels.

We're going to count them.

But why did people go to the trouble of making such complicated balls?

That is what we are going to try to understand in the following chapters.

We have to begin by taking the time to observe the ball in detail.

Even professional illustrators make mistakes.

THE TV STAR

Here's a photo of an old football from around fifty years ago. It is made of leather, shaped by panels sewn together. It was inspired by the ball from the 1970 World Cup in Mexico and has subsequently served as a model for almost all footballs.

Its name is *Telstar* because it was the star of television! During that period World Cup matches were televised for the first time. Perhaps it was also called *Telstar* because that is the name of one of the first telecommunication satellites, which was very famous in the 1960s.

The black panels have five sides, and the white panels have six. How many black panels are there? It's not easy to count them because you only see one side of the ball, and you have to imagine the other side. There is one black panel in the middle and five others around that one. So you see six black panels. How many are there on the other side, the side you don't see at all? Well, there must be the same number. Six panels are hidden. That makes a total of twelve.

The ball contains twelve black panels, each having five sides.

For the white panels, look again at the ball, but put a white panel in the middle.

The middle white panel touches three other white panels, and we can see six more near the edge, which means we see $1+3+6=10$. On the other side, there are ten more.

The ball has twenty white panels, each with six sides.

You might have some doubts about this, because there are white panels we can't completely see, which are a bit on each side. Maybe we counted them twice?

There are twelve black panels. Each one touches five white panels. Does that make $12 \times 5 = 60$ white panels? No, because each white panel touches three black panels. If we count that way, we come up with three times too many white panels. So, we have to divide those 60 by 3. That gives us twenty white panels. The count is good!

But the simplest and most accurate way to find out is to pick up an actual ball and count. Even in geometry, reasoning isn't always enough, and experience is important.

In total, our ball above is formed of $12+20=32$ leather panels that had to be sewn together. Let's count the number of sewn sides. The twelve black panels have $12 \times 5 = 60$ sides, and the twenty white panels have $20 \times 6 = 120$ sides, which comes to a total of 180. Since each seam connects two sides, you have to divide by 2.

That comes to a total of 90 seams.

SOME VERY ASTONISHING MISTAKES

It isn't easy to depict the *Telstar* ball. I asked some kids between the ages of 6 and 12 to draw a ball that was placed in front of them. Some drawings are pretty close, and others are . . . more imaginative.
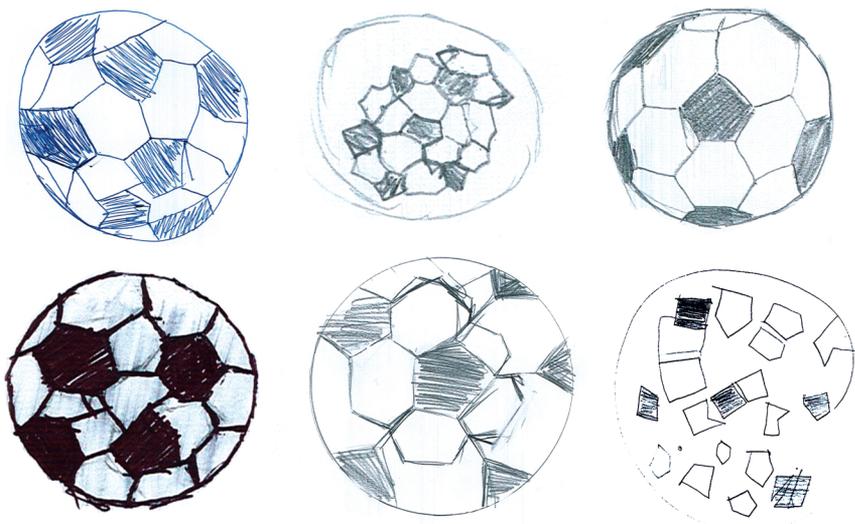
The "football" was invented in England over one hundred fifty years ago. One might therefore assume that the English know their footballs pretty well. And yet . . .

Look at the traffic sign posted in England indicating the way to a football stadium.

Unbelievable! "Shocking!" as the English might say. All the panels, the white and the black, have six sides.

That's impossible, that drawing is a mistake and doesn't depict a true football: the illustrator was wrong.

In 2017, mathematicians protested to the English government and asked that the sign be corrected.

Here's the petition that garnered 22,536 signatures:

> *The football shown on UK street signs (for football grounds) is made entirely of hexagons. But it is mathematically impossible to construct a ball using only hexagons. Changing this to the correct pattern of hexagons and pentagons would help raise public awareness and appreciation of geometry.*

Alas, the English government didn't see it that way. Here is its response:

> *The Department for Transport commissioned research into road user's understanding of traffic signs in 2011. This concluded that respondents "showed a good basic level of understanding as to what different types of sign meant" and recommended that signs should be kept simple.*
>
> *The purpose of a traffic sign is not to raise public appreciation and awareness of geometry which is better dealt with in other ways. If the correct geometry were put onto a sign, it would only be visible close up and not from the distance at which drivers will see the sign. The detail of the geometry would also not be taken in by most drivers who were merely looking at the sign for direction. The higher level of attention needed to understand the geometry could distract a driver's view away from the road for longer than necessary which could therefore increase the risk of an incident.*
>
> *Additionally the public funding required to change every football sign nationally would place an unreasonable financial burden on local authorities. The Department could not justify the spending needed as an exercise to increase public awareness and appreciation of geometry.*
>
> *For the reasons given, we will not be changing the football symbol used on a traffic sign.*
>
> *—Department for Transport*

Don't be misled by the drawing above which appears to show a football, all of whose panels have six sides. By attempting to complete the figure on the back side, you would encounter some problems.

The Champions League ball is also very important. It is attractive, with its twelve five-pointed stars and twenty white six-sided panels. Almost like *Telstar*.
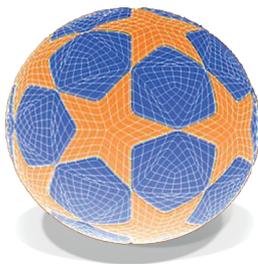
But the Champions League logo isn't correct.

There are indeed five-pointed stars, but on the actual ball, there are three stars that surround a white six-sided panel. On the logo, some stars are correctly grouped three-by-three, but others are grouped four-by-four, which creates eight-sided panels, not the correct six. The design is wrong!

Even professional illustrators make mistakes when they depict a football, even when they have it right before their eyes (or better yet under their feet).

Here's the logo for Qatar's candidacy for the 2022 World Cup:

In the middle, all is well: a five-sided polygon surrounded by five other polygons, like a *Telstar* that one views with a black panel in the center. Around it, there are indeed the requisite thirty-two polygons, arranged in eight spirals, each containing four polygons. But all these polygons have five sides, and we know that there are only twelve of them that have five sides, whereas the others have six sides.

Clearly, no one really observes soccer balls, not even the people drawing them.

A SQUARE BALL?

Contemporary artists often use objects that are familiar and transform them into works of art. It is precisely because we don't really look at them carefully that artists want to show us just how significant they are for us.

Fabrice Hyber created hundreds of objects that he calls *Prototypes d'objets en fonctionnement* (Prototypes of working objects), or *POF*. His square football is POF 65, and I'm told the artist organizes actual football matches using such balls. Players probably have a lot of fun with them. After all, rugby is played with oval balls, so why not square balls?

However, there are two things I would like to point out to Fabrice Hyber.

First, his ball isn't square, it's cubic. You draw a square on a sheet of paper, but in space you talk about a cube.

Second, it would be closer to an actual football if the black panels had five sides, not six.