Contents

Chapter One

Introduction

1.1 DARK MATTER: THE GREATEST MYSTERY

We live at an extraordinary time in scientific history. Never before have we known so much about the Universe and yet been so certain of our ignorance about it. The twentieth century saw the development and refinement of quantum field theory (QFT) and general relativity (GR). QFT is the language in which the Standard Model is written and provides the tools with which it has been proven with great accuracy. GR allows us to conceive a science of the whole Universe and once again make precise calculations about it that have stood up to every conceivable test.

Cosmology, the large-scale application of GR and its development as a precision observational science, is where the cracks show their widest. On the one hand, GR and QFT can be married perfectly to predict the light element abundances and the existence of the cosmic microwave background (CMB). The precise form of the CMB, as expressed in the peaks and troughs of the anisotropy multipoles, however, tells us that the Standard Model and GR cannot be the whole story. A consistent theory of the CMB and galaxy formation requires that the vast majority of the Universe's mass density is composed of a new form of matter: dark matter (DM).

We live in the post-discovery world of dark matter. The total mass density has been measured to percent-level precision. The time at which it was created in the early Universe can be narrowed down to within a few days or even fractions of a second. CMB lensing measurements map the location of dark matter filaments stretching across billions of light-years. Inside galaxy clusters, lensing reveals vast 'halos' of dark matter moving independently of the baryonic gas. Dark matter provides the cosmological conditions for galaxy formation and star formation. We know where dark matter is, how it moves under gravity and how much of it there is. Part I of this book will convince you that dark matter has been discovered at the macroscopic level.

It is often said that we don't know what dark matter is, but that is not really true. The macroscopic theory works perfectly, and the possibilities for microscopic theories are severely restricted. The list of things that dark matter might be can be broken down into three distinct categories, covered in Part II of this book as 'WIMP-like', 'axion-like' and 'macroscopic' or 'primordial black hole (PBH)-like'. In each category, the space of possibilities is restricted and bounded by precise measurement (we know an awful lot about what dark matter isn't), leaving only finite regions in which the dark matter could be hiding. In Part III of this book we show, for each category, where and how to look for evidence that would favour one category of dark matter candidate over another: how to achieve the 'microscopic' discovery of dark matter.

The science of dark matter has progressed in rigour and complexity in the last forty years to arrive at this almost complete picture. The tools are at hand in almost every case to close those final windows of possibility and discern the microscopic nature of dark matter: a task I am personally confident will be achieved within my lifetime. The twenty-first century will be the century of dark matter. The stage is set, the players are ready, the curtain is about to rise.

In honour of the University of Göttingen, where this book was conceived, we are reminded of the saying of its greatest son:

> Wir müssen wissen; wir werden wissen.
> We must know; we will know.
>
> *David Hilbert*

1.2 OVERVIEW OF THIS BOOK

This book is designed as a relatively self-contained introduction to the vast and multidisciplinary topic of dark matter. Theoretical background is kept to a minimum, and we cover only some cosmology (chapter 5) and a tour of the Standard Model (chapter 7). You will need a physics background in special relativity, thermodynamics, classical mechanics (including Lagrangian/Hamiltonian dynamics and classical field theory), electromagnetism and quantum mechanics. The mathematics should pose no problem to anyone with a bachelor's degree in physics. Nonetheless, to really get the most out of this book, you will need accompanying introductory texts on cosmology and particle physics. Recommendations are given in the bibliography.

As pertains to dark matter, in every area of this book we attempt some level of balance between topics that has been absent in other reviews and discussion. This means that each topic alone may receive less detail than in specialised reviews, but we hope that the broad scope is nonetheless valuable.

In Part I we cover astrophysical and cosmological evidence and attempt balance between galactic rotation curves (chapter 3), lensing and dynamics of clusters (chapter 4) and cosmological structure formation (chapter 6). Part II focuses on theories for dark matter, giving balance to WIMP-like models (chapter 9), axions (chapter 10) and primordial black holes (chapter 11).

Part III covers constraints on each type of dark matter where we balance not only the treatment by theory, but across laboratory and astrophysical limits. For WIMPs, we cover both direct (chapter 12) and indirect (chapter 13) constraints. Similarly for axions, we cover direct (chapter 14) and indirect (chapter 15) constraints. Only for PBHs are we forced to address only indirect constraints (chapter 16), the reason being that direct collisions of PBHs with Earth that would allow direct detection are thankfully rare. The book closes in chapter 17 with a brief epilogue on the theory zoo beyond standard cold dark matter.

Each chapter in this book aims to build intuition and methods for approximation, rather than teaching technical methods. There are eighteen short quizzes throughout the book to test ongoing understanding (and assumed background knowledge). Each part ends with a set of original problems. Online material includes worked examples of numerical calculations and plotting using JUPYTER notebooks.

We have not written this book as a review article, filled with references. References given in the text are only to those things we relied on directly in writing the material. The bibliography in the appendix gives only bare-bones suggested further reading. It should help a beginner to get started diving into the vast literature on dark matter, but it is only intended as a pointer. The bibliography is not exhaustive, nor is it historically complete.

1.3 HISTORICAL NOTE

The story told in Part I of this book on the evidence for dark matter follows the well-worn and idealised folk history of the subject, as told by teachers to students and used in the introductions to countless theses, papers, books, television programmes and research seminars. This idealised story begins with Fritz Zwicky in the 1930s, moves on to Vera Rubin, stopping briefly at the Bullet Cluster and eventually reaches the crowning glory of the cosmic microwave background anisotropies in the 2000s. Of course, the history of the discovery of dark matter, and its acceptance by the scientific community, is much more complex than this. An excellent telling of this history is given by Bertone and Hooper [1]. The following short section tells this extended history briefly, relying entirely on Bertone and Hooper (where you can find all of the original sources), along with some other important notes on history that do not appear in the rest of this book.

An early analogy to the present situation can be found with Le Verrier, who, by observing the motions of the planets, recognised that a new perturbing gravitational force was needed. This force was provided by a heretofore dark object, the position of which he predicted precisely and which was later observed as the planet Neptune. The case of the orbit of Mercury provides the counterpoint story: the necessary dark matter in this case, the planet Vulcan, was found not to exist, and the mystery was only solved by replacing Newtonian gravity with GR. This story of dark matter versus modified gravity is one we will meet in this book, and we will see how only dark matter can explain the evidence on all scales.

The history of our dark matter begins 30 years prior to Zwicky. Lord Kelvin and Henri Poincaré used the virial theorem (see chapter 2) in our own galaxy to determine that invisible matter accounted for up to half of the local matter density. Given how approximate he knew their calculations to be, Poincaré concluded that DM was not necessary within the uncertainty. Now we know that their 50-50 estimate is actually quite close to the truth about the local DM density (see section 3.4).

The continued application of Kelvin's ideas, the 'theory of gases', to determine the density of matter in our own galaxy was taken up by many more scientists in the following years, including Oort, Jeans, Kapteyn and Linblad. Their estimates of the density of dark objects were of the order of 0.05 solar mass per cubic parsec, a few times 10^{-24} g per cubic centimetre or, in our favoured units, about 1 GeV per cubic centimetre. These estimates are also on the correct scale of current measurements. As early as 1936, Hubble wrote that he considered the problem of missing mass 'real and important'.

In Zwicky's story, there is yet another success compared to modern theory, which came the same year as his famous work on the Coma Cluster. Likely in reference to Oort and others measuring the local density, Zwicky compared this to the expansion rate of the Universe determined by Hubble and determined an overdensity in galaxies compared to the cosmic density of around 10^5. Zwicky's famous work determining the presence of DM in the Coma Cluster (section 2.1.2) was actually pretty inaccurate for DM density in clusters. The immense distance to clusters required use of the Hubble expansion rate when measuring velocities, which at the time was determined inaccurately to be around ten times higher than the value we know today. This error, and the fact that Zwicky could not observe the X-ray emitting gas in the cluster, led to a vast overestimate of the required amount of DM in Coma.

The story in this book then jumps ahead nearly forty years from Zwicky to Rubin, but the time in between was in reality far from scientifically quiet. There were worries that the virial theorem could not be applied: that the systems under study were not in equilibrium. This can be proven by showing that the out-of-equilibrium expansion of clusters would lead to sizes and ages incompatible with those measured (something we will meet in a different context in sections 15.1.2 and 16.3 as providing constraints on 'fuzzy' DM, and PBHs from the out-of-equilibrium heating of star clusters). There were also worries that individual interactions of stars were important, but Chandrasekhar proved that such interactions were negligible.

Rubin's tool, the rotation curve (see chapter 3), was used by many predecessors, of which we note just a few here. As early as 1939, Babcock measured the rotation curve of Andromeda, M31, out to large distances and noted the need for 'absorption in the outer parts', that is, a presence of more mass than he could see light from. A leap forward for M31 came with the advent of radio astronomy and the measurements of the outer rotation curve observed in 21 cm emission in 1957 by van de Hulst et al., and in 1966 by Roberts.

A notable work in this period that foreshadowed the modern interpretation was in 1963 by Arigo Finzi, who saw a commonality between missing mass implied by galactic rotation curves and the virial theorem in clusters. The work is significant because Finzi also proposed many candidates for baryonic dark matter (dwarf planets and so on) and even modified gravity.

Rubin and Ford's famous work came in 1970 as did many other flat rotation curves and mass-to-light ratios that rise with increasing radius from the centre. In particular, Albert Bosma measured twenty-five flat rotation curves. The significance of such measurements as pertains to dark matter became apparent to Morton Roberts and Ken Freeman.

A further shift to the modern era came in 1974 when key figures in Europe and the United States began to take the evidence from clusters, rotation curves and an apparently flat Universe together as a common problem. The connection to the total density of the Universe is related to the key observational number that all DM theories must today predict: the 'relic density' (see sections 5.1.4 and 6.1). Another key advance at this time came due to computing power, which allowed for simulations with hundreds of mass elements ('particles'), which allowed Peebles and Ostriker to show that DM halos were necessary theoretically for the observed fact that stellar disks in galaxies are stable, which they cannot be if they themselves dominate the gravitational potential.

In chapter 6, we meet our modern mainstay: the cosmic history of DM and the precision evidence cosmology provides for its existence, which was pioneered by Peebles and collaborators in the late 1970s and early 1980s. Peebles introduces the concept of the "primeval" (primordial in modern language) power spectrum, which was starting to be calculated also by particle theorists working on inflation (see sections 5.3 and 11.3). It was only at this time that joined up modern thinking about DM emerged. Prior to the 1980s, astronomers thought of the missing mass as astronomical objects, not as particles created in the early Universe. But Peebles saw this, and his work in the 1980s even gives us a correct lower limit to thermal DM: what we call the 'warm DM' bound that the particle mass should be larger than about 1 keV (see section 13.1). Finally, in 1985, further advances in computing by Davis, Efstathiou, Frenk and White allowed for the first cosmological simulations of cold dark matter, which gave rise to a 'cosmic web' consistent with what was observed in the large galaxy surveys that were made around the same time. Thus it is in the 1980s that we can say that the modern era of dark matter begins.

The story of dark matter from the 1980s to the present day is given in a more complete way throughout this book. Here we give just a brief sense of the historical narrative. On the theory side, the 1970s saw each of our main theories written down. Supersymmetry (section 9.2.2) was developed throughout the decade by a large number of scientists. The year 1971 saw the first discussion of PBHs (chapter 11) by Hawking, and 1977 and 1978 saw the proposal of the QCD axion (section 10.2.1) by Peccei, Quinn, Weinberg and Wilczek.

Axions and supersymmetry (SUSY) came into the DM game in the 1980s, with both theories significantly refined. For SUSY, it was in 1981 that Dimopoulos and Georgi constructed the supersymmetric standard model, while for the axion the modern theories appeared between 1979 and 1981. In remarkable synchronicity, yet again the relic densities of both the QCD axion and the SUSY neutralino were computed in 1983 in a series of papers by different authors.

At this stage the connection between theory and observation was so strong that experiments were built to search for each dark matter candidate. The results of these experiments were published in 1987: from the Homestake Mine (section 12.3) and a series of axion haloscopes (section 14.2). How different the scientific landscape would be if these first detection attempts had been lucky enough to succeed!

The years since 1987, the year of my birth, have seen the hunt for dark matter intensify. In cosmology, the 1990s and 2000s saw the dawn and apex (literally: the first acoustic peak) of the measurement of cosmic microwave background anisotropies, determining the presence and necessity of dark matter with exquisite accuracy. The 2000s saw huge leaps forward in the scale and precision of dark matter direct searches for both axions and WIMPs. The dawn of gravitational wave measurements in 2015 saw the great PBH revival. Now, entering the 2020s, we see a newly invigorated programme to close in on dark matter from all sides and on all fronts. This book will give you the tools to join history here and, after the microscopic discovery of dark matter, to continue the path to dark matter precision science.

David J. E. Marsh, December 2022

Warm-up Problems: Units

In this warm-up problem set we discuss units in particle physics and cosmology, deriving various mass scales, time scales and length scales, while briefly introducing some key concepts for later.

Here are the values of fundamental constants, in SI units, and some unit conversions, that you will need.

$$c = 3.00 \times 10^8 \text{ m s}^{-1}; \qquad \text{speed of light in vacuum,} \tag{1.1}$$

$$\frac{h}{2\pi} = \hbar = 1.05 \times 10^{-34} \text{ J s}; \qquad \text{reduced Planck's constant,} \tag{1.2}$$

$$k_B = 1.38 \times 10^{-23} \text{ m}^2\text{kg s}^{-2} \text{ K}^{-1}; \qquad \text{Boltzmann's constant,} \tag{1.3}$$

$$G = 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}; \qquad \text{Newton's constant,} \tag{1.4}$$

$$1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}; \qquad \text{electron volts,} \tag{1.5}$$

$$1 \text{ Mpc} = 3.09 \times 10^{22} \text{ m}; \qquad \text{megaparsecs.} \tag{1.6}$$

Problem 1. *Natural Units in Particle Physics*

By measuring speeds in units of c, and angular momentum in units of $\hbar$, the fundamental constants related to relativity and quantum mechanics, we can express many dimensionful quantities in units of energy (we will deal with temperature in Problem 3). The unit of energy commonly used is the electron volt.

- The proton mass is $m_p = 1.67 \times 10^{-27}$ kg. What is its mass in eV? In MeV? (1 MeV $= 10^6$ eV.)
- What is the 'nuclear time scale', $1/m_p$, in seconds?
- The Higgs has mass $m_H = 126$ GeV. What is the Higgs mass in kg?
- The maximum energy of the Large Hadron Collider (LHC) is 14 TeV (1 TeV $= 10^9$ eV). Assume that all of this energy can go into pair producing new particles (this is not really the case, since the total energy is distributed among the different constituents of the proton). If the lightest supersymmetric particle (LSP) is *just* accessible via pair production, that is, only two particles are produced in the final state, at the LHC, what is the lightest m_{LSP} can be in kg?

Problem 2. *The Hubble Scale*

The Hubble rate measures the expansion of the Universe. The Hubble rate today is approximately $H_0 = 70\,\text{km s}^{-1}\text{ Mpc}^{-1}$.

- What is H_0 in yr^{-1}? The Hubble time, $1/H_0$, is a good estimate for the age of the Universe; what is it? (We will calculate the age of the Universe more precisely in problem I.9 later.)
- What is H_0 in eV? In kg? Notice how small this is in comparison to typical particle physics energy scales.
- What is the Hubble length, $1/H_0$, in Mpc? This is the length scale associated with the size of the Universe.
- If the Hubble rate as a function of time is given by $H(t) = H_0(t_0/t)$, where t_0 is the age of the Universe, what was the Hubble rate, in eV, when the Universe was just 300,000 years old? What was the Hubble length in Mpc? The CMB (see Problem 3) was first formed at about this time.

Problem 3. *Temperature*

By using units where Boltzmann's constant, k_B, is also set to unity, one can also measure temperature in electron volts. For systems in thermal equilibrium, the temperature sets the scale of the kinetic energy of particles, $\mathrm{KE} \sim k_B T$. Temperatures in SI units are measured on the Kelvin scale, where absolute zero occurs at zero Kelvin, $0\ \mathrm{K} = -273°\mathrm{C}$.

- The CMB is the leftover radiation from the Big Bang. Its temperature was measured by COBE in 1992 to be $T_{CMB} = 2.7$ K. What is this in eV? This is the current temperature of the Universe.
- Big Bang nucleosynthesis (BBN) refers to the formation of the light elements (hydrogen, helium and lithium) in the early Universe. This occurs at around an MeV. What is this in kelvin?
- Einstein's famous formula $E = mc^2$ gives the energy of particles at rest, the 'rest energy'. For particles with relativistic momentum $p = \gamma m v$, moving at speed v and 'boost factor' $\gamma = 1/\sqrt{1 - v^2/c^2}$, it is given by

$$E^2 = m^2c^4 + p^2c^2. \tag{1.7}$$

  The kinetic energy is $\mathrm{KE} = p^2/2m$. Assuming thermal kinetic energy, at what temperature does the kinetic energy equal the rest energy? What is the velocity, in units of c, at this temperature?
- We call particles where the kinetic energy is greater than or equal to the rest energy 'relativistic'. At what temperature, in kelvin, did protons become non-relativistic? What mass of particles are just becoming non-relativistic, out in the cosmos, today?

Problem 4. *The Planck Scale and the String Scale*

Newton's constant is a fundamental constant that characterises the strength of gravity. It appears in general relativity also.

- The 'reduced Planck mass' is defined by $1/M_{\mathrm{Pl}}^2 = 8\pi G$. What is this in GeV? In kg? Notice how much larger this is than energy scales associated with, for example, the LHC. How much larger is M_{Pl} than the maximum energy of the LHC? Measuring masses in units of the reduced Planck mass makes all remaining physical quantities dimensionless.

- Convert the reduced Planck mass to a length, the Planck length ℓ_P. Give your answer in m. This is the length scale associated with quantum gravity.
- What is the Planck time, t_P, in s? Without studying quantum gravity we can know nothing about the evolution of the Universe on time scales shorter than this. What is the current age of the Universe in Planck units?
- String theory has a natural length scale, the string length, l_s. If the mass scale associated with this is $M_s = 1$ TeV, what is the string length in m?
- The extra dimensions of string theory are expected to be on the scale of $l_s \approx 1/M_s$. Experimental limits on the size of extra dimensions are on the scale of microns, where $1\ \mu\text{m} = 10^{-6}$ m. What string scale, M_s, does this correspond to in eV?
- The cosmological constant Λ is given by $\Lambda = \Omega_\Lambda 3H_0^2/8\pi G$, where $\Omega_\Lambda = 0.68$ is approximately the inferred value from cosmological observations. What is this in eV? In Planck units, where $M_{\text{Pl}} = 1$? In string units, if $M_s = 1$ TeV? The smallness of this number is a source of 'fine-tuning' in physics, since Λ is expected to be of order one in the fundamental units.

Index