

Contents

Introduction to the Scientific Writings	ix
1 A Theory of the Foundations of Thermodynamics (1903)	1
2 On a Heuristic Point of View Concerning the Production and Transformation of Light (1905)	18
3 On the Motion of Small Particles Suspended in Liquids at Rest Required by the Molecular-Kinetic Theory of Heat (1905)	33
4 On the Electrodynamics of Moving Bodies (1905)	43
5 Does the Inertia of a Body Depend upon Its Energy Content? (1905)	72
6 On the Theory of Light Production and Light Absorption (1906)	75
7 Planck's Theory of Radiation and the Theory of Specific Heat (1907)	82
8 On the Relativity Principle and the Conclusions Drawn from It (1907)	93
9 On the Present Status of the Radiation Problem (1909)	147
10 On the Development of Our Views Concerning the Nature and Constitution of Radiation (1909)	165
11 On the Influence of Gravitation on the Propagation of Light (1911)	181
12 Thermodynamic Proof of the Law of Photochemical Equivalence (1912)	191
13 Experimental Proof of Ampère's Molecular Currents (1915) with W. de Haas	198
14 Explanation of the Perihelion Motion of Mercury from the General Theory of Relativity (1915)	202

15	The Foundation of the General Theory of Relativity (1916)	212
16	Approximative Integration of the Field Equations of Gravitation (1916)	265
17	On the Quantum Theory of Radiation (1916)	275
18	Hamilton's Principle and the General Theory of Relativity (1916)	289
19	Cosmological Considerations in the General Theory of Relativity (1917)	296
20	On Gravitational Waves (1918)	307
21	On the Foundations of the General Theory of Relativity (1918)	323
22	The Law of Energy Conservation in the General Theory of Relativity (1918)	326
23	Does Field Theory Provide Possibilities for the Solution of the Quantum Problem? (1923)	339
24	Quantum Theory of the Monatomic Ideal Gas (1924)	346
25	Quantum Theory of the Monatomic Ideal Gas. Second Paper (1925)	354
26	Unified Field Theory of Gravitation and Electricity (1925)	369
27	Patent of Refrigerator (1927) with L. Szilard	376
28	Unified Field Theory Based on Riemannian Metric and Distant Parallelism (1930)	384
29	On the Cosmological Problem of the General Theory of Relativity (1931)	398
30	On the Relation between the Expansion and the Mean Density of the Universe (1932) with W. de Sitter	402
31	Elementary Derivation of the Equivalence of Mass and Energy (1935)	404

32 Can Quantum-Mechanical Description of Physical Reality Be Considered Complete? (1935) with B. Podolsky and N. Rosen	412
33 The Particle Problem in the General Theory of Relativity (1935) with N. Rosen	420
34 Lens-like Action of a Star by the Deflection of Light in a Gravitational Field (1936)	431
35 On a Generalization of Kaluza's Theory of Electricity (1938) with P. Bergmann	433
36 The Gravitational Equations and the Problem of Motion [Excerpt] (1937) with L. Infeld and B. Hoffmann	455
37 On the Non-Existence of Regular Stationary Solutions of Relativistic Field Equations (1943) with W. Pauli	486
38 Relativistic Theory of the Non-symmetric Field (1955)	494
Bibliography	521
Index	527

1. A Theory of the Foundations of Thermodynamics (1903)

Annalen der Physik 11 (1903): 170–187. [Collected Papers Vol. 2, Doc. 4].

Unaware of Josiah Willard Gibbs's writings at the time, Einstein here gives an alternative view on the foundations of statistical mechanics, supplementing Gibbs's better-known and more influential work and effectively introducing the concept of a canonical ensemble himself.

In a recently published paper,^[a] I showed that the laws of thermal equilibrium and the concept of entropy can be derived with the help of the kinetic theory of heat. The question that then arises naturally is whether the kinetic theory is really necessary for the derivation of the above foundations of the theory of heat, or whether, perhaps, assumptions of a more general nature may suffice. In this article, it shall be demonstrated that the latter is the case, and it shall be shown by what kind of reasoning one can reach the goal.

§1. On a general mathematical representation of the processes in isolated physical systems

Let the state of some physical system that we consider be uniquely determined by very many (n) scalar quantities $p_1, p_2 \dots p_n$, which we call state variables. The change of the system in a time element dt is then determined by the changes $dp_1, dp_2 \dots dp_n$ that the state variables undergo during that time element.

Let the system be isolated, i.e., the system considered should not interact with other systems. It is then clear that the state of the system at a given instant of time uniquely determines the change of the system in the next time element dt , i.e., the quantities $dp_1, dp_2 \dots dp_n$. This statement is equivalent to a system of equations of the form

$$\frac{dp_i}{dt} = \varphi_i(p_1 \dots p_n) \quad (i = 1 \dots i = n), \quad (1)$$

where the φ 's are unique functions of their arguments.

In general, for such a system of linear differential equations, there does not exist an integral of the form

$$\psi(p_1 \dots p_n) = \text{const.},$$

which does not contain the time explicitly. However, for a system of equations that represents the changes of a physical system closed to the outside, we must assume that at least one such equation exists, namely the energy equation

$$E(p_1 \dots p_n) = \text{const.}$$

At the same time, we assume that no further integral of this kind that is independent of the above equation is present.

§2. On the stationary distribution of state of infinitely many isolated physical systems of almost equal energies

Experience shows that, after a certain time, an isolated system assumes a state in which no perceptible quantity of the system undergoes any further changes with time; we call this state the stationary state. Hence, it will obviously be necessary for the functions φ_i to fulfill a certain condition so that equations (1) may represent such a physical system.

If we now assume that a perceptible quantity is always represented by a time average of a certain function of the state variables $p_1 \dots p_n$ and that these state variables $p_1 \dots p_n$ keep on assuming the same systems of values with always the same unchanging frequency, then it necessarily follows from this condition, which we shall elevate to a postulate, that the averages of all functions of the quantities $p_1 \dots p_n$ must be constant; hence, in accordance with the above, all perceptible quantities must also be constant.

We will specify this postulate precisely. Starting at an arbitrary point of time and throughout time T , we consider a physical system that is represented by equations (1) and has the energy E . If we imagine having chosen some arbitrary region Γ of the state variables $p_1 \dots p_n$, then, at a given instant of time T , the values of the variables $p_1 \dots p_n$ will lie within the chosen region Γ or outside it; hence, during a fraction of the time T , which we shall call τ , they will lie in the chosen region Γ . Our condition then reads as follows: If $p_1 \dots p_n$ are state variables of a physical system, i.e., of a system that assumes a stationary state, then, for each region Γ , the quantity $\frac{\tau}{T}$ has a definite limiting value for $T = \infty$.

For any infinitesimally small region, this limiting value is infinitesimally small.

The following consideration can be based on this postulate. Let there be very many (N) independent physical systems, all of which are represented by the same system of equations (1). We select an arbitrary instant t and inquire after the distribution of the possible states among these N systems, assuming that the energy E of all systems lies between E^* and the infinitesimally close value $E^* + \delta E^*$. From the postulate introduced above, it follows immediately that the probability that the state variables of a system randomly selected from among N systems will lie within the region Γ at time t has the value

$$\lim_{T \rightarrow \infty} \frac{\tau}{T} = \text{const.}$$

The number of systems whose state variables lie within the region Γ at time t is thus

$$N \cdot \lim_{T \rightarrow \infty} \frac{\tau}{T},$$

i.e., a quantity independent of time. If g denotes a region of the coordinates $p_1 \dots p_n$ that is infinitesimally small in all variables, then the number of systems whose state variables fill up the arbitrarily chosen infinitesimally small region g at an arbitrary time will be

$$dN = \varepsilon(p_1 \dots p_n) \int_g dp_1 \dots dp_n. \quad (2)$$

The function ε is obtained by expressing, in symbols, the condition that the distribution of states expressed by equation (2) is a stationary one. Specifically, the region g shall be chosen such that p_1 shall lie between the definite values p_1 and $p_1 + dp_1$, p_2 between p_2 and $p_2 + dp_2$, $\dots$ p_n between p_n and $p_n + dp_n$; then we have, at the time t ,

$$dN_t = \varepsilon(p_1 \dots p_n) \cdot dp_1 \cdot dp_2 \dots dp_n,$$

where the subscript of dN denotes the time. Taking into account equation (1), one obtains furthermore, at time $t + dt$ and the same region of the state variables,

$$dN_{t+dt} = dN_t - \sum_{v=1}^{v=n} \frac{\partial(\varepsilon \varphi_v)}{\partial p_v} \cdot dp_1 \dots dp_n \cdot dt.$$

However, since $dN_t = dN_{t+dt}$, because the distribution is stationary, we have

$$\sum \frac{\partial(\varepsilon \varphi_v)}{\partial p_v} = 0.$$

This yields

$$-\sum \frac{\partial \varphi_v}{\partial p_v} = \sum \frac{\partial(\log \varepsilon)}{\partial p_v} \cdot \varphi_v = \sum \frac{\partial(\log \varepsilon)}{\partial p_v} \cdot \frac{dp_v}{dt} = \frac{d(\log \varepsilon)}{dt},$$

where $\frac{d(\log \varepsilon)}{dt}$ denotes the change of the function $\log \varepsilon$ with respect to time for an individual system, taking into account the changes with time of the quantities p_v . One obtains further

$$\varepsilon = e^{-\int dt \sum_{v=1}^{v=n} \frac{\partial \varphi_v}{\partial p_v} + \psi(E)} = e^{-m + \psi(E)}.$$

The unknown function ψ is the time-independent integration constant which may depend on the variables $p_1 \dots p_n$, but can contain them, according to the assumptions made in §1, only in the combination in which they appear in the energy E .

However, since $\psi(E) = \psi(E^*) = \text{const.}$ for all N systems considered, the expression for ε reduces, in our case, to

$$\varepsilon = \text{const. } e^{-\int dt \sum_{v=1}^{v=n} \frac{\partial \varphi_v}{\partial p_v}} = \text{const. } e^{-m}.$$

According to the above, we now have

$$dN = \text{const. } e^{-m} \int_g dp_1 \dots dp_n.$$

For the sake of simplicity, we now introduce new state variables for the system considered; they shall be denoted by π_v . We then have

$$dN = \text{const. } \frac{e^{-m}}{\frac{D(\pi_1 \dots \pi_n)}{D(p_1 \dots p_n)}} \int_g d\pi_1 \dots d\pi_n,$$

where the symbol D denotes the functional determinant.—We now want to choose the new coordinates such that

$$e^{-m} = \frac{D(\pi_1 \dots \pi_n)}{D(p_1 \dots p_n)}.$$

This equation can be satisfied in infinitely many ways, e.g., by setting

$$\begin{aligned}\pi_2 &= p_2 \\ \pi_3 &= p_3 \\ \dots & \quad \pi_1 = \int e^{-m} \cdot dp_1. \\ \pi_n &= p_n\end{aligned}$$

Using the new variables, we thus obtain

$$dN = \text{const.} \int d\pi_1 \dots d\pi_n.$$

Henceforth, we will always suppose that such variables have been introduced.

§3. On the distribution of states of a system in contact with a system of relatively infinitely large energy

We now assume that each of the N isolated systems is composed of two partial systems, Σ and σ , in interaction. Let the state of the partial system Σ be determined by the values of the variables $\Pi_1 \dots \Pi_\lambda$, and that of the system σ by the values of the variables $\pi_1 \dots \pi_l$. Further, let the energy E —which, for each system, shall lie between the values E^* and δE^* , i.e., shall equal E^* up to the infinitesimally small—be composed of two terms, of which the first, H , shall be determined only by the values of the state variables of Σ , and the second η only by the state variables of σ so that, except for the relatively infinitesimally small, one has

$$E = H + \eta.$$

Two systems in interaction that satisfy this condition will be called two systems in contact. We also assume that η is infinitesimally small compared with H .

For the number dN_1 of the N -systems whose state variables $\Pi_1 \dots \Pi_\lambda$ and $\pi_1 \dots \pi_l$ lie between Π_1 and $\Pi_1 + d\Pi_1$, Π_2 and $\Pi_2 + d\Pi_2$, $\dots$ Π_λ and $\Pi_\lambda + d\Pi_\lambda$, and π_1 and $\pi_1 + d\pi_1$, π_2 and $\pi_2 + d\pi_2$, $\dots$ π_l and $\pi_l + d\pi_l$, we get the expression

$$dN_1 = C \cdot d\Pi_1 \dots d\Pi_\lambda \cdot d\pi_1 \dots d\pi_l,$$

where C can be a function of $E = H + \eta$.

However, since, according to the above assumption, the energy of each of the systems considered up to the infinitesimally small has the

value E^* , we can replace C by $\text{const. } e^{-2hE^*} = \text{const. } e^{-2h(H+\eta)}$ without causing any changes in the result, where h is a constant still to be defined precisely. Hence, the expression for dN_1 becomes

$$dN_1 = \text{const. } e^{-2h(H+\eta)} \cdot d\Pi_1 \dots d\Pi_\lambda \cdot d\pi_1 \dots d\pi_l.$$

The number of systems whose state variables π lie between the indicated limits while the values of the variables Π are not subjected to any restrictive condition may thus be represented in the form

$$dN_2 = \text{const. } e^{-2h\eta} \cdot d\pi_1 \dots d\pi_l \int e^{-2hH} d\Pi_1 \dots d\Pi_\lambda$$

where the integral is to be extended over all values of Π to which correspond values of the energy H lying between $E^* - \eta$ and $E^* + \delta E^* - \eta$. Had the integration been carried out, we would have found the distribution of the state of the systems σ . This is, in fact, possible.

We put

$$\int e^{-2hH} \cdot d\Pi_1 \dots d\Pi_\lambda = \chi(E),$$

where the integral on the left-hand side is to be extended over all values of the variables for which H lies between the definite values E and $E + \delta E^*$. The integral that appears in the expression dN_2 then assumes the form

$$\chi(E^* - \eta),$$

or, since η is infinitesimally small compared with E^* ,

$$\chi(E^*) - \chi'(E^*) \cdot \eta.$$

Thus, if h can be chosen such that $\chi'(E^*) = 0$, the integral reduces to a quantity that is independent of the state of σ .

It is possible to put, up to the infinitesimally small,

$$\chi(E) = e^{-2hE} \int d\Pi_1 \dots d\Pi_\lambda = e^{-2hE} \cdot \omega(E),$$

where the integration limits are the same as above and where ω denotes a new function of E .

The condition for h now assumes the form

$$\chi'(E^*) = e^{-2hE^*} \cdot \{\omega'(E^*) - 2h\omega(E^*)\} = 0;$$

consequently:

$$h = \frac{1}{2} \frac{\omega'(E^*)}{\omega(E^*)}.$$

If h is chosen in this way, the expression for dN_2 will assume the form

$$dN_2 = \text{const. } e^{-2h\eta} d\pi_1 \dots d\pi_l. \quad (3)$$

With suitable choice of the constant, this expression represents the probability that the state variables of a system in contact with another system of relatively infinitely large energy will lie within the indicated limits. The quantity h depends only on the state of the above system Σ of relatively infinitely large energy.

§4. On absolute temperature and thermal equilibrium

Thus, the state of the system σ depends only on the quantity h and the latter only on the state of the system Σ . We call the quantity $\frac{1}{4}h\kappa = T$ the absolute temperature of the system Σ , where κ denotes a universal constant.

If we call the system σ “thermometer,” then we can immediately advance the following propositions:

1. The state of the thermometer depends only on the absolute temperature of the system Σ and not on the kind of contact of the systems Σ and σ .
2. If, in case of contact, two systems Σ_1 and Σ_2 impart the same state to a thermometer σ , then they have the same absolute temperature and will also impart the same state to another thermometer σ' in case of contact.

Further, suppose two systems Σ_1 and Σ_2 are in contact and Σ_1 is also in contact with a thermometer σ . The distribution of states of σ depends then only on the energy of the system $(\Sigma_1 + \Sigma_2)$, i.e., on the quantity $h_{1,2}$. If the interaction between Σ_1 and Σ_2 is imagined to decrease infinitely slowly, this does not change the expression for the energy $H_{1,2}$ of the system $(\Sigma_1 + \Sigma_2)$, which can be readily seen from our definition of contact and the expression for the quantity h that we formulated in the last section. Finally, if the interaction had ceased completely, the distribution of states of σ , which does not change during the separation of Σ_1 and Σ_2 , will now depend on Σ_1 , i.e., on the quantity h_1 ,

where the index denotes association with the system Σ_1 alone. Hence, we have

$$h_1 = h_{12}.$$

By an analogous line of argument, one could have obtained

$$h_2 = h_{12};$$

hence

$$h_1 = h_2,$$

or, in words: If one separates two systems Σ_1 and Σ_2 in contact, which form an isolated system ($\Sigma_1 + \Sigma_2$) of absolute temperature T , then the now-isolated systems Σ_1 and Σ_2 will have the same temperature after separation. We imagine a given system in contact with an ideal gas. This gas shall be completely describable in terms of the kinetic theory of gases. As the system σ , we consider a single monoatomic gas molecule of mass μ , whose state shall be completely determined by its orthogonal coordinates x, y, z and the velocities ξ, η, ζ . In accordance with §3, we obtain, for the probability that the state variables of this molecule lie between the limits x and $x + dx, \dots \zeta$ and $\zeta + d\zeta$, the well-known Maxwellian expression

$$dW = \text{const. } e^{-h\mu(\xi^2 + \eta^2 + \zeta^2)} \cdot dx \dots d\zeta.$$

By integration, one obtains from this, for the mean kinetic energy of this molecule,

$$\overline{\frac{\mu}{2}(\xi^2 + \eta^2 + \zeta^2)} = \frac{1}{4h}.$$

However, the kinetic theory of gases teaches that, at a constant volume of the gas, this quantity is proportional to the pressure exerted by the gas. The latter is, by definition, proportional to the quantity designated in physics as absolute temperature. Thus, the quantity we designated as absolute temperature is nothing else but the temperature of a system measured by the gas thermometer.

§5. On infinitely slow processes

Until now, we have only considered systems that are in a stationary state. Now, we are also going to investigate changes of stationary states, though only those that proceed so slowly that the distribution of states

existing at an arbitrary instant differs only infinitesimally from the stationary distribution; or, more precisely, that, up to the infinitesimally small, the probability that the state variables lie in a certain region G can be represented, at any moment, by the formula found above. We call such a change an infinitesimally slow process.

If the functions φ_v (equation (1)) and the energy E of a system are specified, then, according to the above, its stationary state distribution is also specified. An infinitely slow process will thus be specified either by a changing E , by the functions φ_v containing the time explicitly, or by both circumstances simultaneously, but in such a way that the corresponding differential quotients, with respect to time, are very small.

We assumed that the state variables of an isolated system change according to equations (1). However, conversely, if there exists a system of equations (1), according to which the state variables of a system are changing, this system does not always have to be an isolated one. For it can happen that a system under consideration is influenced by other systems in such a way that this influence depends only on such functions of the variable coordinates of the influencing systems, which do not change when the distribution of states of the influencing system is constant. In this case, the change of the coordinates p_v of the system considered can also be represented by a system having the form of equations (1). However, the functions φ_v will then depend not only on the physical nature of the system in question, but also on certain constants that are defined through the influencing systems and their distributions of states. This kind of influence on the system under consideration we call adiabatic. It is easy to see that as long as the distributions of state of the adiabatically influencing systems do not change, there exists an energy equation for the equations (1) in this case as well. If the states of the adiabatically influencing systems do change, then the functions φ_v of the systems considered change explicitly with time, with equations (1) maintaining their validity at all times. Such a change of the distribution of states of the system under consideration we call an adiabatic one.

We now consider a second kind of changes of the state of a system Σ . Consider a system that can be influenced adiabatically. We assume that, at time $t = 0$, the system Σ enters into such an interaction with a system P of a different temperature that we called “in contact” above,

and we remove the system P after the time necessary for the equalization of the temperatures of Σ and P . The energy of Σ has then changed. The equations (1) of Σ are invalid during the process but valid before and after it, while the functions φ_ν are the same before and after the process. Such a process we call “isopycnic,” and the energy supplied to Σ , “heat supplied.”

It is evident that, up to the infinitesimally small, it is possible to construct each infinitely slow process from a succession of infinitesimally small adiabatic and isopycnic processes, so that in order to get a general overview, we have to study the latter ones only.

§6. On the concept of entropy

Let there be a physical system whose instantaneous state shall be completely determined by the values of the state variables $p_1 \cdots p_n$. Let this system undergo a small, infinitely slow process, in which the systems that influence this system adiabatically experience an infinitesimally small change of state; energy is being supplied to the system considered by systems in contact. We take account of the adiabatically influencing systems by stipulating that in addition to the $p_1 \cdots p_n$, the energy E of the system considered shall also depend on some parameters $\lambda_1, \lambda_2 \cdots$, whose values shall be determined by the distributions of states of the systems that influence adiabatically the system considered. In purely adiabatic processes, there holds, at any instant, a system of equations (1) whose functions φ_ν depend not only on the coordinates p_ν , but also on the slowly changing quantities λ ; for adiabatic processes, too, there will hold, at any instant, the energy equation, whose form is

$$\sum \frac{\partial E}{\partial p_\nu} \varphi_\nu = 0.$$

We now investigate the energy increase of the system during an arbitrary infinitesimally small, infinitely slow process.

For each element dt of the process, we have

$$dE = \sum \frac{\partial E}{\partial \lambda} d\lambda + \sum \frac{\partial E}{\partial p_\nu} dp_\nu. \quad (4)$$

For an infinitesimally small isopycnic process, all $d\lambda$ vanish in each time element, and thus the first term of the right-hand side vanishes too.

However, since, according to the previous section, in an isopycnic process, dE is to be considered as heat supplied for such a process, the heat supplied dQ is represented by the expression

$$dQ = \sum \frac{\partial E}{\partial p_v} dp_v.$$

However, for an adiabatic process, during which equations (1) are always valid, we have, according to the energy equation,

$$\sum \frac{\partial E}{\partial p_v} dp_v = \sum \frac{\partial E}{\partial p_v} \varphi_v dt = 0.$$

On the other hand, according to the previous section, $dQ = 0$ for an adiabatic process, so that one can put

$$dQ = \sum \frac{\partial E}{\partial p_v} dp_v$$

for an adiabatic process as well. Hence, this equation must be considered as valid for any arbitrary process during each time element. Thus, equation (4) becomes

$$dE = \sum \frac{\partial E}{\partial \lambda} d\lambda + dQ. \quad (4')$$

This expression represents the energy change of the system occurring during the whole infinitesimally small process at changed values of $d\lambda$ and dQ as well.

At the beginning and the end of the process, the distribution of states of the system considered is stationary, and when the system is in contact with a system of relatively infinitely large energy before and after the process, this assumption having formal significance only, this distribution is defined by the equation having the form

$$\begin{aligned} dW &= \text{const. } e^{-2hE} \cdot dp_1 \dots dp_n \\ &= e^{c-2hE} \cdot dp_1 \dots dp_n, \end{aligned}$$

where dW denotes the probability that the values of the system's state variables lie within the limits indicated at any arbitrarily chosen moment. The constant c is defined by the equation

$$\int e^{c-2hE} \cdot dp_1 \dots dp_n = 1, \quad (5)$$

where the integration has to be extended over all values of the variables.

Specifically, if equation (5) holds before the process under consideration, then afterwards we have

$$\int e^{(c+dc)-2(h+dh)\left(E+\sum \frac{\partial E}{\partial \lambda} d\lambda\right)} \cdot dp_1 \dots dp_n = 1, \quad (5')$$

and the two last equations yield

$$\int \left(dc - 2Edh - 2h \sum \frac{\partial E}{\partial \lambda} \cdot d\lambda \right) \cdot e^{c-2hE} \cdot dp_1 \dots dp_n = 0,$$

or, since the expression in parentheses can be taken as a constant during integration because the system's energy E never differs markedly from a fixed average value before and after the process, and taking into account equation (5),

$$dc - 2Edh - 2h \sum \frac{\partial E}{\partial \lambda} d\lambda = 0. \quad (5'')$$

However, according to equation (4'), we have

$$-2hdE + 2h \sum \frac{\partial E}{\partial \lambda} d\lambda + 2hdQ = 0,$$

and, by adding these two equations, one obtains

$$2h \cdot dQ = d(2hE - c),$$

or, since $\frac{1}{4h} = \kappa \cdot T$,

$$\frac{dQ}{T} = d \left(\frac{E}{T} - 2\kappa c \right) = dS.$$

This equation states that dQ/T is a total differential of a quantity that we will call the entropy S of the system. Taking into account equation (5), one obtains

$$S = 2\kappa(2hE - c) = \frac{E}{T} + 2\kappa \log \int e^{-2hE} dp_1 \dots dp_n,$$

where the integration has to be extended over all values of the variables.

§7. On the probability of distributions of states

In order to derive the second law in its most general form, we have to investigate the probability of distributions of states. We consider a very large number (N) of isolated systems, all of which can be represented

by the same system of equations (1) and whose energies coincide up to the infinitesimally small. The distribution of states of these N systems can then be represented by an equation of the form

$$dN = \varepsilon(p_1 \dots p_n, t) dp_1 \dots dp_n, \quad (2')$$

where in general ε depends explicitly on the state variables $p_1 \dots p_n$ and also on time. Here, the function ε completely characterizes the distribution of states.

It follows from §2 that when the distribution of states is constant—which, according to our assumptions, is always the case at very large values of t —we must have $\varepsilon = \text{const.}$, so that, for a stationary distribution of states, we will have

$$dN = \text{const. } dp_1 \dots dp_n.$$

From this, it follows immediately that the expression for the probability dW , for the values of the state variables of a system randomly chosen from among the N systems to lie in the infinitesimally small region g of the state variables located within the assumed energy limits, is given by

$$dW = \text{const. } \int_g dp_1 \dots dp_n.$$

This proposition can also be formulated as follows: If the whole pertinent region of state variables that is determined by the assumed energy limits is divided into l partial regions $g_1, g_2 \dots g_l$ such that

$$\int_{g_1} = \int_{g_2} = \dots = \int_{g_l},$$

and if one denotes by W_1, W_2 , etc., the probabilities that the values of the state variables of the arbitrarily chosen system lie within $g_1, g_2 \dots$ at a certain instant, then

$$W_1 = W_2 = \dots = W_l = \frac{1}{l}.$$

The probability that, at a given moment, the system considered will belong to a specific region from among these $g_1 \dots g_l$ regions is thus just as great as the probability that it will belong to any other of these regions.

The probability that, at a randomly chosen time, ε_1 of N systems considered will belong to the region g_1 , ε_2 to region g_2 , and $\dots \varepsilon_l$ to

region g_l is hence

$$W = \left(\frac{1}{l}\right)^N \frac{N!}{\varepsilon_1! \varepsilon_2! \dots \varepsilon_n!},$$

or also, since $\varepsilon_1, \varepsilon_2 \dots \varepsilon_n$ are to be thought of as very large numbers:

$$\log W = \text{const.} - \sum_{\varepsilon=1}^{\varepsilon=l} \varepsilon \log \varepsilon.$$

If l is sufficiently large, one can put, without noticeable error,

$$\log W = \text{const.} - \int \varepsilon \log \varepsilon dp_1 \dots dp_n.$$

In this equation, W denotes the probability that a given distribution of states, which is expressed by the numbers $\varepsilon_1, \varepsilon_2 \dots \varepsilon_l$ or else, by a specific function ε of $p_1 \dots p_n$ according to equation (2'), prevails at a given time.

If, in this equation, ε were constant, i.e., independent of the p_v 's within the energy limits considered, then the distribution of states considered would be stationary and, as can easily be proved, the expression for the probability W of the distribution of states would be a maximum. If ε depends on the values of the p_v 's, then it can be shown that the expression for $\log W$ for the distribution of states considered does not have an extremum, i.e., that there exist distributions of states differing infinitesimally from the considered one for which W is larger.

If we follow the N systems considered for an arbitrary time interval, the distribution of states, and thus also W , will continually change with time, and we will have to assume that always more probable distributions of states will follow upon improbable ones, i.e., that W increases until the distribution of states has become constant and W a maximum.

It will be shown in the following sections that the second law of thermodynamics can be deduced from this proposition.

First of all, we have

$$- \int \varepsilon' \log \varepsilon' dp_1 \dots dp_n \geq - \int \varepsilon \log \varepsilon dp_1 \dots dp_n,$$

where the function ε determines the distribution of states of the N systems at a certain time t , the function ε' determines the distribution of states at a certain later time t' , and the integration on both sides is to be

extended over all values of the variables. Further, if the quantities $\log \varepsilon$ and $\log \varepsilon'$ of the individual systems from among the N systems do not differ markedly from each other, then, since

$$\int \varepsilon dp_1 \dots dp_n = \int \varepsilon' dp_1 \dots dp_n = N,$$

the last equation becomes

$$-\log \varepsilon' \geq -\log \varepsilon. \quad (6)$$

§8. Application of the results obtained to a particular case

We consider a finite number of physical systems $\sigma_1, \sigma_2 \dots$ that together form an isolated system, which we shall call a total system. The systems $\sigma_1, \sigma_2 \dots$ shall not interact markedly with each other thermally, but they might affect each other adiabatically. The distribution of states of each of the systems $\sigma_1, \sigma_2 \dots$, which we shall call partial systems, shall be stationary up to the infinitesimally small. The absolute temperatures of the partial systems may be arbitrary and different from each other.

The distribution of states of the system σ_1 will not be markedly different from the distribution of states that would hold if σ_1 were in contact with a physical system of the same temperature. We can therefore represent its distribution of states by the equation

$$dw_1 = e^{c_{(1)} - 2h_{(1)}E_{(1)}} \int_g dp_1^{(1)} \dots dp_{(n)}^{(1)},$$

where the indices (1) indicate affiliation with the partial system σ_1 .

Analogous equations hold for the other partial systems. Since the instantaneous values of the state variables of the individual partial systems are independent of those of the other systems, we obtain, for the distribution of states of the total system, an equation of the form

$$dw = dw_1 \cdot dw_2 \dots = e^{\sum c_v - 2h_v E_v} \int_g dp_1 \dots dp_n, \quad (7)$$

where the summation is to be extended over all systems, and the integration over the arbitrary region g , which is infinitesimally small in all the variables of the total system.

We now assume that, after some time, the partial systems $\sigma_1, \sigma_2 \dots$ enter into some arbitrary interaction with each other, but that during that process, the total system always remains an isolated one. After the lapse of a certain time, there shall arise a state of the total system in which the partial systems $\sigma_1, \sigma_2 \dots$ do not affect each other thermally and, up to the infinitesimally small, exist in a stationary state.

Then, an equation completely analogous to that holding before the process will hold for the distribution of states of the total system:

$$dw' = dw'_1 \cdot dw'_2 \dots = e^{\sum c'_v - 2h'_v E'_v} \int_g dp \dots dp_n. \quad (7')$$

We now consider N such total systems. Up to the infinitesimally small, equation (7) shall hold for each of these systems at time t , and equation (7') at time t' . Then, the distribution of states of the N total systems considered at times t and t' will be given by the equations

$$\begin{aligned} dN_t &= N \cdot e^{\sum (c_v - 2h_v E_v)} \cdot dp_1 \dots dp_n \\ dN_{t'} &= N \cdot e^{\sum (c'_v - 2h'_v E'_v)} \cdot dp_1 \dots dp_n. \end{aligned}$$

To these two distributions of states, we now apply the results of the previous section. Neither the

$$\varepsilon = N \cdot e^{\sum (c_v - 2h_v E_v)},$$

nor the

$$\varepsilon' = N \cdot e^{\sum (c'_v - 2h'_v E'_v)}$$

for the individual systems among the N systems are here markedly different so that we can apply equation (6), which yields

$$\sum (2h'E' - c') \geq \sum (2hE - c),$$

or, noting that, according to §6, the quantities $2h_1 E_1 - c_1, 2h_2 E_2 - c_2, \dots$ are identical with the entropies $S_1, S_2 \dots$ of the partial systems up to a universal constant,

$$S'_1 + S'_2 + \dots \geq S_1 + S_2 + \dots, \quad (8)$$

i.e., the sum of the entropies of the partial systems of an isolated system, after some arbitrary process, is equal to or larger than the sum of the entropies of the partial systems before the process.

§9. Derivation of the second law

Let there be an isolated total system whose partial systems shall be called W , M , and $\Sigma_1, \Sigma_2 \dots$. Let the system W , which we shall call heat reservoir, have an energy that is infinitely large compared with the system M (engine). Similarly, the energy of the systems $\Sigma_1, \Sigma_2 \dots$, which interact adiabatically with each other, shall be infinitely large compared with that of M . We assume that all the partial systems $M, W, \Sigma_1, \Sigma_2 \dots$ are in a stationary state.

Suppose that the engine M passes through a cyclic process, during which it changes the distributions of states of the systems $\Sigma_1, \Sigma_2 \dots$ infinitely slowly through adiabatic influence, i.e., performs work and receives the amount of heat Q from the system W . The reciprocal adiabatic influence of the systems $\Sigma_1, \Sigma_2 \dots$ at the end of the process will then differ from that before the process. We say that the engine M has converted the amount of heat Q into work.

We now calculate the increase in entropy of the individual partial systems during the process considered. According to the results of §6, the entropy increase of the heat reservoir W equals $-\frac{Q}{T}$ if T denotes the absolute temperature. The entropy of M is the same before and after the process because the system M has undergone a cyclic process. The systems $\Sigma_1, \Sigma_2 \dots$ do not change their entropies during the process at all because these systems only experience an adiabatic influence that is infinitely slow. Hence, the entropy increase $S' - S$ of the total system has the value

$$S' - S = \frac{Q}{T}.$$

Since, according to the results of the last section, this quantity $S' - S$ is always ≥ 0 , it follows that

$$Q \leq 0.$$

This equation expresses the impossibility of the existence of a perpetuum mobile of the second kind.

Bern, January 1903.

(Received on 26 January 1903.)

Editorial Note

[a] Einstein 1902.

Index

- aberration, law of, 63, 95, 107
- Abraham, Max, 122
- absolute temperature and thermal equilibrium (physical systems), 7–8
- accelerated reference system
 - and gravitational field, 96, 137–38
 - space and time in, 138–42
- addition theorem for velocities, 56–58, 67, 103–5, 107, 170
- Ampère’s molecular currents, experimental proof, 198–202
- angular momentum, and magnetic moments, 198–202
- antisymmetrical tensors, 225–26
- approximative integration of field equations of gravitation, 265–74
- atomistic theory of matter and electricity, 417–27
- atoms, actual size, 42–43
- “ausstrahlung” (spontaneous emission), 278

- Bargmann, Valentin, 485
- Bauer, Hans, 327, 328
- Bergmann, Peter, 430
- Besso, Michele, 71
- β —rays, Kaufmann’s experiments, 119–21
- Bianchi identities, 345, 495, 496, 508, 514
- Birkhoff, George David, 408
- black-body radiation (“black radiation”)
 - and Boltzmann’s theory of the second law, 82–83
 - difficulty concerning theory of, 19–21, 22, 75, 76
 - energy quanta, 28, 75
 - entropy of, 23–24
 - law of, 24
 - and light quanta, 76–79
 - moving, 96
 - “non-Wien radiation,” 29
 - Planck’s theory. *See* Planck’s radiation theory
- Bohr, Niels, 281, 288, 341
- Boltzmann, Ludwig, 152–53
- Boltzmann’s constant, 277
- Boltzmann’s principle, 25, 277, 358–59, 361
- Boltzmann’s theory of gases, 175, 176, 298, 347, 349, 358–59
- Boltzmann’s theory of the second law, 82–83
- Bose, Satyendra Nath, 347, 355, 357
- Bose-Einstein condensate, 354
- Bose-Einstein statistics, 346, 354
- Bose’s theory of radiation, 357
- boundary conditions, general theory of
 - relativity, 299–302, 331–32, 334
- Boyle-Gay-Lussac law, 27
- Brownian particle motion, 33–43, 152, 157, 281
- Bucherer, Alfred Heinrich, 122

- canonical distribution of states, 277–78
- Cartan, Élie, 384
- cathode rays, 173
 - generation by illumination of solid bodies, 30–32
 - Kaufmann’s experiments, 119–21
 - primary and secondary, 174
- Christoffel, Elwin Bruno, 213, 233, 234, 241–43
- Christoffel symbols, 266, 390, 432, 466
- Chwolson, Orest, 166
- clocks
 - coordinate system and time, 96–98
 - gravitational field effect on, 142
 - inferences from the transformation equations concerning rigid bodies and clocks, 102–3
 - moving, and moving rigid bodies, 54–56
 - and relativity of lengths and times, 47–49
 - and rods, behaviour in static gravitational field, 260–64
 - and simultaneity, 45–47
 - space and time in a uniformly accelerated reference system, 138–42
- closed system
 - energy and momentum, 328
 - gravitational mass, 336–39
- closed universe, conservation theorems
 - applicability, 330–34
- Cohn, Emil, 95
- collisions of particles, 408–9
 - elastic eccentric, 405–7
 - inelastic, 407–8
- commutation rule of differentiation, 391
- Compton effect, 340

- conservation law
 - conservation of momentum and energy, principles, 403–4
 - conservation of momentum, principle, 117–18, 127, 133
 - and divergence law, 515
 - in the general case, 250–51
 - in general theory of relativity, 326–39
 - Hamiltonian form, 246–48
 - for matter, 251–52
- contraction, of curvature tensor, 503
- contravariant
 - four-vectors, 222–23, 224
 - fundamental (metric) tensor, 229
 - tensors, 224
 - vector, “divergence,” 239
- coordinate system
 - and relativity of lengths and times, 47–49
 - and simultaneity, 45–47
 - See also* reference system; transformation of coordinates and time
- coordinates, measurement 218
- corpuscular theory of light, 172
- cosmological considerations in the general theory of relativity, 296–307
 - additional term for the field equations of gravitation. *See* cosmological constant
 - boundary conditions, 299–302
 - calculation and result, 306–7
 - Newtonian theory, 297–99
 - spatially finite universe with a uniform distribution of matter, 302–4
- cosmological constant, 296, 305–308, 325–326, 331, 334–335, 395, 399
- cosmological problem, general theory of relativity, 395–98
- covariant
 - differential quotients of the fundamental tensor, 389–90
 - differentiation, 388–89
 - four-vectors, 223–24
 - fundamental (metric) tensor, 228–29, 386–87
 - general theory of, 222–43
 - requirement for equations expressing general laws of nature, 216, 218–19
 - tensors, 224–25
 - vector, “curl,” 239
- Curie-Langevin law, 198
- curvature tensor, 501–3. *See also* Riemann curvature tensor
- De Broglie, Louis, 362
- De Haas, Wander, 198, 199
- De Sitter, Willem, 265, 269, 296, 300, 301, 312, 399, 453
- determinant of the fundamental (metric) tensor, 229–30
- diamond, 91–92
- diffusion coefficient, 38–42
- diffusion theory
 - and disordered motion of particles suspended in a liquid, 39–42
 - of small suspended spheres, 37–39
- dilute solutions, entropy of, 25–27
- Dirac delta-function, 415, 476
- disordered motion of particles suspended in a liquid and its relation to diffusion, 39–42
- distant parallelism, 384–98
 - differential relations, 387–92
 - field equations, 392–95
 - first approximation, 395–97
 - mathematical description of the spatial structure, 385–87
 - structure of the continuum, 385
- divergence
 - and conservation law of momentum and energy, 515
 - tensors and vectors, 239–41, 389
- Doppler effect (Doppler’s principle), 62–63, 95, 103, 106–7, 188, 285, 395–98
- dragging of light by moving bodies, 95, 97, 108
- Drude, Paul, 19, 87, 89, 365
- Dulong-Petit’s law, 87
- Eddington, Arthur, 369
- Ehrenfest, Paul, 357
- EIH paper, 452–82
- Einstein-de Sitter model, 399. *See also* cosmological considerations in the general theory of relativity
- “einstrahlung” (induced absorption), 279–80
- elastic collisions, 405–7
- electric field, external, strength of, 116
- electrical particles, fitting into framework of general theory of relativity, 424–26, 427
- electricity, Kaluza’s theory, generalization, 430–52
- electrodynamics in relativity theory, 108
 - and Planck’s radiation laws, 147–60
 - transformation of Maxwell-Lorentz equations, 108–13, 148–49
- electrodynamics of moving bodies, 43–73
 - Doppler’s principle and of aberration, 62–64

- dynamics of the slowly accelerated
 - electron, 68–71
- kinematics, 45–58
- Lorentz's theory, 93–95
- Maxwell's theory, 44
- transformation of the energy of light
 - rays, 64–66
- transformation of Maxwell-Hertz
 - equations for empty space, 58–61
- transformation of Maxwell-Hertz
 - equations when convection currents are taken into account, 66–67
- electromagnetic phenomena, gravitational
 - field effect on, 143–47
- electromagnetic radiation
 - nature and constitution, 165–81
 - and radiation pressure, massless hollow body, 133–34
 - See also* radiation
- electromagnetic theory, 18, 19, 20, 22, 72, 75, 76, 150, 166; process theory (Maxwell), 18, 19, 20
- electromotive forces arising due to motion in
 - a magnetic field, 58–61
- electron rays (β -rays), Kaufmann's
 - experiments, 119–21
- electron theory, 19, 22, 75, 76; of metals, 365–66
- electron yield curves, 120–22
- electrons, angular momentum and magnetic
 - moments, 198–202. *See also* material point
- elementary quanta. *See* energy quanta
- elementary quantum of electricity, 163–64, 176
- elementary structures
 - energy of, 86, 88–89
 - specific atomic heats, 87–91
- energy
 - components, gravitational field, 310–14
 - content and gravitational mass, 147, 185–86
 - content and inertial mass, 72–75, 96, 147, 183–84
 - and electromagnetic phenomena, 18
 - exchange by radiation, hypothesis concerning, 278–80
 - gravitation, 183–86
 - of gravitational waves by mechanical systems, 316–19
 - loss of material systems by emission of gravitational waves, 271–74
 - and mass equivalence, 72–75, 95–96, 122–26, 401–9
 - transformation of light rays, 64–66
- energy and momentum
 - of a closed system, 328
 - of a moving system, 126–30
- energy and momentum conservation in the
 - general theory of relativity, 326–39
- energy of the spherical world, 334–36
- formulation of the theorem and
 - objections raised, 327–28
- gravitational mass of a closed system, 336–39
- integral theorem for a closed world, 330–34
- in which respect energy and momentum are independent of choice of coordinates, 328–30
- See also* conservation law of momentum and energy
- energy quanta, 19, 28, 75, 176
 - and ionization of gases by ultraviolet light, 32
 - light as, 28–30, 32
 - Planck's determination, 22, 176
 - Regener's determination, 176
 - Rutherford and Geiger's determination, 176
 - and Stokes's rule, 28–29
 - See also* light quanta
- entropy, 10–12, 17, 35
 - dilute solutions, 25–27
 - molecular-theoretical investigation of the dependence on volume of gases and dilute solutions, 25–27
- monochromatic radiation at low
 - radiation density, 24–25
- and probability of state, 175
- radiation, 23–24
- and state probability, monatomic ideal gas, 347–48
- statistical independence of gas
 - molecules, 358–59
- and temperature dependence on state of motion, 96
- and temperature of moving systems, 134–36
- Eötvös, Loránd, 216
- EPR paradox, 409–17
- Epstein, Paul, 281
- equation of motion, and mutually gravitating particles, 452–82

- equation of state
 - saturated ideal gas, 364–66
 - unsaturated ideal gas, 366–68
- equations of motion
 - Hamilton's, 118–19
 - Lagrange's, 118
 - of the mass point and principles of mechanics, 116–19
 - of a material point
 - derivation, 68–71, 95, 113–16
 - in the gravitational field, 243–44
 - mutually gravitating particles, 452–82
 - Newton's, 93, 116
 - of a point mass in a gravitational field, 207–9
 - volume and pressure of a moving system, 130–33
- equivalence hypothesis, 72–75, 93, 137–47
 - and gravitational light deflection, 181–89
 - principle, 126, 324, 419
- Euclidean geometry 217; non-Euclidean, 218, 262
- Euclidean space, 334, 385, 483, 490
 - four-dimensional, 304, 333, 342
 - three-dimensional, 497
- Euler's equations for a frictionless adiabatic fluid, 253–54
- expanding universe, 395, 396–97; and mean density, 399–401
- extra-galactic nebulae, 395–96, 400
- Faraday, Michael, 497
- ferromagnetism, theory of, 198
- field equations
 - compatibility and strength of systems of, 491–96
 - contravariant, 222–24, 229, 239
 - distant parallelism, 395–95; first approximation, 395–97
 - of gravitation
 - in the absence of matter, 244–46
 - for any chosen field of co-ordinates, 305
 - approximative integration, 265–74
 - general form, 248–50
 - and laws of conservation in the general case, 250–51
 - and laws of momentum and energy (Hamiltonian form), 246–48
 - and laws of momentum and energy for matter, 251–52
 - properties based on the theory of invariants, 292–95
 - and variational principle, 290–91, 443–46
 - of gravitation and matter, variational principle, 290–91
 - relativistic, non-existence of regular stationary solutions, 483–91
 - topological solution, 417–27
 - and variational principle, 508–10
- field theory and solution of the quantum problem, 339–46
 - derivation of system of overdetermined equations, 343–46
 - generalities, 340–43
 - See also* unified field theory of gravitation and electromagnetism
- FitzGerald, George Francis, 94, 95, 169
- five-dimensional space, 430–452; modified Kaluza's theory, 436–47
- Fizeau, Hippolyte, 97, 108
 - light interference experiment, 167–68
 - Fraunhofer diffraction effect, 364
- Freundlich, Erwin, 203, 206, 212, 262
- frictionless adiabatic fluid, Euler's equations for, 253–54
- Friedman, Alexander, 396, 397–98
- fundamental tensor. *See* metric tensor
- Galilean coordinate system, 329, 330, 331
 - space, 329, 336, 337, 461
 - system of reference, 215, 217–18, 362
- Galilei, Galileo, 213, 215
- gases
 - entropy, 25–27
 - ionization by ultraviolet light, 32
 - kinetic theory, 8, 19, 20, 43
 - molecules, mutual statistical independence, 358–60
 - monatomic ideal, quantum theory, 346–68
 - partially dissociated, thermodynamic equilibrium with radiation, 192–97
 - radiation and kinetic energy, 177–79
 - saturated ideal, 355–57
 - viscosity at low temperatures, 363–64
 - See also* ideal gas
- Gauss, Carl Friedrich, 213, 222, 450, 460, 497; theorem, 487
- Gaussian coordinate system, 385–86
- Geiger, Hans, 176
- general theory of relativity
 - boundary conditions, 299–302, 331–32
 - cosmological considerations, 296–307
 - cosmological problem, 395–98
 - equation of motion of the point mass in a gravitational field, 207–9
 - foundations, 212–64, 323–26

- gravitational field theory, 221, 243–52
 - and Hamilton's principle, 289–95
 - law of energy conservation in, 326–39
 - and Mach's principle, 324, 325–26
 - material phenomena, 252–57
 - particle problem, 417–27
 - and perihelion motion of Mercury, 202–12
 - relation of the four coordinates to measurement in space and time, 219–21
 - response to Kretschmann's objection, 323–25
 - and space and time, 216
- generally covariant equations, 222–43
 - cases of special importance, 237–41
 - contravariant and covariant four-vectors, 222–24
 - equation of the geodesic line, 232–34
 - formation of tensors by differentiation, 234–37
 - fundamental (metric) tensor, 228–32
 - multiplication of tensors, 226–28
 - tensors of the second and higher ranks, 224–26
- geodesic equation, 232–34
- geometry. *See* Euclidean geometry
- Gibbs, Josiah Willard, 1, 151; Gibbs's paradox, 354, 363
- gravitation
 - of energy, 183–86
 - expression for the field-components, 243–44
 - field equations, 244–52, 265–74, 290–95, 305
 - and general theory of relativity, 221
 - influence on propagation of light, 181–91
 - mutually gravitating particles, and equations of motion, 452–82
 - and principle of relativity, 137–47
- gravitation and electromagnetism
 - Kaluza's five-dimensional theory, 430–52
 - unified field theory, 339, 369–76, 384–98, 491–518
- gravitational equations, 373, 417, 420, 425
 - axially symmetric static solutions, 419
 - for empty space, 494–96
 - free from singularities, 436
 - no solutions for stationary fields free of singularities, 483–91
 - spherically symmetric static solution (Schwarzschild), 421–24
- gravitational equations and problem of motion, 452–82
 - application of the general theory, 482; approximation, 482
- general theory
 - alternative form of the equations when singularities are absent, 467–69
 - expansion properties of field quantities, 465–67
 - field equations and coordinate conditions, 455–59
 - fundamental integral properties of the field, 459–61
 - method of approximation, 461–68
 - splitting the equations when singularities are absent, 469–72
 - when singularities are present, 472–79
 - zero coordinate condition, 479–81
- gravitational field
 - and accelerated reference system, 96, 137–38, 143
 - effect on clocks, 142
 - effect on electromagnetic phenomena, 143–47
 - energy components, 310–14
 - equations of motion of a material point in, 243–44
 - Hamiltonian function, 246–48
 - light ray deflection, 189–91, 203, 206, 260–64
 - Newton's theory as a first approximation, 257–60
 - and perihelion motion of Mercury, 203–7
 - physical nature, hypothesis, 182–83
 - pure, as a special case, 372–73
 - separate existence, 291–92
 - solutions of approximate equations by means of retarded potentials, 308–10
 - and space-time variability, 221
 - static, behaviour of rods and clocks, 260–64
 - theory, 243–52
 - time and velocity of light, 186–89
- gravitational lensing, 428–30
- gravitational mass
 - of a closed system, 336–39
 - dependence on its energy content, 147, 185–86
 - proportionality to inertial mass, 126
 - and relativity principle, 96

- gravitational waves, 307–23
 - effect on mechanical systems, 320–21
 - energy of by mechanical systems, 316–19
 - energy loss of material systems by
 - emission of, 271–74
 - plane, 269–71, 314–16
 - and quantum theory, 273, 319
 - response to objection by Levi-Civita, 321–23
 - solutions of the approximation equations
 - for the gravitational field by means of retarded potentials, 308–10
- Grommer, Jakob, 300, 346, 367, 376, 483
- Günther, Paul, 364
- Hamiltonian function for the gravitational field, 246–48
- Hamiltonian integral, 441, 443, 445
- Hamilton's equations of motion, 118–19
- Hamilton's principle and general theory of relativity, 289–95, 304, 372
 - properties of the field equations of gravitation based on the theory of invariants, 292–95
 - separate existence of the gravitational field, 291–92
 - variational principle and the field equations of gravitation and matter, 290–91
- Heckmann, Otto, 399
- Helmholtz, Hermann von, 453
- Hertz, Heinrich, 278
- Hilbert, David, 252, 290
- Hoffmann, Banesh, 452
- Hubble, Edwin, 395–98
- Huygens principle, 189
- ideal gas
 - fluctuation properties, 360–63
 - gas theory compared to mutual statistical independence of gas molecules, 357–60
 - monatomic, quantum theory, 346–68
 - saturated, 355–57, 364–66
 - unsaturated, 366–68
- inelastic collisions, 407–8
- inertial mass of a body
 - dependence upon its energy content, 72–75, 96, 147, 183–84
 - proportional to its gravitational mass, 126, 324
- inertial systems, 496–98
- Infeld, Leopold, 452
- infinitesimal displacement field, 499–501
- inner multiplication of tensors, 227–28
- integral theorem for a closed world, 330–34
- integration of the approximative equations of the gravitational field, 266–69
- invariant theory, and properties of the field equations of gravitation, 292–95
- ionization of gases by ultraviolet light, 32
- isolated physical systems
 - absolute temperature and thermal equilibrium, 7–8
 - and derivation of the second law, 13, 17
 - distribution of states in contact with two systems in interaction, 5–7
 - entropy concept, 10–12
 - infinitely slow processes, changes in stationary states, 8–10
 - mathematical representation of processes, 1–2
 - partial systems, 15–17
 - probability of distributions of states, 12–15
 - stationary distribution of state, 2–5
- isopycnic process, 10
- Jacobi's theorem, 231
- Jeans, James Hopwood, 148, 150, 151, 153, 161, 162, 164
- Kaluza, Theodor, theory of electricity of, 369, 430–52, 484, 486
 - generalization, 436
 - general remarks about space structure, 436–38
 - space structure, 438–41
 - tensor analysis with respect to the special coordinate system, 441–43
 - variational principle and field equations, 443–46
 - original theory, 431–35
 - special coordinate system, 433–36
- Kaufmann, Walter, experiment on β -rays, 119–21
- Kepler's laws, 209
- Killing, Wilhelm, equation of, 432
- kinematic foundations of relativity theory, 95, 96–108
 - addition theorem of velocities, 103–5
 - application of transformation equations to some optics problems, 105–8
 - general remarks concerning space and time, 98–99
- inferences from the transformation equations concerning rigid bodies and clocks, 102–3

- principle of constancy of the velocity of light, 96–98
- principle of relativity, 96–98
- time definition, 96–98
- transformation of coordinates and time, 99–102
- kinematics of moving bodies
 - addition theorem for velocities, 58
 - definition of simultaneity, 45–47
 - physical meaning of the equations
 - obtained as concerns moving rigid bodies and moving clocks, 54–56
 - principles, 47
 - on the relativity of lengths and times, 47–49
 - transformations of coordinate and time
 - from the rest system to a system in uniform translational motion relative to it, 49–54
- kinetic energy, gases and radiation, 177–79
- kinetic theory
 - of gases, 8, 19, 20, 43
 - and light waves, 167
 - of specific heat, 161
 - See also* molecular-kinetic theory of heat
- Klein, Oskar, 431
- Kretschmann, Erich, 323–26
- Kronecker tensor, 448, 503
- Lagrange theorem, 367
- Lagrange's equations of motion, 118
- lambda (λ)-transformation, 503, 505–6, 508
- lambda (λ)-constant. *See* cosmological constant
- Langer, Rudolph Ernest, 409
- Langevin, Paul, 199
- Laplace's equation, 489
- Laub, Jakob Johann, 95
- Laue, Max, 95, 107, 249, 317
- law of aberration, 63, 95, 107
- law of conservation of momentum and energy. *See* conservation law of momentum and energy
- law of constancy of mass, 125
- law of parallel displacement. *See* distant parallelism
- law of photochemical equivalence, thermodynamic proof, 191–97
- Le Verrier, Urbain, 203, 245, 264
- Lenard, Philipp, 30–32
- lens-like action of a star by the deflection of light in a gravitational field, 428–30
- Levi-Civita, Tullio, 213, 224, 387, 419, 498
- Einstein's answer to objection by (general theory of relativity), 321–23
- tensor densities, 447–48, 512
- light
 - corpuscular theory, 172
 - deflection in a gravitational field, and
 - lens-like action of a star, 428–30
 - as electromagnetic process, 166–67
 - as energy quanta, 28–30, 32
 - heuristic view of production and transformation, 18–32
 - interference and refraction, 165–66, 167–68
 - and matter, Lorentz's theory on
 - interaction of, 168–69
 - nature of and theory of relativity, 172–73
 - Newton's emission theory, 166, 170
 - principle of the constancy of the velocity, of 47, 96–98, 171
 - production of cathode rays by, 30–32
 - structure of, 172
 - wave-particle duality, 147, 160–61, 165, 167–80
 - wave theory, 18–19, 166, 173–74, 179
- light absorption theory, 75–82
- light production theory, 75–82
- light propagation
 - gravitation influence on, 181–91
 - and luminiferous ether, 166–70
- light quanta
 - absorption and emission (elementary processes), 160
 - and elementary quantum of electricity, 163–64
 - experimental investigation of
 - consequences, 160–61
 - hypothesis, 176–77
 - and Maxwell-Lorentz theory, 161–62
 - and Planck's theory of radiation, 76–79, 160, 162–63, 164
- light rays
 - bending in gravitational field, 189–91, 203, 206, 260–64
 - transformation of energy, 64–66
- light waves
 - Doppler's principle, 62–63, 95, 106–7
 - kinetic theory, 167
- Lorentz, Hendrik A., 95, 97, 148, 151, 199, 290, 322, 335, 365, 408
 - electrodynamics of moving bodies, 93–94
 - and principle of relativity, 170, 171
 - theory on interaction of light and matter, 168–69
 - transformation law, 308, 402, 404, 409

- low temperatures
 - degeneration of specific heat at, 340
 - viscosity of gases, 363–64
- luminiferous ether, 166–70
- Mach, Ernst, 214, 497
- Mach's principle, 296, 323, 324, 325–26
- magnetic moment, and angular momentum, 198–202
- magnetomotive force, 61
- Mandl, Rudi W., 428
- many-particle problem, 417–27
- mass action law, 192–94
- mass-energy equivalence, 72–75, 95–96, 122–26; elementary derivation, 401–9
- mass-point, 405
- material point
 - derivation of equations of motion, 68–71, 113–16
 - equations of motion in a gravitational field, 243–44
 - Kaufmann's experimental test of motion, 119–22
 - mechanics, 95, 113–22
 - motion of mass point and the principles of mechanics, 116–19
- matter
 - field equations of gravitation in the absence of, 244–46
 - and gravitation, field equations, 290–91, 443–46
 - laws of momentum and energy for, 251–52
 - and light, Lorentz's theory of interaction, 268–69
 - motion of, gravitational equations, 452–82
- Maxwell, James Clark, 275; light as electromagnetic process, 166
- Maxwell-Boltzmann distribution law, 77
- Maxwell-Hertz equations, for empty space, 72
 - transformation, 58–61
 - transformation when convention currents are taken into account, 66–67
- Maxwell-Lorentz theory, 95, 110, 148–49, 340–41, 402, 455
 - equations, transformation of, 108–13
 - and light quanta, 161–62
- Maxwell-Poynting expressions, 257
- Maxwell's electromagnetic theory, 18–20, 22, 44–72, 75–76, 150, 166, 369, 395, 417, 497, 512; relation to unified theory of gravitation and electromagnetism, 373–76
- Maxwell's equations, 344, 345, 397, 401, 418
 - for empty space, 254–57, 374–75, 453, 493–94
 - in a vacuum, 397
 - See also* Maxwell-Hertz equations
- Maxwell's theory of electricity. *See* Maxwell's electromagnetic theory
- Maxwell's velocity distribution, 277, 281, 353–54, 359, 365, 366
 - and curve of chromatic distribution of thermal radiation, 275, 276
 - See also* Maxwell-Boltzmann distribution law
- Mayer, Walther, 420
- mean displacement formula for suspended particles, 42–43
- mechanical systems
 - energy of mechanical waves, 316–19
 - gravitational waves effect on, 320–21
- mechanics of the material point (electron), 95
 - derivation of the equations of motion, 68–71, 113–16
 - Kaufmann's investigation, 119–22
 - motion of the mass point and principles of mechanics, 116–19
- mechanics of systems, 95–96, 113–22
 - dynamics of systems and principle of least action, 136–37
 - energy and mass equivalence, 72–75, 122–26
 - energy and momentum of a moving system, 126–30
 - examples, 133–34
 - volume and pressure of a moving system, and equations of motion, 130–33
- Mercury, perihelion motion, 202–7
- metals, electron theory of, 365–66
- metric tensor, 228–32, 237–38
 - contravariant, 229
 - covariant, 228–29, 386–87
 - covariant differential quotients, 389–90
 - determinant, 229–30
 - and formation of new tensors, 231–32
- Michelson-Morley experiment, 94, 98, 169
- Mie, Gustav, 341
- Minkowski, Hermann, 212, 220, 255, 419, 483; space, 490
- mirrors, perfect, radiation pressure on, 64–66, 157–59
- mixed multiplication of tensors, 227–28

- mixed tensors, 225
 - “contraction,” 226–27
 - divergence of mixed tensor of the second rank, 240–41
- molecular currents, Ampère’s, 198–202
- molecular-kinetic theory of heat
 - and motion of small particles suspended in liquids at rest, 33–43
 - and osmotic pressure, 35–37
 - and Planck’s radiation theory, 83–92
 - See also* kinetic theory
- momentum
 - fluctuations, and radiation pressure, 157–59
 - of the material point, 117–18
 - spatial distribution of momentum of radiation, 179–80
- momentum and energy
 - conservation laws. *See* conservation laws of momentum and energy
 - and energy of a moving system, 126–30
- monatomic ideal gas, quantum theory, 346–68
 - cells, 347
 - classical theory as a limiting case, 351–52
 - deviation from the gas equation of classical theory, 352–54
 - state probability and entropy, 347–48
 - thermodynamic equilibrium, 349–51
 - See also* saturated ideal gas
- monochromatic radiation
 - dependence of entropy on volume, interpretation using Boltzmann’s principle, 27–28
 - entropy at low radiation density, 24–25
- Mosengeil, Friedrich, 96, 283
- motion
 - of material point (electron), 68–71, 95, 113–22; in gravitational field, 243–44
 - of matter, gravitational equations, 452–82
 - of molecules in a field of radiation, 281–83
 - calculation of Δ^2 , 206–7
 - calculation of R , 283–85
 - result, 287–89
 - of a particle, 232–34
 - of planets, 207–12
 - of small particles suspended in liquids at rest, required by molecular-kinetic theory of heat, 33–43
- moving bodies
 - contraction in direction of motion, 94
 - dragging of light, 95, 97, 108
 - electrodynamics, 43–73, 93–94
 - entropy and temperature dependence on state of motion, 96
 - kinematics, 45–58
- moving systems
 - dynamics, and principle of least action, 136–37
 - energy and momentum, 126–30
 - entropy and temperature, 134–36
 - volume and pressure, 130–33
- multiplication of tensors, 226
 - “contraction” of a mixed tensor, 226–27
 - inner and mixed, 227–28
 - outer, 226
- mutually gravitating particles, and equations of motion, 452–82
- nature, general laws of, requirement of
 - general covariance for equations expressing, 216, 218–19
- Nernst, Walther, 364; theorem of, 352, 359–60
- Neumann-Kopp’s rule, 87, 90
- Newcomb, Simon, 211
- Newton, Isaac, 497
- Newton-Poisson theory of gravitation, 395
- Newtonian mechanics, and special theory of relativity, 213, 214–16, 257–60
- Newtonian theory, 203, 296, 297–99, 301; as a first approximation, 257–60
- Newton’s emission theory of light, 166, 170, 175
- Newton’s equations of motion, 93, 95, 116, 207–9, 454
- non-accelerated reference systems, 96, 137, 138, 143, 145
- non-symmetrical field, relativistic theory, 491–517
- Nordström, Gunner, 313, 327
- Oersted, Hans Christian, 198
- Oort, Jan, 401
- optical properties of solids, 87–92
- optical problems of relativity, 95, 105–8. *See also* aberration; Doppler’s principle
- optics, radiation pressure exerted on perfect mirrors, 64–66, 157–59
- oscillating magnetic fields, ferromagnetic substances in, 198–202

- osmotic pressure
 - ascribed to suspended particles, 34–35
 - from the standpoint of the
 - molecular-kinetic theory of heat, 35–37
- overdetermined equations system, 343–46, 372
- Palatini, Attilio, 449, 485
- paradox. *See* rotating disc paradox; Gibbs, Josiah Willard
- partial systems, 5, 15–17
- particle collisions, 405–9
- particle problem, in general theory of relativity, 417–27
- particle-pair, 404, 405, 406, 407
- particles
 - quantum-mechanical description of behavior, 411–12
 - as singularities in the field, 419–27
 - See also* electrons; material point
- Pauli, Wolfgang, 483
- perfect mirrors, radiation pressure on, 64–66, 157–59
- perihelion motion
 - of Mercury, 202–7
 - of planetary orbit, 207–12, 260–64
- photochemical equivalence, law of, 173, 191–97; thermodynamic equilibrium between radiation and a partially dissociated gas from the standpoint of the mass action law, 192–97
- photoelectric diffusion, and Volta effect, 79–82
- photoelectric effect, 18–33
- physical reality, quantum-mechanical description, 409–17
- physical systems, isolated, 1–17
- Planck, Max, 20, 121, 125, 135, 348
 - black-body radiation theory. *See* Planck's radiation theory
 - determination of elementary quanta, 22
 - dynamics of moving systems and principle of least action, 136–37
- Planck's constant, 411
- Planck's function, 356
- Planck's probability, 357
- Planck's radiation formula (law), 22, 76, 153–54, 159, 160, 161, 162, 174, 191, 275, 276, 355, 360, 361
 - and currently accepted theoretical foundations, 151–53
 - derivation, 175–76, 280–81
 - Einstein's insights into implications of, 147–60
 - Jeans's interpretation, 150–51, 153
 - and laws of electrodynamics, 148–60
 - and Maxwell-Lorentz differential equations, 148
 - and quantum theory, 160, 176–77
 - Ritz's views, 148, 149
- Planck's radiation theory, 22, 151–53, 174–77
 - and light quanta, 76–79, 160, 162–63, 164
 - modification of foundations, 153–57
 - and the theory of specific heat, 82–92
- Planck's resonator, 278–79, 288; derivation of mean energy, 83–86
- plane gravitational waves, 269–71, 314–16
- planetary orbit, perihelion motion, 207–12, 260–64
- Podolsky, Boris, 409
- point coincidence, 219
- Poisson's equation, 249, 296, 298–99, 305, 398
- postulate of relativity
 - extension to overcome defect in, 214–16
 - observations on the special theory of relativity, 213–14
 - relation of the four coordinates to measurement in space and time, 219–21
 - space-time continuum, 216–19
 - See also* relativity principle
- Precht, Julius, 125
- pressure and volume of a moving system, 130–33
- principle of general covariance, 218
- principle of the conservation of mass, 183
- principle of the conservation of momentum, 117–18, 127, 133. *See also* conservation law
- principle of the constancy of the velocity of light, 47, 96–98, 171, 189
- principle of least action, and dynamics of systems, 136–37
- principle of relativity. *See* relativity principle
- probability of distributions of states, 12–15
- pure gravitational field, 372–73
- quantum energy. *See* energy quanta
- quantum theory
 - Ampère's molecular currents, experimental proof, 198–202
 - applied to solids, 82–92
 - field theory and solution of the quantum problem, 339–46
 - fundamental hypothesis, 277–78

- and gravitational waves, 273, 319
- monatomic ideal gas, 346–68
- photoelectric effect, 18–33
- and Planck's formula, 160, 176–77, 276
- of radiation, 275–89
- theory of light production and light absorption, 75–82
- quantum-mechanical description of physical reality, 409–17
- radiation
 - absorption and emission, 160–61
 - entropy of, 23–24
 - gases and kinetic energy, 177–79
 - hypothesis concerning energy exchange by, 278–80
 - interaction with resonator electrons, 20–21
 - monochromatic, 24–25, 27–28
 - nature and constitution, 165–81
 - problem of, Einstein's insights into, 147–64
 - quantum theory, 275–89
 - spatial distribution of momentum, 179–80
 - thermodynamic equilibrium with partially dissociated gas, 192–97
 - undulatory structure and quantum structure, 180
 - See also* black-body radiation; Planck's radiation formula (law)
- radiation field, motion of molecules in, 281–89
- “radiation friction,” 177–78
- radiation pressure
 - exerted on electromagnetic radiation enclosed in massless hollow body, 133–34
 - exerted on perfect mirrors, 64–66, 157–59
 - and fluctuations of momentum, 157–59
 - gases and kinetic energy, 177–79
 - and wave structure of radiation, 179
- radioactive decay, 125–26
- Rayleigh, Lord (John William Strutt), 281; radiation formula, 85, 275
- reality, quantum-mechanical description of physical reality, 409–17
- red shifts, 395, 396, 400
- reference system, 98–99
 - accelerated, and gravitational field, 137–38
 - electromagnetic radiation in a massless hollow body, 133–34
 - energy and momentum of a moving system, 126–30
 - entropy and temperature of moving systems, 134–36
 - inferences from the transformation equations concerning rigid bodies and clocks, 102–3
 - transformation of coordinates and time, 99–102
 - transformation of Maxwell-Lorentz equations, 111–12, 113
 - uniformly accelerated, 138–42
 - volume and pressure of a moving system, 130–33
- refrigerator patent, 376–83
- Regener, Erich, 176
- Reissner, Hans, 418
- relativistic field theory of non-symmetric field, 491–517
 - curvature tensor, 501–3
 - divergence law and conservation law of momentum and energy, 515
 - general remarks, 496–99, 516–17
 - identities, 510–12
 - infinitesimal displacement field Γ , 499–500
 - λ – transformation, 503, 505–6, 508
 - pseudotensor, 505–8
 - requirement of “transposition invariance,” 504–5
 - strength of the system of equations, 513–15
 - variational principle and field equations, 508–10
- relativity principle, 47, 59, 169–70, 324
 - and conclusions drawn from it, 93–147
 - electrodynamical part, 108–13
 - energy-mass equivalence, 72–75, 95–96, 122–26, 401–9
 - general remarks concerning space and time, 98–99
 - and gravitation, 137–47
 - gravitation influence on propagation of light, 183–91
 - influence on acceleration and gravitation, 96, 137–47
 - kinematic part, 95, 96–108
 - and Lorentz and FitzGerald's hypothesis, 94–95
 - and Lorentz's theory, 170, 171
 - mechanics of the material point (electron), 95, 113–22
 - mechanics and thermodynamics of systems, 95–96, 122–37
 - merger with Lorentz's theory, 95, 97

- relativity principle (cont.)
 - and nature of light, 171–72
 - and principle of constancy of the velocity of light, 96–98
 - time definition, 96–98
 - transformation of coordinates and time, 99–102
 - transformation equations, application to optics problems, 95, 105–8
 - transformation equations concerning rigid bodies and clocks, inferences, 102–3
 - transformation of Maxwell-Lorentz equations, 95, 108–13
 - velocity addition theorem, 103–5, 107, 170
 - See also* general theory of relativity; Mach's principle; postulate of relativity; special theory of relativity
- relativistic field equations, non-existence of regular stationary solutions, 483–91
- resonators (electrons), 19–20, 153, 175–76, 278–79
 - interaction with radiation in space, 20–21, 76–79
 - Planck's. *See* Planck's resonator
- rest-energy, 403, 405, 407
- rest-mass, 405
- "rest system"
 - relativity of lengths and times, 48–49
 - and simultaneity, 45–47
 - transformations of coordinate and time from the rest system to a system in uniform translational motion relative to it, 49–54
- Ricci, Gregorio, 213, 224
- Riemann, Bernhard, 213, 437, 497
- Riemann-Christoffel tensor, 241–43
- Riemann curvature tensor, 305, 342, 343, 370, 390, 449, 485, 490
- Riemannian geometry, 342, 372, 388
- Riemannian manifold, 384, 385, 390
- Riemannian metric and distant parallelism, unified field theory based on, 384–98
- rigid bodies, 45, 96
 - inferences from the transformation equations concerning rigid bodies and clocks, 102–3
 - moving, and moving clocks, 54–56
 - in nonaccelerated motion, 98–99
- Ritz, Walter, 148, 149
- Robertson, Howard Percy, 482
- Rosen, Nathan, 409, 417
- rotating disc paradox, 217
- Rutherford, Ernest, 176
- saturated ideal gas, 355–57; equation of state, 364–66
- scalar wave equation, 492–93
- Schrödinger's equation, 413
- Schwarzschild, Karl, 345, 418, 421–24
- second law of thermodynamics
 - derivation, 17
 - and probability of distributions of states, 12–15
- Serini, Rocco, 490
- Silberstein, Ludwik, 419
- simultaneity, definition of, 45–47, 49
- singularities from the field, 417–27, 436
 - combined field-fitting electricity into the framework, 424–26
 - and gravitational equations
 - singularities absent, 467–69
 - singularities present, 472–79
- Schwarzschild solution, 421–24
- special kind of singularity and its removal, 419–21
- six-vector
 - antisymmetrical extension, 239
 - divergence, 239–40
- solid body illumination, and cathode ray generation, 30–32
- solids
 - oscillations of elementary structures, 88–91
 - quantum hypothesis applied to, 82–92
 - thermal and optical behavior, 87–92
- Sommerfeld, Arnold, 281; quantization rule, 362
- space and time
 - general remarks, 98–99
 - and general theory of relativity, 216
 - Kaluza's theory, 430–52
 - requirement of the four coordinates to measurement, 219–21
 - and special theory of relativity, 214, 216–19
 - topological shortcuts, 417–27
 - in uniformly accelerated reference system, 138–42
- space-time continuum, 216–19, 230–31
- spatial distribution of the momentum of radiation, 179–80
- special theory of relativity, 401–2
 - electrodynamics of moving bodies, 43–72

- equivalence of energy and inertial mass, 72–75, 324
- and Newtonian mechanics, 213, 214–16, 257–60
- observations, 213–14
- review, 93–147
- and space and time, 214
- and theory of gravitation, 137–47
- specific heat theory, 87–92; and Planck's radiation theory, 82–92
- spherical world, energy of, 334–36
- Stark, Johannes, 160
- stars, lens-like action by deflection of light in a gravitational field, 428–30
- stationary state, isolated physical systems, 2–5
- statistical independence of gas molecules, 358–60
- statistical mechanics, foundations, 1–17. *See also* Bose-Einstein statistics
- Sterling's theorem, 348
- Stokes's law, 28–29, 31
- summation convention, 223, 291
- suspended particles in liquids
 - disordered motion in relation to diffusion, 39–42
 - formula for mean displacement, 42–43
 - motion of required by molecular-kinetic theory of heat, 33–43
 - osmotic pressure ascribed to, 34–35
 - and theory of diffusion, 37–39
 - See also* Brownian particle motion
- symmetrical tensors, 225
- Szilard, Leo, 376–80
- Taylor expansion, 493–94
- temperature
 - and entropy dependence on state of motion, 96
 - and entropy of moving systems, 134–36
- tensors, 222, 223, 390–91
 - antisymmetrical, 225–26
 - contravariant, 224
 - covariant, 224–25
 - curvature, 501–3
 - densities, 420, 447–52, 512
 - divergence, 239–41
 - formation by differentiation, 234–37
 - fundamental, 228–32, 237–38
 - identities, 392
 - Kronecker, 448, 503
 - mixed, 225, 240–41
 - multiplication, 226–28
 - products, differentiation, 390
 - Riemann curvature, 305, 342, 343, 370, 390, 449, 485, 490
 - Riemann-Christoffel, 241–43
 - of the second and higher ranks, 224–26
 - symmetrical, 225
 - thermal behavior of solids, 87–92
 - thermodynamic equilibrium
 - and absolute temperature (physical systems), 7–8
 - between radiation and a partially dissociated gas from the standpoint of the mass action law, 192–97
 - monatomic ideal gas, 349–51
 - thermodynamic theory, and motion of suspended particles in liquids, 33–43
 - thermodynamics
 - foundations of, 1–17
 - second law, 14, 17
 - of systems, 96, 134–36
 - time
 - definition, 96–98
 - gravitational dilation 218
 - and simultaneity, 45–47
 - and space. *See* space and time
 - and velocity of light in the gravitational field, 186–88
 - transformation of coordinates and time, 99–102; from the rest system to a system in uniform translational motion relative to it, 49–54
 - transformation equations
 - application to optics problems, 105–8
 - concerning rigid bodies and clocks, inferences, 102–3
 - transformation of Maxell-Lorentz equations, 95, 108–13
 - transformation of Maxwell-Hertz equations
 - for empty space, 58–61
 - when convection currents are taken into account, 66–67
 - transposition invariance, 504
 - ultraviolet light, ionization of gases, 32
 - unified field theory of gravitation and electromagnetism, 339, 369–76, 483
 - based on Riemannian metric and distant parallelism, 384–98
 - fifth dimension, 430–52
 - general theory, 370–72
 - generalization of Kaluza's theory of electricity, 430–52

- unified field theory of gravitation and electromagnetism (cont.)
 - non-existence of regular stationary solutions of relativistic field equations, 483–91
 - pure gravitational field as a special case, 372–73
 - relation to Maxwell theory, 373–76
 - relativistic theory of the non-symmetric field, 491–517
- uniform translational motion, 49–54
- universe
 - expanding, 395, 396–97
 - relation between expansion and mean density, 399–401
 - spatial curvature, 399, 400, 401
- unsaturated ideal gas, equation of state, 366–68
- van der Waals, Johannes, 365
- variational principle
 - and field equations, 508–10
 - and field equations of gravitation and matter, 290–91, 443–46
- vectors
 - contravariant, 223–24, 239
 - covariant, 223–24, 239
 - divergent, 239–41, 389
- velocity addition theorem, 56–58, 67, 103–5, 107, 170
- velocity of light
 - principle of constancy, 47
 - and time in a gravitational field, 186–88
- viscosity of gases at low temperatures, 363–64
- Volta effect, and photoelectric diffusion, 79–82
- volume and pressure of a moving system, 130–33
- Warburg, Emil, 193
- wave function in quantum mechanics and physical reality, 410–17
- wave-particle duality of light, 147, 160–61, 165, 167–80
- wave systems, interaction, 413–16
- wave theory of light, 18–19, 166, 173–74, 179–80
- Weber, Heinrich Friedrich, 91–92
- Weiss, Pierre, 198
- Weitzenböck, Roland, 384
- Weyl, Hermann, 335, 344, 369, 419, 430
- Wiedemann-Franz, deviations from, 82
- Wien, Wilhelm
 - displacement law, 175, 280
 - radiation formula, 28, 76, 162, 19
 - radiation law, 23–24, 29, 160, 179, 191, 197, 275, 276, 355, 357
- “wormhole,” 417
- X-rays/X-ray production, 173, 174
- zero-coordinate condition, equation of motion, 479–81
- “zero-point theory,” 199