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CHAPTER I

The Abbaco Master

The starless and ominous night came to an end. French troops surrounded the city ready to launch the attack, while the persistent tolling of the bells summoned the population to take up arms against the enemy. The date was February 19, 1512—Fat Thursday, a time to celebrate Carnival—and Brescia was about to witness one of its most tragic days.

The city that would later be known as "the Lioness of Italy" had already fallen under the French yoke in May 1509, after having been ruled for more than eighty years by the Most Serene Republic of Venice. Under the wise and liberal Venetian administration Brescia had become one of the most prosperous cities in Lombardy, only to fall prey to the arrogant and oppressive French rule, an increasing source of popular discontent. Seeking to restore the previous political order, some prominent citizens—Count Luigi Avogadro and other members of the local aristocracy among them—had thus begun to conspire against the French authorities, and later led the successful uprising of February 3, 1512. That day, with the help of Venetian soldiers—and thanks to the fact that most of the French troops had been redeployed to the siege of Bologna—Brescia chased away the foreign occupiers, forcing the remaining French soldiers to take refuge in the city fortress, known as the Castle.[1] The joy of the Brescians would, alas, prove short-lived.

Called back from Bologna, the twenty-three-year-old French general Gaston de Foix promptly regained Lombardy, and on February 17 reached with his army the walls of Brescia. In no time, the foreign forces surrounded the city. De Foix called on the insurgents to surrender, promising them the clemency of Louis XII, king of France, but his offer was curtly rejected. In the night of February 18, the young commander, together with some 500 lancers and 6,000 infantrymen ready for action, entered the castle, where those French soldiers who had escaped the uprising were still barricaded. The order to attack was given as soon as day broke, and shortly after this the carnage began.

The French garrison coming out of the castle pierced the first Venetian lines, and after joining the other units commanded by de Foix pushed toward the city center. Brescian fighters and Venetian troops offered a desperate resistance, but the mismatch of forces and the superior organization of the French soon overcame all defense efforts and the attack ended in a bloodbath. By the time the sun set on that gory Fat Thursday Brescia was back in French hands, its streets scattered with corpses.

But it was not over yet: an exemplary punishment was handed down to the city by the French troops, in the form of widespread destruction of unheard of ferocity. They looted and burned down houses, slaughtered men and children, and raped women in a maddening spree that lasted almost two days. Many leaders of the revolt were publicly and cruelly executed, further adding to the horror. Count Avogadro, the soul of the insurrection, “had his head savagely cut off and exhibited as a trophy on top of the People’s Tower,” and his remains “were hanged on the city gates, low enough so that dogs could feed on them.”[2]

Not even the doors of places of worship stopped the invaders’ rampage, as churches were ransacked and stripped of their treasures and valuable furnishings. To flee the violence, terrified citizens sought shelter in the cathedral, among them a poor widow and her two children: a twelve-year-old boy, Niccolo, and a younger girl. But the aggressors had no scruples in storming the temple to continue their depredation. During the assault, a French soldier targeted Niccolo and dealt him a blow to the head with his sabre. And then a second, and one more still: as a stream of blood started to gush from the boy’s skull, he was hit twice in the face. Mercilessly, the blade cut through his mouth and teeth, fracturing his jaw and palate. He suffered five injuries in all; any of them could have been fatal, and yet Niccolo survived.

In the following weeks, unable to afford a doctor, the mother cared for her son by herself. Niccolo was incapable of speaking or eating, other than swallowing with great effort some liquid foods, and he remained in such a condition for several months. Thanks to his mother's care, he finally recovered from his wounds. Later, as an adult, he would grow a thick beard to conceal the deep and permanent scars that disfigured his face. Little by little he also regained the ability to speak, but the injuries to his mouth had left him with a stammer. Because of this his playmates gave him a mocking nickname that he adopted as his last name, today written in golden letters in the history of mathematics: Tartaglia. [In Italian, *tartagliare* means "to stammer."]

Born in Brescia, probably in 1499, Niccolo Tartaglia was one of the sons of "Micheletto cavallaro," a humble postal courier who delivered mail on horseback and from whom the famous mathematician inherited his short stature, if nothing else.[3] In an autobiographical page of his *Quesiti et inventioni diverse* (first published in 1546),[4] Niccolo affectionately describes his father; and in a dialogue with Gabriele Tadino, Knight of Rhodes and Prior of Barletta,[5] he mentions the composition of his family of origin.

PRIOR: Tell me again, what was your father's name?

NICCOLO: His name was Michele (Michael). And because Nature did not endow him with an adequate height, he was known as Micheletto (Little Michael).

PRIOR: Certainly, if Nature was not prodigal with regard to your father's height, it has not been more generous with yours.

NICCOLO: And I'm glad for that, because being so short proves to me that I am really his son. Even if he left us—my brother, my two sisters, and myself—almost nothing except fond memories of him, I have heard from many people well acquainted with my father that he was a good person. And this is for me a source of greater joy than inheriting a fortune from a disreputable parent.

PRIOR: What was your father's occupation?

NICCOLO: My father possessed a horse, which he rode to deliver post at the service of notables of Brescia; letters from Brescia to Bergamo, Crema, Verona, and other such places.[6]

Niccolo then declares not to know his father's last name, adding that his father passed away when he was five or six years old, leaving his family in the most dire poverty.

PRIOR: What was your father's family name?

NICCOLO: God knows I do not know. I don't remember his family name or his first name, except that as a child I always heard him called Micheletto Cavallaro. He may have had some other name, but not to my knowledge. The reason is that my father died when I was about six, leaving our family—my brother (slightly older than me), my sister (younger than me), and my mother—without any financial means. We went through very hard times, of which I will spare you the details. Under such circumstances, inquiring about my father's family name was the least thing in my mind.[7]

However, years later Tartaglia will mention in his testament "Zuampiero Fontana" as his "legitimate carnal brother."[8] This prompted several historians to consider "Fontana" as Tartaglia's real last name, but in fact none of the attempts to verify this interpretation were conclusive.[9] What is certain, though, is that Tartaglia wished to adopt the surname "Tartaglia" as a reminder of the personal drama he suffered during the 1512 sack of Brescia—"as a good memory of such a disgrace of mine,"[10] he writes—and perhaps also to remember the devoted and tender care his mother had provided him in those days of pain and suffering.[11]

During his conversation with the prior, Tartaglia reveals a few details of his early training. In particular, we learn that between the ages of five and six, shortly before his father's death, he was sent to a "reading school" run by a teacher whose name he did not remember.[12] Later, when he was around fourteen, he went "voluntarily and for about fifteen days to the writing school of a teacher called Francesco," who taught him "how to write a, b, c, and so on up to k in a script called 'mercantesca' [of merchants]." This was a cursive script used in several cities of northern Italy to write documents and commercial books in the vernacular; its principal characteristics were the roundness of the letters and the richness of the joins.[13] To the next, predictable question of the prior, "why only up to the letter k?" Niccolo replies:

> Because it was agreed that I would pay the teacher one-third of his fee at the beginning, another third after I had learned the letters up to k, and the rest upon having learned the whole alphabet. And I didn't have the money to honor the last part of the agreement. However, since I wished to learn, I procured some alphabets and examples of letters written by the teacher's hand and never went back, because from these I learned by myself. And from that day on I never went to another teacher, and my only company was that daughter of poverty called Industriousness. I have continually studied the works of departed men.[14]

In a historical period and social context in which, save for rare exceptions, free public instruction did not exist,[15] the young Tartaglia had to work doggedly in order to cope with his dire financial situation and acquire the desired educational training. He achieved this by teaching himself, and mathematics was one of his first subjects. In his last book, *General trattato di numeri, et misure* [General Treaty of Numbers and Measures], Niccolo recalls having started the study of the discipline in 1514 and making such rapid progress that he soon was able to improve the rule to extract arithmetic roots.[16]

On the whole, Tartaglia built and perfected his own scientific training through the study of the works of various masters of the past (the "departed men"): from the great Greek thinkers, in particular Euclid, Archimedes, and Apollonius, to the medieval and Latin authors. He was thus obliged to become proficient in what was at the time the universal language of the learned, Latin, an indispensable tool for gaining access to the scholarly texts and their treasure trove of knowledge.

From what can be gleaned from the few available autobiographic notes about his youth, Tartaglia lived in Brescia until the age of eighteen or nineteen. Between 1516 and 1518, after spending some time wandering around "young and bachelor" in Crema, Bergamo, and Milan,[17] he left his native town and settled in Verona, where he stayed until 1534. We ignore the reasons for his going there, but whatever the case, Niccolo kept pleasant memories of the city of the Scala family: "Not only was it my first home away from the nest in which I was born," he writes in his *General trattato*, "but it always nourished me, caressed me, and honoured me."[18]

In Verona, Tartaglia, now in his twenties, married Domenica, a woman fourteen years his elder and mother of Benvenuta, an eight-year-old girl. The couple would later have their own child, Margherita, in 1527. For a while, family duties prevented Niccolo from devoting as much time to his studies as he would have wished. He nevertheless acquired a certain reputation, in Verona and other places of northern Italy, in his new role teaching practical mathematics or, more precisely, as an abbaco master.

During the thirteenth century, many Italian cities experienced a thriving increase in commercial activity. With the development of trading companies and the expansion of international trade, the new merchants were confronted with having to run ever-larger companies, whose administration required novel and more complex accounting procedures. In addition, the pressing need to master certain calculation methods was a common concern of shopkeepers, craftsmen, artists, and architects—in short, of all those involved in buying and selling goods, who had therefore to deal with, for example, currency equivalence, conversion from one unit (of weight, length, or area) to another, evaluation of assets and profits, and calculation of interest.

To satisfy the demand for training, in Italy around the middle of the thirteenth century flourished so-called abbaco schools—"institutions perhaps unique in late-medieval and Renaissance Europe,"[19] similar to present-day trade schools—where students learned those elements of practical mathematics that future merchants and technicians would require. At the time, therefore, the term "abbaco" not only referred to the wooden board with carved grooves within which beads could be moved around that ancients used to carry out arithmetical calculations, but it also denoted "the accounting operations and problems related to commercial practice."[20]

Altogether different, on the other hand, was the mathematics studied in those days at university: an abstract and speculative mathematics, whose paradigm was the elegant geometry of Euclid, and which aimed almost exclusively at satisfying theoretical and philosophical interests. Surely, academic geometry also "offered useful techniques for measuring heights, widths and areas," but their main purpose was "a quest for harmony."[21] Similarly, the arithmetic taught at the medieval lecture halls sought to "find harmonious numerical relations" that could be of interest to other disciplines, such as metaphysics or theology. But the principles of order, unity, and harmony so dear to university mathematicians "did not help merchants solve the disorderly problems of money exchange."[22] It should be noted, though, that abbaco mathematics was not entirely foreign to the academic world: there is evidence that at the end of the fourteenth century lectures on mathematics "*di minor guisa*" (in a smaller manner) were given at the University of Bologna, addressing the solution of practical and utilitarian problems "related to the exercise of professions or to events of civil life."[23]

Originally established in Tuscany, abbaco schools—which could be public or private—spread to most Italian cities during the last decades of the thirteenth century, starting a pedagogical tradition that continued until the end of the sixteenth century. Throughout this period, these schools provided a kind of secondary level education, after a first cycle of elementary schooling during which pupils learned to read and write and acquired basic arithmetic skills. At the age of ten or eleven, they could normally choose to enter either "grammar school," to study Latin language and literature and logic, or abbaco school, where commercial mathematics was taught in the vernacular (whereas the language of instruction of academic mathematics was Latin). While grammar schools gave access to university, abbaco schools—after about two years of training, depending on the place and the requirements of pupils—led straight to an apprenticeship with a merchant or craftsman as preparation for the exercise of a trade. As historians of science Enrico Gamba and Vico Montebelli observe, "Abbaco schools were attended either by nobles whose interests lay in commerce and the political rewards that it was expected to provide, or by members of lower classes seeking to improve their social and economic status through the acquisition of a professional qualification." [24]

Grammar and abbaco schools were not necessarily parallel and alternative educational paths, as "grammar could also be chosen subsequent to abbaco learning."[25] A fitting example of this possibility is the schooling of the celebrated Florentine philosopher, writer, and politician Niccolo Machiavelli. In 1480, at the age of eleven, he attended an abbaco school in his native city for one year and ten months, after which he went on to grammar school.[26]

The first public abbaco teacher was probably the eminent Pisan mathematician Leonardo Fibonacci,[27] whose life straddled the twelfth and thirteenth centuries, and to whom the authorities of Pisa awarded in 1241 an annual stipend of twenty pounds as consultant in commercial arithmetic for the Tuscan Municipality and its officials.* The young Fibonacci followed his father Guglielmo, a notary who assisted Pisan merchants with customs procedures, to the Algerian port city of Bugia. It was there that he became familiar with the avant-garde mathematics of the Arabs, which he would later further study during his journeys to Egypt, Syria, Greece, Sicily, and Provence. The culmination of all those years of peregrinations and research was a 1202 manuscript, the *Liber abaci* (Book of Calculation). This fundamental treatise written in Latin,[28] together with his numerous other works, earned Fibonacci the reputation of "most important medieval mathematician in the West."[29]

Divided into fifteen chapters, the *Liber abaci* is a voluminous collection of the arithmetic and algebraic knowledge of the Arab world, enriched with the author's own original contributions. The first part of the book introduces the Indo-Arabic numerals (that is, the usual ten numerals from 0 to 9) and positional notation.[30] If these are today familiar notions, they were practically unknown in the Europe of the time, where the cumbersome and inefficient Roman numeral system was still being used. Fibonacci then presents in the new notation the algorithms for the four arithmetic operations—addition, subtraction, multiplication, and division—on integers and fractions. The second part of the book discusses a variety of topics, including commercial mathematics—problems on purchases, sales, exchanges, and currencies; extraction of square and cubic roots; the theory of geometric proportions; and finally algebra.[31]

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* See, for example, [103c], pp. 256–57.

Even if it was considerably advanced with respect to the mathematical knowledge prevailing in the West at the beginning of the thirteenth century, it took nearly one hundred years before the *Liber abaci* was fully understood and appreciated. In Italy, it was through the abbaco schools that the new ideas proliferated, notably with the production of mathematical manuscripts based on Fibonacci's book, the "abbaco treatises," which began in the late thirteenth century. Written in the dialect of the various Italian regions, these volumes presented essentially the contents of the *Liber abaci*[32] in an elementary way, leaving out its most abstract parts.

There are some three hundred extant handwritten abbaco treatises,[33] the majority of them from the fifteenth century, written mostly by abbaco teachers for their students, or more generally for the benefit of anyone "wishing to have a handbook to which to refer for the solution of everyday problems arising in commercial operations."[34] Some of the authors are nonprofessional mathematicians, bankers or merchants, and, occasionally, even artists, as in the case of a 1480 *Trattato d'abaco* written by the great Tuscan painter Piero Della Francesca.[35]

At the turn of the fifteenth century, after the invention of the printing press by the German typographer Johannes Gutenberg, the first printed abbaco books made an appearance, their number increasing during the following decades and retaining essentially the same characteristics as handwritten ones. The anonymous work known as *Aritmetica di Treviso*, published in 1478 in Treviso, as its title indicates, is not only the oldest printed abbaco treatise but also the first ever printed book on a mathematical topic.[36]

The abbaco books of the Middle Ages and the Renaissance were fundamentally different from present-day mathematics textbooks, both in style and structure. An introduction explaining rules and definitions was followed by a long series of problems, each accompanied by an extremely detailed solution procedure. All this was expressed in narrative terms, as mathematics had not yet developed a symbolic notation—which does not make for easy reading, even for those with some mathematical knowledge. Here is an example:

> A soldo of Provence is worth 40 denari of Pisa and a soldo imperiale is worth 32 of Pisa. Tell me how much will I have of these two monies mixed together for 200 lire of Pisa? Do it thus: add together 40 and 32 making 72 (denari), which are 6 soldi, and divide 200 lire by 6, which gives 33 lire and 6 soldi and 8 denari, and you will have this much of each of these two monies, that is 33 lire 6 soldi 8 denari for the said 200 lire of Pisa. And it has been done.*

Abbaco books and teachers showed how to solve particular problems by analyzing them step by step, and for each problem a specific solution technique was provided. They did not teach general solution methods, as is the case in modern mathematics education. Quite the contrary: for a minor variation in the structure of a problem a different solution procedure was often proposed, the one regarded as the most appropriate and efficient for that particular case. Paul Grendler, an American specialist in schooling in Renaissance Italy, explains:

> The abbaco book collected individual problems and their solutions for reference use. A teacher found in it the day's problems and solutions to teach; students copied down what the teacher explained. The slow, literary statement of the problem and solution may have helped the students to understand and remember. If he faithfully copied enough problems and solutions, he had his own abbaco book. When as an adult merchant, banker, or clerk he came across a problem that he could not solve, he looked into his student abbaco book to find an

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* This example is taken from [56], p. 315. The problem requires the conversion of 200 lire into numerically equal amounts of the two monies. Readers who would like to check the solution should keep in mind that in the then prevailing monetary system, adopted in most of Europe and based on lire (singular, lira), soldi (singular, soldo), and denari (singular, denaro), one soldo was worth 12 denari, and one lira 20 soldi.

almost identical problem and applied its method. Joining problem and solution together made the learner's task easier.[37]

The primary goal of abbaco schools was essentially the solution of problems, to which students were led not by logical deduction from general theoretical principles but through memorization of typical cases and preestablished rules. Considered as an "inseparable unit of a statement and its solution procedure,"[38] each problem was for the student a reference, a model to be followed in order to unravel a strictly similar practical question. Therefore, as Gamba and Montebelli observed in the foregoing quotes, abbaco teaching instilled in the student "a mnemonic, analogical, and practical mental attitude rather than a logico-deductive one, well-suited to the exercise of commerce, where situations that required rapidly counting one's and other's money abounded."[39]

One of the principal skills required of abbaco students was precisely the rapid execution of written, mental, and hand calculations. After learning the Indo-Arabic numerals and the arithmetic operations on integers and fractions, students had to memorize long multiplication tables to be able to perform complex calculations fast; they were also expected to check the correctness of the results. Tartaglia himself recommended "to verify diligently every step and operation, not just once but two and three times, because to err is human."[40] For the rest, the curriculum of the abbaco schools, which can be inferred from abbaco treatises, was standard and included commercial arithmetic (buying and selling, costs, profits, interest, discounts, currency conversions, conversions from one unit of measure to another, exchanges, and creation of capital companies, among others), practical geometry (especially the calculation of areas and volumes of concrete objects), and bookkeeping.

Even if the abbaco tradition failed to directly produce original mathematical results of any significance, it is a fact—and not a marginal one—that in late Medieval and Renaissance Italy it notably increased mathematical literacy through the gradual adoption of the Indo-Arabic number system and the introduction of mathematics into many professional activities. It would be a mistake, however, to reduce abbaco mathematics solely to its practical dimension. In the abbaco treatises—and foremost in Fibonacci's *Liber abaci*—one can find plenty of problems with no immediate practical application that fall in the category of "recreational" mathematics (games, riddles, and curiosities), and even algebra.[41]

It was precisely in the field of algebra that the contribution of abbaco mathematics was crucial, by paving the way to the most important mathematical discovery of the sixteenth century, one that marked the first real step forward of mathematics with respect to the ancient knowledge of the Greeks and the Arabs. It was a historic turning point, which had as one of its main protagonists the abbaco master from Brescia whose story we interrupted at the point where he moved to Verona.

According to archival sources, as early as 1284 the City of Verona had established a public abbaco school, and the following year a certain Lotto, from Florence, was appointed as a teacher with an annual stipend of fifty Veronan lire and free lodging.[42] Between the fourteenth and fifteenth centuries abbaco teaching thrived in Verona thanks to the pedagogical efforts of other Tuscan teachers and local tutors, appointed by the podesta (the highest judicial and military magistrate) and remunerated by the Merchants Guild, a powerful and influential arts and crafts corporation.

Niccolo Tartaglia was certainly a public abbaco teacher in Verona at least since 1529, as confirmed by official records held in the Archives of the State of Verona,[43] but it is likely that his appointment dated back to 1521 or shortly after. Other official documents, from the early 1530s, indicate that Tartaglia held classes at the Mazzanti Palace, located near Piazza delle Erbe, and that his financial situation was modest. This is confirmed by the 1531 Tax Roll, where it is stated that the tax assessed to "*Nicolas brixiensis magister abbachi*" (Niccolo from Brescia, abbaco teacher) was the meagre sum of zero lire and six soldi.[44] On the other hand, low pay and the resulting economic hardship was the lot of many abbaco and grammar teachers all over the country, who were often forced to move from city to city in search of better working conditions.

To supplement their insufficient income and make the most of their technical skills, abbaco teachers carried out various professional activities besides teaching; these included consultancy work, account auditing, land surveying, commercial advising, and assisting architects and engineers. Tartaglia was no exception: he once acted as accounting expert in a judicial dispute involving gemstone merchants in Verona;[45] on another occasion he was given the task of verifying the tables used by bakers to determine the weight of one soldo of bread as a function of the price of flour, which had undergone a sudden increase during a period—going back to 1531—when the city was hit by a severe famine.[46]

On the strictly mathematical front, the numerous problems in algebra, arithmetic, and geometry posed to Tartaglia by the most disparate people from Verona and elsewhere were clear evidence of his reputation as a talented abbaco master. For his part, Niccolo was well aware of the importance of dialogue with other experts or curious amateurs for his own investigations, and he stressed the benefits of having such conversations: "The questions and problems posed by wise and judicious interlocutors often prompt us to consider many things, and be acquainted with countless others which, had not been by the query, we would never have learned or considered."[47]

People of every social standing—merchants, engineers, architects, humanists, churchmen—turned to Tartaglia for advice or help with mathematical problems. He was also approached by amateur mathematicians and other abbaco teachers wishing to challenge him. And this is not just a manner of speaking: contests in which mathematicians challenged one another, true scientific duels carried out in a way reminiscent of chivalry tournaments, were very much in vogue in Italy in those times. A mathematician or scholar would send another a list of problems to be solved in a given amount of time—the "challenge gauntlet"—after which the recipient would propose a further set of problems to his rival. Tradition required that in case of disagreement a public debate should be held in which the contenders would discuss the disputed problems and solutions in front of judges, notaries, government officials, and a large crowd of spectators. It was not unusual in those duels for tempers to flare, and personal abuse take the place of scientific argument. Admittedly, the stakes could be very high: the winner of a public mathematical duel—whoever had solved the largest number of problems—gained not only glory and prestige but possibly also a monetary prize, new fee-paying disciples, appointment (or confirmation) to a chair, a salary increase, and, often, well-paid professional commissions. The defeated contender's future career, on the other hand, risked being seriously compromised.

Sixteenth-century mathematical duels had a long history and some illustrious predecessors. One famous example is the debate over mathematical, physical, and philosophical beliefs that took place in Ravenna on Christmas Day of the year 980 and that opposed the French humanist monk and scholar Gerbert of Aurillac and the German philosopher Otrich von Magdeburg. A large crowd, notably including Holy Roman Emperor Otto II, witnessed the day-long debate, with the contenders becoming "increasingly frenetic" and "showing no signs of stopping" as the hours passed.[48] At a certain point the emperor himself interrupted the discussion "because it was getting late and the audience was tired."[49]

In the end, it was the Frenchman who came out victorious, and not long after was appointed by Otto II, abbot at Bobbio, a small town in northern Italy. In 999, Gerbert of Aurillac would become pope and take the name Sylvester II. He was the first French pope in history and one of the most learned men of his time.

During the first half of the thirteenth century several mathematical contests had as protagonist the great and often-remembered Leonardo Fibonacci. Among the most remarkable ones are those in Pisa that placed him in competition against Johannes of Palermo, a scholar at the imperial court of Holy Roman Emperor Frederick II, in the presence of the emperor. Fibonacci later published the solutions to some important problems in algebra and arithmetic proposed to him by his rival during these contests.

Medieval university life was marked by scholarly debates—not only on mathematical questions—in which academics were required to participate. Ettore Bortolotti, a mathematics historian, writes:

> A 1474 directive ordered lecturers[50] at the University of Bologna to gather in the public square or under the porticos at the end of classes

> and therein engage in debates and challenge one another; some of the most famous ones, held at Square Santo Stefano, are still remembered today. These daily confrontations served as preparation for the real contests that took place on designated days in the presence of the entire academic body and before huge crowds, the contenders having to conform to rules set out in the statutes. Every lecturer [at the University of Bologna] was required to take part in these public disputes at least twice a year.[51]

The Bolognian debates "were followed with great deference and admiration," and they were so popular that often "there wasn't a hall big enough to accommodate everybody."[52]

An unwritten rule of Renaissance mathematical duels was that a challenger should not propose to his rival any problem he was not able to solve himself. This was precisely the reason for Tartaglia's skepticism and annoyance in receiving, in 1530, two problems from a certain Zuanne de Tonini da Coi,[53] a mathematics teacher from Brescia: "Find me a number that multiplied by its root plus 3 makes 5. Similarly, find me three numbers, such that the second is greater by 2 than the first and the third is also greater by 2 than the second; and such that the first multiplied by the second, and this product multiplied by the third, makes 1000."[54]

Tartaglia soon realized that the problems led to two algebraic equations that in modern notation are written $x^3 + 3x^2 = 5$ and $x^3 + 6x^2 + 8x = 1000$ (they are solved by finding the value of the unknown number x that makes them true).*

The cause of Tartaglia's puzzlement was simple: these are third-degree equations (also called *cubic* equations), which means that the unknown x is raised to the third power (or *cube*, $x^3 = x \times x \times x$), but at the time no general formula for solving all equations of this type was known. Actually, certain very particular cases of the equation could be solved by approximation methods, but Tonini da Coi's were not among these. A few decades earlier, in his monumental and influential *Summa* published in 1494,[55] the famous Tuscan mathematician Luca Pacioli—a Franciscan monk who taught in several Italian cities—had considered it "impossible" to solve the general equation by means of an algebraic formula with the algorithmic tools then available.[56] According to Pacioli, "art"—that is, algebra—could not have yet "formed" general rules for solving this kind of equation "except by feeling one's way in the dark, [. . .] in some particular cases."[57]

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* For the interested reader: in the first problem, the equation is obtained by letting x^2 be the unknown number and x its square root; then, the expression $x^2\,(x+3) = 5$ follows from the statement of the problem, and the final equation is arrived at by algebraic manipulation. The equation in the second problem results from denoting the unknown numbers by x, $x+2$, and $x+4$, respectively, so that we have $x(x+2)(x+4) = 1000$, from which the final equation can be obtained.

Was it then possible for such a not particularly gifted mathematician as Zuanne de Tonini da Coi to have suddenly pushed back the boundaries of algebraic knowledge when the ancient masters had failed? Tartaglia refused to believe it, even for a second. With his characteristic stubborn and caustic disposition, he replied to Messer Zuanne* in rather polemic terms, accusing him of bragging and ignoring Pacioli's highly respected opinion:

> Messer Zuanne, you have sent me these two questions of yours as something impossible to solve, or at least as being unknown to you, since proceeding by algebra the first leads to a cube plus 3 cenno equal to 5 [$x^3 + 3x^2 = 5$] and the second to a cube plus 6 cenno plus 8 things equal to 1000 [$x^3 + 6x^2 + 8x = 1000$].† These chapters[58] have until now been considered as impossible to solve with a general rule by Fra Luca and others. You believe with these questions to place yourself above me and appear as a great mathematician, as I heard you have done with all the other professors of this science in Brescia, who fearing your questions do not dare speak to you. Perhaps they understand this science better than you without knowing all the answers, but believing you do, they concede everything to you.[59]

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* The word Messer is an archaic title of courtesy prefixed to the first name (sometimes with the surname also) of an Italian man. It is more or less equivalent to the modern Italian Signor, which in turn is equivalent to the English Mr.

† In the algebraic language of the time, which was nonsymbolic, the term "thing" [cosa, in Italian] designated the unknown x, whereas "cenno" [censo] indicated the square of the unknown, that is, x^2. These terms later disappeared from the mathematical vocabulary, while "cube" [cubo], to express x^3, is still used today.

It appears then that this was not the first time Tonini da Coi had tried to impress fellow mathematicians, who, unaware of his inclination for boasting, might have been intimidated by his questions. But Tartaglia proved too clever for him, and da Coi received an abrupt and harsh rebuke. Niccolo finished his letter by declaring himself ready to bet that his challenger was incapable of solving the problems he had so daringly proposed:

> Hence, to correct the vain opinion you have of yourself and induce you to seek honors through knowledge rather than by posing questions you know nothing about, I reply to you that asking others for solutions you cannot find yourself should make you blush. And to prove to you how certain I am of this, I am willing to wager you ten ducats against five that you will not be able to solve with a general rule the two cases you proposed to me.[60]

His bluff being called, Messer Zuanne nevertheless took the blow, and in a subsequent letter asked Tartaglia if he, too—as his reply suggested—considered it impossible to find an algebraic formula to solve the cubic equations in question.

Niccolo replied with an unexpected admission: "I am not saying that such cases are impossible. On the contrary, for the first one, that of cube and cenno equal to number [$x^3 + 3x^2 = 5$ in modern notation] I am convinced to have found the general rule, but for the time being I prefer not to reveal it for several reasons [. . .]"[61]

Not to reveal it? Why would Tartaglia wish to keep secret a discovery of historic significance, which would be certain to bring him enormous prestige in mathematical circles? What could be his reasons for choosing silence? Had he really discovered a "general rule" for solving those types of cubic equations, or was he, too, indulging in unfounded boasting?

Only Niccolo knew the truth. In any case, in those times it was customary among mathematicians to keep their methods and results secret as long as possible, either for fear that once revealed, students or potential clients would no longer require their services, or for the purpose of eventually using them to their advantage in duels with other scholars. Seen in this light, Tartaglia's attitude merely conformed to the customs of his time. Besides, in his second letter in reply to Tonini da Coi, Niccolo admitted to not having yet found a general formula for solving the second equation proposed by Zuanne, adding, however, that it should not be impossible to obtain: "[. . .] as regards the second case, that of cube and cenno and thing equal to number [the equation $x^3 + 6x^2 + 8x = 1000$], I must admit not having been able until now to find a general rule, but I am not saying that one is impossible to find, even if none has so far been discovered."[62] The letter ends with yet another harsh scolding addressed to Tonini da Coi, and a repeated invitation to "blush" for having flaunted knowledge he did not possess.

And yet, it was precisely thanks to the questions posed by that braggart Messer Zuanne in 1530 that Niccolo began to turn his attention to those elusive cubic equations with the intention of finally cracking their mystery—an event that mathematics had been awaiting for millennia.

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