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FOREWORD TO THE PRINCETON SCIENCE LIBRARY EDITION vii

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CHAPTER ONE

Introduction

In the physics section of the University of Chicago catalog for 1898–99, one reads the following:

> While it is never safe to affirm that the future of the Physical Sciences has no marvels in store even more astonishing than those of the past, it seems probable that most of the grand underlying principles have been firmly established and that further advances are to be sought chiefly in the rigorous application of these principles to all the phenomena which come under our notice. . . . An eminent physicist has remarked that the future truths of Physical Science are to be looked for in the sixth place of decimals.

This catalog description was almost surely written by Albert A. Michelson, who was then head of the physics department and who had spoken very nearly the same words in a convocation address in 1894. The eminent gentleman whom he quotes may well have been Lord Kelvin. That 1894 talk proved to be well timed for contradiction. In quick succession, beginning soon afterward, there came the discovery of X-rays, radioactivity, the electron, special relativity, and the beginnings of quantum mechanics—all of this within a decade centered around the turn of the century. Indeed, it was Michelson himself, working together with E. W. Morley, who in 1881 had carried out the crucial experiment that was later recognized as a foundation stone of special relativity. Both Michelson and Kelvin received Nobel Prize awards in the early years of the twentieth century.

In short, all the grand underlying principles had *not* been firmly established by the end of the nineteenth century. This cautionary tale should not be told with any sense of mockery. Those distinguished scientists—and there were others who spoke along the same lines—were looking back on a century of extraordinary accomplishment, an epoch that had carried the physical sciences to a state of high development by the late years of the century. The wavelike character of light had been demonstrated; the laws of electricity and magnetism were discovered and placed together in a unified framework; light was shown to be the manifestation of electric and magnetic field oscillations; the atomic hypothesis had increasingly taken hold as the century moved on; the laws of thermodynamics were successfully formulated and—for atomists—grounded in the dynamics of molecular motion; and more. To be sure, although the gravitational and electromagnetic force laws seemed well understood, it remained yet to learn whether other kinds of forces come into play at the atomic level. That is, there was work yet to be done, and not just at the sixth place of decimals. But a clocklike Newtonian framework seemed assured. In this *classical* picture of the physical world, space and time are absolute; and every bit of ponderable matter is at every instant at some definite place, moving with some definite velocity along some definite path, all governed by the relevant force laws according to Newton.

This classical outlook in fact continues to provide an excellent description of the physical world under conditions where velocities are small compared to the speed of light and relevant dimensions large compared to the size of atoms. But our deeper conceptions of space-time have been transformed by relativity; and of objective reality, by quantum mechanics. Both run counter to everyday experience, to our common sense of the world. This is especially so for quantum mechanics, which is the focus of the present book.

Overview

Before we embark on our journey, it may be good in advance to sketch out very roughly some of the contrasts that will be encountered between the classical and quantum modes. For the most part here, we will be considering a system of point particles moving under the influence of interparticle and perhaps external force fields characterized by a potential energy function.

Quantization

Classically, a particle might be anywhere a priori; and it might have any momentum (momentum = mass × velocity). Correspondingly, its angular momentum—a quantity defined in terms of position and momentum—might a priori have any value. So too the particle's energy, kinetic plus potential, might have any value above some minimum determined by the potential. Quantum mechanically, however, angular momentum can take on only certain discrete values. It is *quantized*. Energy is sometimes quantized too, depending on details of the force field. It is this classically inexplicable discretization that provides the adjective "quantum" in quantum mechanics.

Probability

A much sharper and more profound contrast with classical mechanics has to do with the probabilistic character of quantum mechanics. For a classical system of particles, the state of affairs is completely specified at any instant by the position and momentum variables of all the particles. The data on positions and momenta at any instant constitute what we may call the *state* of the system at that instant. It tells all that can be known dynamically about the system. Other quantities of interest, such as energy, angular momentum, and so on, are defined in terms of the position and momentum variables. Classical mechanics is deterministic in the sense that future states of the system are fully and uniquely determined if the state is specified at some initial instant. The present determines the future. Of course, in practical situations the initial data will inevitably be compromised to some greater or lesser extent by measurement uncertainties. Depending on the system under consideration, the future may or may not be sensitive to this uncertainty. But there is no limit *in principle* to the accuracy that can be imagined. There is no bar in principle, that is, to precise knowledge of the position and momentum of each particle, and therefore no bar to anticipating future developments. When wearing our classical, commonsense hats, we do not doubt that every bit of matter is at every instant at some definite place, moving with some definite momentum, whether or not we are there to observe.

The notion of state also arises in quantum mechanics. Here again the *state* of a system connotes *all that can possibly be known about the system at any instant*. Also, just as in the classical case, the system develops deterministically in that future states are fully determined if the state at some initial instant is given. In this sense, here too the present determines the future. But there is a very profound difference. A quantum state does not precisely specify particle positions and momenta, it only specifies probabilities. Quantum mechanics, that is, is probabilistic! For example, there are states for which the probability distribution of a particle's position is sharply localized, so that the position may be said to be almost definite (at the instant in question). But there are other states for which the probability distribution is broad, so that upon measurement the particle might be found almost anywhere. And there are infinitely many possibilities in between. So too for momentum: for some states the momentum is almost definite, for others it is broad, and there are infinitely many possibilities in between.

This probabilistic description obtains not because we have imperfect information about the state of the system, but is intrinsic. Moreover, the rules of probability composition have some very peculiar features. We will, of course, go into these things more fully later on, but it is important already at this early stage to emphasize a point that may be illustrated with the following example. Suppose one sets up detectors at various locations to determine the position of a particle known (somehow) to be in a certain quantum state at a certain instant. If a particular detector clicks, we will have learned that the particle was in the volume occupied by that detector at the instant in question. That is, there *will* be a definite finding of location. But if the experiment is repeated over and over, always with the particle arranged to be in exactly the same state, there will be a spread of outcomes. On different runs different detectors will click. Full knowledge of the quantum state does not allow one to predict the outcome event by event, only the probability distribution.

The Uncertainty Principle

It is the case that any state that has a very localized probability distribution for position measurements will inevitably have a broad distribution for momentum measurements, and vice versa. There is a limit to how well one can jointly localize both position and momentum. So too for certain other pairs of *observables* (as measurable quantities are called). This is enshrined in the celebrated Heisenberg uncertainty principle. That principle is not some add-on to quantum mechanics; it is a technical consequence that flows from the structure of quantum mechanics. As must of course be the case, for the macroscopic objects of everyday life the Heisenberg limit is not at all a practical restriction. We can, for example, know both the position and momentum of a moving jelly bean quite accurately enough for all everyday purposes. However, at the atomic level the uncertainty principle comes fully into play.

Identical Particles

In the macroscopic world, we never encounter two or more objects that are strictly identical in every possible respect: mass, composition, shape, color, electric charge, and so on. But even if we did—and we do at the microscopic level, where, for example, one electron is exactly the same as another—this would pose no conceptual problem for classical science. One can in principle keep separate track of the objects by, so to speak, pointing: object 1 is the one that's at this place, object 2 is the other one over there, and so on. For quantum mechanics this approach has its limits. It is not possible to keep track in this way since locations are a probabilistic matter. Rather, there is a distinctly quantum mechanical approach to dealing with identity, one without classical analog. The implications are sometimes quite unintuitive, and they are profound. What is most remarkable is that all the known particles indeed come in strictly identical copies—all electrons are the same, all protons the same, and so on. Quantum field theory provides the only natural explanation for this striking fact of identity.

Radioactivity

This term refers to processes in which an atom *spontaneously* emits one or more particles: an alpha particle (helium nucleus) in the case of one class of processes, α decay; an electron (plus neutrino as we now know) in another class, β decay; an energetic photon in yet another class, γ decay. In α and β radioactivity, the parent atom is transmuted in the process into a daughter atom of a different chemical species. There is no such transmutation in γ radioactivity. One speaks of any of these spontaneous events as a *decay* process. In the case of α and β radioactivity there really is decay, the disappearance of the parent atom and its replacement by an atom of a different ilk. In γ radioactivity the atom does not change its chemical species membership; but as we will see later, it does undergo a change from one energy level to another. In that sense, here too there is decay—of the occupancy of the initial energy level.

Not all atomic species are radioactive, but many are. When radioactivity was first discovered around the end of the nineteenth century, there was great wonder and bafflement. Many questions were raised, among them the question: where in the atom (*if* in the atom) do the ejected particles come from? This was clarified only after Rutherford formulated his famous model of the atom, picturing it as a swarm of electrons orbiting around a positively charged nucleus that is very tiny but that nevertheless carries most of the mass of the atom. With that, it soon became clear that radioactivity is a *nuclear* phenomenon. Two other questions among many remained, and they were especially puzzling: (1) The emitted particles typically carry a lot of energy. Where does that energy come from? (2) How does the nucleus decide when to decay? As to the first of these questions, the answer was already available in Einstein's 1905 formula $E = mc^2$; but it took a while before this sank in conceptually and before sufficiently accurate mass measurements of parent and daughter nuclei could be made to test the concept.

The deeper question (2) had to await the interpretative apparatus of quantum mechanics. If you take a collection of identical atoms of some radioactive species, you will find that the atoms do not all decay at some one characteristic instant but, rather, at various times—randomly. If the emissions are being detected by a counter, you may hear individual clicks as one or another atom decides to decay. As time goes by there will of course be fewer and fewer surviving parent atoms. As it turns out, the population of survivors decreases with time in an essentially exponential fashion, the average time (or, briefly, the *lifetime*) being characteristic of the particular species under consideration. On the classical outlook, the problem is this. The atoms of the given species are presumed to be identical. If they are governed by the clockwork regularity of classical science, why don't they all decay at the same instant, whatever may be the mechanism that causes radioactive decay?

The quantum mechanical answer is that the world is a probabilistic place. An ensemble of identical atoms starting in identical conditions will distribute their decays in a probabilistic way over time. One cannot predict what will happen event by event, atom by atom. What *can* be deduced quite generally is the exponential character of the decay curve. But the mean lifetime varies from species to species and depends sensitively on details of the underlying quantum dynamics. It should be said here that the traditional classes of nuclear instability, α, β, and γ, are only three among a much wider range of decay processes that occur in nature, including hordes of reactions involving subnuclear particles: pi meson decay, muon decay, and so on. The average lifetimes vary over an enormous range, from roughly 10^{-24} seconds for certain subnuclear particles to billions of years and more for certain α emitters (among these, U^{238}, whose half-life happens to be about the same as the age of the earth).

Tunneling

The probabilistic structure of quantum mechanics incorporates the possibility that a particle can be found in locations that are absolutely forbidden to it classically. For example, it can happen classically that there is an energy barrier that separates one region of space from another, so that particles below some energy threshold cannot penetrate the barrier and thus cannot move from one region to the other (it may take more energy than you've got to climb the hill that intervenes between where you are and where you want to go). Quantum mechanically, there is a finite probability that such strange things can happen. Particles can be found in, and can *tunnel* through, classically forbidden regions.

Antimatter

In attempting to find a relativistic generalization of Schroedinger's quantum equation for the electron, P. A. M. Dirac devised a theory that was spectacularly successful in its application to the hydrogen atom but that carried with it some seemingly bizarre baggage: among other things, negative energy states for the free electron. When properly reinterpreted this transformed itself into the prediction of a new particle having the same mass as the electron but opposite (that is, positive) charge. The antielectron, or *positron* as one calls it, was soon discovered experimentally. The situation has since become generalized. Relativistic quantum theory predicts that particles having electric charge must come in pairs with opposite charges but identical masses (and identical lifetimes if unstable). One member of the pair is called the particle, the other the antiparticle. Which is called by which name is a matter of history and convenience. It turns out that there are other kinds of "charge" in addition to electric charge; for example, so-called baryon number charge. The necessity of particle-antiparticle pairs obtains for charges of any kind. Thus, not only is there an antiproton to the proton, there is an antineutron to the neutron. The neutron is electrically neutral but it has baryon number charge. On the other hand, the photon and π^0 meson among others do not have antiparticles; or as one says, each is its own antiparticle.

Creationism, Destructionism

Our notion of what it means to say that something is made of other things has undergone a revolutionary transformation in this century. When you take a clock apart you find gears, springs, levers, and so on (or maybe a quartz crystal and battery). You say the clock is made of these parts. If you take apart the parts in finer and finer detail, you eventually get to atoms. If you take apart atoms, there are electrons and nuclei of various sorts. Going on, you find that the nuclei are made of protons and neutrons, and then that these are made of quarks and gluons. At the microscopic level, incidentally, taking apart means zapping the target with a projectile and looking at the pieces that emerge. In earlier years the surprise may have been that deconstruction did not stop at the atom. Still, the ancient notion could persist that, eventually, one comes to the immutable ingredients of the world, building blocks that can arrange and rearrange themselves in various combinations but that are themselves eternal and indestructible.

Thus, for example, the nuclear reaction $d + t \rightarrow \mathrm{He} + n$ can be pictured as a mere rearrangement of the neutron (n) and proton (p) ingredients of the deuterium (d) and tritium (t) nuclei, the ingredients reemerging as the helium nucleus (He) with one neutron left over. The particle reaction (i) $\pi + p \to \Lambda + K$ might be taken to indicate that the particles involved here—pion, proton, lambda particle, kaon—are made of tinier things, perhaps quarks, that are similarly rearranging themselves. But if so, what does one make of the reaction (ii) $\pi + p \to \Lambda + K + \pi$, in which an extra pion appears on the right? Haven't the quarks already been conceptually "used up" to account for reaction (i), so that there are no ingredients left over to explain reaction (ii)? And what does one make of the reaction $p + p \to p + p + \pi^0$? No amount of rearrangement can explain how it is that the final system contains the same objects as the initial system *plus* something else. There is no getting around it, the π^0 is simply created here de novo; or at any rate its ingredients are. In short, down at the subnuclear level one is simply forced to acknowledge that particles can be created and destroyed!

This creation and destruction of matter is not something of everyday experience. It is a phenomenon that comes into play at high-energy particle accelerators, in the collisions induced by cosmic rays (high-energy particles that rain on the earth from outer space), in the stars and wider cosmos, and in certain radioactive decay processes. The transactions underlying most of science, technology, and everyday life have mostly to do with the "mere" motions and rearrangements of electrons and nuclei. However, there is one very notable exception to this, even in everyday life. It involves a thoroughly familiar phenomenon interpreted in a modern light, namely, light! A beam of light is nothing but an assemblage of massless particles, *photons*, moving at (what else?) the speed of light. Because they are massless, photons are easy to create. They are created whenever the light switch is turned on. Regarded microscopically, what happens is that they are produced in electron and atomic collision processes taking place in the light source when the latter is heated or otherwise "excited." Photons are destroyed when they impinge on and are absorbed by nontranslucent material bodies (walls, books, the retina of the eye, etc.).

Photon creationism-destructionism actually entered the world when Einstein proposed his particle-like interpretation of electromagnetic radiation. But the photon concept had a protracted birth, and the photon is anyhow such a special particle. It is massless; it is the quantum of a field we have known classically. Somehow, for photons, the enormity of creation-destruction as such did not seem to attract much philosophical discussion in the early years of this century. In any case, for a while one could still cling to the idea that “real” *ponderable* particles, particles with nonzero mass such as electrons, protons, and neutrons, are truly immutable. But there is no such immutability for them either. This first became apparent with the discovery of the neutron and the recognition of its role in nuclear beta decay. The basic beta decay reaction is

$$\text{neutron} \rightarrow \text{proton} + \text{electron} + \text{antineutrino}.$$

The neutron is destroyed, the proton, electron, and antineutrino created. The antineutrino, which is highly unreactive, easily escapes the nucleus and passes through the earth, the solar system, the galaxy, and into outer space without leaving much of a scratch. But that’s another story.

Where does quantum theory fit in? The quantum theory of the electromagnetic field got its start in the heroic period of the mid 1920s when the foundations of quantum mechanics were being established. Quantum electrodynamic theory was designed from the beginning to account for photon creation and destruction. The photon emerges naturally in the theory as a quantum of the electromagnetic field. Since that time physicists have brazenly invented other fields, fields not known to us in their classical guise but that are invented for the purpose of being quantized to yield other particles as well. So, for example, there is a field that makes and destroys electrons. The older theories used to have separate fields as well for protons, neutrons, pions, and so on. We have now reached a more basic level involving, among other entities, quarks and gluons. But these too can be created and destroyed.

Beginnings

In its modern form, the structure of quantum theory was laid down in the middle of the 1920s in a concentrated burst of creativity and transformation that is perhaps without parallel in the history of scientific thought. Mainly, the creators were very young: Werner Heisenberg, Paul Dirac, Pascual Jordan, and Wolfgang Pauli were all in their twenties. The elders included Erwin Schroedinger, who published his famous wave equation at age thirty-nine, and Max Born, who at the age of forty-three recognized and helped elaborate what Heisenberg had wrought. The new outlook brought with it an unintuitive concept of reality along with a number of attendant oddities of various sorts. Among contemporary physicists, some could not readily absorb the new doctrine. They grumbled and fell out. But already the earliest applications to phenomena met with convincing success. Informed dissidents, Albert Einstein foremost among them, soon accepted the effective correctness of quantum mechanics. They were reduced to hoping that classical reality prevails at some deeper level of nature not readily accessible to observation. That deeper level, if there is one, is still today nowhere in sight. As far as the eye can see, the principles of quantum mechanics stand irreducible and empirically unchallenged. In cases where the difficult experiments and corresponding theoretical calculations can be carried out with high precision, quantitative agreement is spectacular. As often happens in intellectual revolutions, it was the younger generation that could adapt to the new ways of thinking somewhat more easily than the older one. Succeeding generations have had an even easier time of it; they simply grew up with the subject. Nevertheless, the world view of quantum mechanics is odd; and the oddest thing of all is that, still today, many decades after its foundation, quantum mechanics continues to seem odd even to scientific practitioners who work with the subject every day and who know and operate confidently in its framework. Their wonderment expresses itself not so much at the operational level as at a philosophical one. Deep questions persist at that level. We will surely not resolve them here. The more modest aim here is simply to convey some notion of what quantum mechanics is: its principles and some of its consequences and oddities.

Many questions within the classical framework were still unresolved toward the end of the nineteenth century, especially questions having to do with the nature of atoms—and for some diehards, even the very existence of atoms. But the Newtonian framework was not in doubt. It is possible today in hindsight to recognize hints of quantum effects, empirical departures from classical expectation that should have been pounced on by our nineteenth century ancestors. However, this is only in hindsight. They *did* in fact encounter anomalies and *did* fret over them, but it was far from clear at the time that these could not be resolved within the still developing classical picture.

There are vast stretches of contemporary macroscopic science and engineering that still do very well today without any reference at all to the quantum mechanical basis of nature. This is so because classical Newtonian behavior emerges for the most part as a very good approximation to quantum mechanics for macroscopic systems. But this assertion has to be understood in a qualified sense. The qualification can be illustrated by means of an example. Consider the flow of oil through a smooth cylindrical pipe, the flow being driven by a pressure differential that is established between the ends of the pipe. If the pressure differential is not too large the flow will be smooth; and it is then an easy matter, a standard textbook problem in classical fluid dynamics, to compute the flow rate, the volume of oil transported per unit time. The answer depends on the length and diameter of the cylinder and on the pressure differential. These are parameters of experimental choice or circumstance. But the answer also depends on the viscosity of the oil. If the value of that parameter is simply accepted as a given fact of nature, as a quantity to be determined empirically, then the computation of flow rate may be said to proceed along purely classical lines without reference to quantum mechanics. However, to understand why oil has the viscosity and other properties that it has, one has to move down to the atomic level. And there the differences between quantum and classical science are as striking as can be.

Another qualification should be noted. The quantum mechanical rules, the concrete equations, are definite and well established. *In principle* one can compute the structure of oil molecules and work out the way these molecules interact among themselves in bulk oil and thence go on to the viscosity of oil. But a completely detailed calculation that traverses the whole route from the individual molecule and its ingredients all the way up to the astronomical number (about 10^{24}) of molecules present in even a small drop of oil is utterly unthinkable. The single molecule is already complicated enough. Thus, approximations and aggregate treatments have to be adopted along the way, relying on various rich and active fields of scientific inquiry; for example, the field of statistical mechanics. A pumper who wants highly accurate predictions of flow rate is well advised to adopt the empirical value of viscosity. But that same pumper may also share with others a curiosity about why things are the way they are. Moreover, there is the possibility of learning enough at the microscopic level to design molecular additives that can alter the viscosity in wanted directions.

As with viscosity, so too for other kinds of information that enter in parametric form into the various branches of classical science and engineering: tensile strength of materials, thermal conductivity, electrical resistance, equations of state (the relation of pressure to density and temperature) for various gases and liquids, optical reflection coefficients, and so on. The different fields have their independent methodologies and concepts. None suffers any shortage of engaging intellectual and practical challenges within its own framework. But so far as we know, science is seamless. At a deeper level the different fields share in common the science of atoms, where the quantum reigns. Deeper still is the fantastic world of the subatomic particles; and farther out, the world of the cosmos.

Quantum mechanics first began to intrude itself on mankind's attention in the very first year of the twentieth century. It did not by any means spring up full grown. The beginnings can be sharply placed within a rather esoteric corner of the scientific scene of those times; namely, the physics of *blackbody radiation*. The blackbody question has to do with the frequency spectrum of electromagnetic radiation that fills any volume of space surrounded by material walls in thermal equilibrium. That seems an awfully specialized topic. However, it had been established decades earlier through elegant thermodynamic reasoning that the spectrum, the radiation intensity as a function of frequency, must be of a fundamental character. It can depend only on frequency and temperature, not on the shape of the vessel nor, more strikingly, on the kinds of materials that the walls are made of. Deep issues therefore appeared to be at stake. Experimental measurements over various parts of the frequency spectrum were actively pursued toward the end of the century. The challenge on the theoretical side was to predict the spectrum. It was the German physicist Max Planck who succeeded. That was in the fall of 1900. We will describe the scientific issues more fully later on; but briefly, what happened was this. Presented with the latest experimental results on the blackbody spectrum, Planck sat down at one point and in not much more than an evening's work so far as we know, he devised—stumbled upon—an empirical formula that fit the spectral data remarkably well. This was something more than a case of raw curve fitting, however, since he brought to the task some guiding ideas that had emerged from earlier work by himself and others. Nevertheless, his formula was essentially empirical. Over the succeeding months he sought to deduce it within the framework of the classical theory of his times. This required some statistical mechanics reasoning. But the statistical mechanics aspects of classical science were still somewhat in flux and Planck did not recognize, or at any rate did not choose to follow, a simple path to the blackbody spectrum that was available to him. Had he taken that path (noticed slightly earlier by Lord Rayleigh), he would have encountered catastrophic disagreement with the data. Instead, he followed a more complicated route that was mostly classical in its outlines, but then did some fiddling that we will describe later on. Out came the empirical Planck blackbody formula! From this small seed came the quantum revolution.

There was no immediate commotion in the streets. Only a small band of scientists were participating in or paying close attention to these developments. Among those few it was pretty clear that something new was afoot, but it was far from clear what that new thing was. A decisive insight was provided by Albert Einstein in 1905, the miracle year in which, among other things, he published his papers inaugurating the special theory of relativity. What Einstein drew from Planck's discovery was the startling hypothesis that electromagnetic radiation of frequency f can exist only in discrete energy bundles, *quanta*, and that the energy of each such bundle is proportional to the frequency: energy $= hf$, where the proportionality constant h is the new parameter of nature that had entered into Planck's blackbody formula. These quanta of Einstein are particle-like entities that have since come to be called *photons*. However, light is nothing but a form of electromagnetic radiation; and one of the triumphs of nineteenth century science had been the discovery that light is a wavelike phenomenon. Here then, with Einstein's quanta, was the beginning of the celebrated wave-particle duality conundrum that hovered over physics during the next two decades.

Quantum ideas were soon extended from radiation to ponderable matter. In fact, Planck's work had already suggested some sort of energy quantization for ponderable matter; but, excusably for that pioneering effort, the suggestion was rather murky. Following up on these hints, in 1907 Einstein developed a simple quantum model of the specific heat of material bodies. Specific heat is a parameter that characterizes the temperature change induced in a material body when it absorbs a given quantity of heat energy. Einstein proceeded as follows. Material bodies can of course sustain sound waves over some range of frequencies f. For these he adopted the same quantization hypothesis that he had adopted for electromagnetic radiation; namely, the assumption that the energy in a sound wave disturbance of frequency f can come only in bundles of energy hf. He was content to take a single representative frequency. Others soon generalized to cover the whole frequency range. The model provided a qualitatively successful account of certain anomalies, departures from the expectation of classical theory, that had been known empirically for some time. The band of scientists paying attention to quantum developments began to grow.

In 1913 the young Danish physicist Niels Bohr turned to the inner workings of the atom. What might the developing quantum ideas have to say on this subject? For the content and structure of the atom he took up a model that had been convincingly proposed only a couple of years earlier by the great experimentalist Ernest Rutherford. In it the atom is pictured as a kind of miniature solar system: a tiny, positively charged nucleus at the center (analog of the sun), and very much lighter, negatively charged electrons (the planets) orbiting around the nucleus. Rutherford came to this picture of the atom through a celebrated experiment in which his colleagues H. Geiger and E. Marsden bombarded a thin metal foil with fast alpha particles and observed, to their wonderment and Rutherford's, that the alpha particles occasionally scattered through large angles. Collisions with the atomic electrons, which are very tiny in mass, could not be expected to produce substantial deflections of the fast, heavier alpha particles. But a heavy, highly concentrated positive charge, an atomic nucleus, would do the trick. On this picture, Rutherford could work out the expected distribution of scattering angles, proceeding along classical Newtonian lines based on the Coulomb law of force between charged particles. The result agreed well with experiment and confirmed Rutherford in his model of the atom.

But the Rutherford atom presented a conundrum. To illustrate, consider the simplest atom, hydrogen. It has a single electron moving around a proton nucleus. The electron, acted on by the Coulomb force of the nucleus, is in a state of accelerated motion. According to the classical laws of electricity and magnetism, an accelerating charge must constantly be emitting electromagnetic radiation and thereby losing energy. Suppose for a moment that this energy loss can be ignored. Then, classically, the electron travels in an elliptical orbit with a revolution frequency that depends on the electron energy among other things. It radiates at the frequency of that orbital motion. But there are infinitely many *possible* orbits, just as in the case of objects (planets, comets, asteroids, spaceships) moving around the sun. Given a macroscopic collection of hydrogen atoms, it would be surprising if the electrons in the different atoms were not traveling in a whole range of different orbits. That is, on this picture one would expect an essentially continuous spread of radiation frequencies. In fact, however, atoms radiate only at certain discrete frequencies, in a characteristic pattern that distinguishes one species of atom from another (one speaks of the characteristic frequencies as "lines" since they show up as lines in a spectrographic display). An even more serious problem for the classical Rutherford atom is that one is not really allowed to ignore the fact that the electron is losing energy as it radiates. Instead of traveling steadily on an elliptical orbit, therefore, a classical electron must eventually spiral into the nucleus, its orbital frequency and thus the radiated frequency changing all the while as the orbit shrinks in size. Empirically, however, nothing like this makes either spectroscopic or chemical or common sense. Confirmed atomists had in fact been confronted with these paradoxes for a long time, trying to figure out how it is possible, classically, to stabilize atoms against radiative collapse; also, how to account for their discrete line spectra.

Here, presented in a series of steps, is what Bohr did to resolve the conundrum, at least for the one-electron atom. Step 1: Ignore radiation for the moment and work out the electron orbits using purely classical dynamics, as discussed above. Bohr restricted himself to circular orbits. Step 2: Now impose a "quantum condition" devised by Bohr to determine which orbits are quantum mechanically "allowed," all others simply being forbidden! A consequence of this will be that only certain energies are possible. Instead of spanning a continuous range of possible values the allowed energies now form a discrete set; they are *quantized*. Step 3: Assert that the electron does not radiate while moving in one of these allowed orbits. But when the electron happens to be in an excited level of energy E and "decides" to jump to a lower level of energy E', it emits a photon of frequency f determined by the equation $hf = E - E'$. This equation is arranged to insure energy conservation, since according to Einstein hf is the photon energy.

Bohr invented his rules very soon after learning of a remarkably simple empirical formula that the Swiss schoolteacher, Johann Jakob Balmer, had devised many years earlier for the frequencies of the hydrogen atom. Balmer's formula, which involved only a single adjustable parameter (the "Rydberg"), predicted that there should be infinitely many hydrogen lines. Only several of the lines were known in Balmer's time, many more when Bohr turned to the subject. There can be no doubt that Bohr tailored his quantum rules to fit the facts. But the remarkable thing is that he *could* fit the facts, that his simple but classically inexplicable rules worked. Bohr could determine the Rydberg solely in terms of basic parameters that were already known and over which he had no freedom to make adjustments; namely, the charge and mass of the electron, and Planck's constant h. The agreement with experiment was very good indeed.

A vigorous and greatly broadened era of quantum theory now got under way as physicists sought to expand Bohr's beachhead to cover the effects of external electric and magnetic fields on the energy levels of hydrogen, to incorporate relativistic effects, to apply quantum ideas to multielectron atoms, and so on. Bohr's quantum conditions were speculatively generalized to cover this wider range of questions. Just as in Bohr's original formulation, the generalized rules had an ad hoc character: quantum conditions superimposed on top of classical reasoning without any deeper understanding of where those quantum conditions come from. To a considerable extent, developments were guided by the so-called *correspondence principle*, which had been formulated and exploited by Bohr and then taken up by others. Roughly, it is the notion that quantum behavior must resemble classical behavior for large energies. This idea was adopted and then ingeniously (and nervily) pressed into service for all energies. There were failures, but there were also many successes. It was a zany era of progress and confusion, a hodgepodge of inexplicable quantum rules and classical dynamics. It flourished for about a dozen years, the interval between Bohr's 1913 papers and the birth of modern quantum theory. The physicist Isidor Rabi, looking back, described it as a time of "artistry and effrontery."

The modern theory began along two seemingly unrelated lines, one opened up by Heisenberg, the other independently by Schroedinger. The pace was breathtaking. The first steps were taken by Heisenberg on a holiday in the spring of 1925. Although constrained and indeed guided to some extent by the correspondence principle, he broke sharply with the concepts of classical mechanics at the atomic level. He argued for abandoning the notion of definite positions and momenta on the ground that these are basically unobservable at that microscopic level. But atomic energy levels *are* observable through their role in determining the frequencies of atomic lines. Heisenberg set up a new mechanics aimed at that target. What he postulated seemed to come out of the blue; and it was expressed in a mathematical language that was unfamiliar to many, even to Heisenberg himself. However, it had the air of being on the right track. Heisenberg's mentor at Göttingen, Max Born, received the paper favorably, puzzled a while over the mathematics, then recognized it for what it was. Within a few brief months, by September, he and another assistant, Pascual Jordan, completed a paper extending Heisenberg's ideas and identifying his mathematical objects as *matrices*. The story is told—if true, it says something about the times—how the then unknown Jordan came to work with Born. The young man found himself traveling in a railroad compartment with Born and a colleague of Born's. Born was talking to his colleague about matrices. Jordan overheard, introduced himself, and said that he knew about matrices and maybe could help. Born signed him on, just like that! Their joint paper was produced not much later. Soon after, in November, Heisenberg joined Born and Jordan to produce the celebrated "three-man" paper (*Dreimanner Arbeit*) which set out in an extended and logical framework Heisenberg's quantum theory, now dubbed *matrix mechanics*. Meanwhile, basing himself only on Heisenberg's original paper and unaware of the work of Born and Jordan, Paul Dirac in Cambridge similarly extended Heisenberg's ideas, in a different, elegant mathematical language. It brought out the formal similarities between quantum and classical mechanics, and also the differences. Before the year was out Pauli had already applied the new quantum theory to the hydrogen atom. In particular, he successfully worked out the effect of an electric field on the energy levels of hydrogen, a problem that could not be tackled in the old quantum theory.

All of this was in the space of not much more than half a year! And then, in the very first month of the next year, 1926, there came the first of Schroedinger's papers laying out what looked to be an entirely different quantum theory. Schroedinger built on an idea that had been introduced several years earlier in the doctoral dissertation of Louis de Broglie, who was by then almost an elder at age thirty! What de Broglie suggested was that just as light had been shown to be both wavelike and particle-like, so too perhaps there are "matter waves" somehow associated with ponderable matter, for example, electrons. Einstein recognized the promise in this idea and gave it his influential blessing. Schroedinger extended it into a full-blown theory. Pursuing analogies with classical mechanics and optics, he introduced the idea of a wave function that is to be associated with any system of material particles; and he wrote down an equation that the wave function must satisfy, all of this even though the physical meaning of this function was initially quite vague. No matter that it was vague, however. The equation passed a first and by now mandatory test. It produced the right energy levels for the nonrelativistic hydrogen atom. Except for some initial reserve, even grumpiness, on the part of Heisenberg and others at Göttingen, Schroedinger's papers quickly captivated the world of physics. Unlike matrix mechanics, his wave mechanics was expressed in a familiar mathematical language; and, initially, it had about it the air of a theory that might be reconciled with classical notions of reality. That latter proved to be an illusion.

If a vote had been taken at the time to choose between the two theories it is probable that most physicists would have boycotted the election altogether (a pox on both of these newfangled quantum theories!). Among the voters, however, it is likely that the majority would have opted for wave over matrix mechanics. But it soon transpired that these two theories are really one and the same, as Schroedinger could demonstrate convincingly enough and as others could soon prove to higher standards of mathematical rigor. The two theories, that is, are just two different mathematical *representations* among an infinite number of other, possible representations of the same physics. This is not altogether unlike the case of different coordinate systems being used to describe the same phenomena but from different vantage points. The principles of quantum theory can in fact be formulated in highly abstract terms that do not commit to any particular representation. However, both for practical calculations and for purposes of developing an intuitive feel for quantum mechanics, it is usually best to come down from the abstract heights. It will be most convenient in the present exposition to proceed along the Schroedinger line.

Quantum mechanics was taken up widely and quickly following the papers of the founders. The earliest applications concentrated on various energy level problems. It was possible to address this class of problems without facing up to broader interpretative questions; in particular, questions having to do with the physical significance of the Schroedinger wave function. The modern interpretation was supplied soon enough, however, beginning with a remark made by Born in a 1926 paper on the quantum theory of scattering. This was swiftly elaborated. Above all others, it was Niels Bohr who presided over development of the general interpretative principles of quantum mechanics. What emerged was the picture of a probabilistic structure of nature and hence a sharp break with intuitive notions of reality. Among the giants, Schroedinger himself resisted, as did Einstein. Einstein watched with "admiration and suspicion." For a time he pressed his antiprobabilistic outlook ("God does not play dice") in a celebrated series of debates with Bohr. Bohr won. Einstein eventually accepted the correctness of quantum mechanics as far as it goes; but for the rest of his life he held out for the existence of a deeper, not yet accessible, level of classical reality.

What does the wave function signify? Everything. According to the principles of quantum mechanics the wave function incorporates all that can be known about the state of the system at any instant. But it does not in general tell where the particles are located or what their momenta are. What it gives us, and that's all we can know, are *probabilities* concerning the outcomes of various kinds of measurements that might be made on the system, measurements of position, momentum, energy, angular momentum, and so on.

The contrast with classical language is interesting here. For example, a classical scientist will write "let x denote the position of the particle," rather than "let x denote the outcome of a *measurement* of the position of the particle." Classically, unless one is concerned with the practicalities of a measurement, it will be understood that the particle surely *is* somewhere. Yes, its position variable *can* in principle be measured, but there is no need to emphasize the latter point or speak of measurement. Quantum mechanically, on the other hand, the particle is *not* at some definite place, not unless a measurement reveals it to be at that place. One can speak only of probabilities in connection with a measurement of position and other variables. The notion of measurement, therefore, is nearer to the surface in quantum mechanics. Heisenberg: "We can no longer speak of the behavior of the particle independently of observation." Bohr: "An independent reality can neither be ascribed to the phenomena or the agencies of observation." Three baseball umpires: First umpire, "I calls them the way I sees them." Second umpire, "I calls them the way they *are*." Third umpire, "They ain't nothing till I calls them."

Let us return briefly to the historical story. Schroedinger's version of quantum mechanics brought out clearly the wave-particle duality aspect of ponderable matter. Wave-particle duality for electromagnetic radiation, whose particle-like aspect is the photon, found its proper quantum basis in 1927 with the application of quantum principles to the electromagnetic field. This was the work of Paul Dirac, who inaugurated *quantum electrodynamics* in a paper published that year. Dirac struck again in the following year, 1928, with his relativistic wave equation of the electron. Apart from an unsuccessful early attempt to marry his quantum ideas to special relativity, Schroedinger's quantum theory had addressed itself to nonrelativistic situations, situations where velocities are small compared to the speed of light. Dirac succeeded in constructing a relativistic quantum theory of the electron, a theory that incidentally (!) predicted the existence of antiparticles—although Dirac did not initially recognize that implication.

By the end of 1928 the foundations of quantum theory were firmly settling in.

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