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Introduction

ONE DAY MY daughter's fourth grade teacher announced that the following Friday was going to be a movie day. Everyone would come to school in pajamas, bring their favorite stuffed animal, and curl up to watch one of three options: *Bolt*, *Incredibles 2*, or *Coco*. To pick the movie, a vote would take place at the beginning of the day.

That morning, as my daughter was getting ready for school, I asked her to try to remember how the vote turned out. When she returned home, she duly reported that *Bolt* received 7 votes, *Incredibles 2* got 6 votes, and *Coco* got 4 votes. The teacher declared *Bolt* to be the winner and the class settled in for a movie afternoon.

Nothing against *Bolt*, but this was a terrible way to determine the winner. Most of the kids, ten of them, didn't give *Bolt* as their first choice. The will of the minority (7) was imposed on the remaining majority (10).

What could the teacher have done differently? She could, for example, have told the four kids who voted for *Coco* that their movie didn't make it, but they could cast another vote, this time between *Bolt* and *Incredibles 2*. The four new votes would have been added to the existing tallies for those two movies, with the upshot that the winner would now necessarily have majority support. If any two of the four kids who originally voted for *Coco* had voted for *Bolt*, that would have been the winner with at least 9 votes, but if—in a nail-biter twist—three had voted for *Incredibles 2*, that's the movie all seventeen kids would have watched, with *Bolt* dethroned after a 9–8 loss.

We will never know what would have happened. The plurality vote the teacher conducted asked only for the kids' top choice and nothing else. When so little information is asked for, only the coarsest tallying method is possible: count up the votes and the candidate with the most votes wins. The nuance of any preferences beyond the first choice is lost, resulting in a winner who does not necessarily represent the true will of the people.

And yes, this was just a bunch of kids choosing what movie to watch, so what's the big deal? But several months earlier, in the 2018 Democratic primary election in the 3rd District of Massachusetts, a few miles north of my daughter's school, you would have seen the same scenario playing out. Lori Trahan carried the nomination with 21.7% of the vote. Fast forward to the 2020 Republican primary in Florida District 3, far to the south, and you would see Kat Cammack winning with only 25.2% of the vote. Fast forward again, zagging back north to the 2022 Ohio Republican primary in the U.S. Senate race, and you would see J. D. Vance winning with 32.2% of the vote. You get the picture—all around us, people who have earned the support of only a minority of voters represent all of the voters.* This scenario is replicated all over the United States and the world in elections of all magnitudes, at all levels, deciding matters big and small.

What we're seeing is, at its root, a problem in mathematics.

Matters of politics have become mired in personalities and partisanship. Although we recognize that problems exist, we're getting worse at identifying them and increasingly paralyzed when it comes to constructing and assessing solutions. However, democracy is not just a human forum, it is also a system, a piece of civic infrastructure that runs on mathematics. Mathematics powers our basic democratic processes in ways that spread well beyond the seemingly simple matter of voting. Determining the size of representative bodies, distributing legislative seats, districting, and gerrymandering—all of these procedures rest on mathematical foundations.

* In heavily partisan districts, as most of them are, primaries are typically the real contests. All of these victors went on to be elected to office in their general elections.

Just as camera filters and lenses can reproduce an image faithfully or manipulate it intentionally or output a garbled mess, the mathematics of democracy can give the people a voice or silence some and amplify others or lead to results too fragile to trust. And indeed, a closer look at the manifestations of mathematics in our democracy reveals that the ways we use it are flawed, and archaic, and often serve discriminatory intent. They have murky, dubious, or politically motivated origins that few know about and even fewer remember.

The good news—the hopeful news—is that mathematics is also transparent, with no agenda or spin. It lets us see what's under the hood—we just have to look. If our politics are a screaming toddler and we are a parent incapacitated by the severity of the tantrum, then the math of those politics is the deep breath, a grounding mechanism that helps us understand that the child is just tired or hungry and we actually know how to fix that. Math is a clarifying way of looking at the world. It provides empowering confidence and is accessible to anyone. It is ready to reveal the deficiencies of our current democratic processes and recommend which new or updated ones can work better.

I have proof. For several years, I have witnessed the transformative effect of political numeracy education through teaching a college-level Math and Politics course. Students come to the class intrigued by the odd couple in the course title and hoping to earn a math credit needed for graduation. By the end, they are outraged that no one ever showed them how terrible our voting methods are, how blatantly devious gerrymandering is, how dysfunctional the U.S. Electoral College is. They are fired up about all the inequalities and discriminatory practices built mathematically into our system and are ready to get out there and do something about it. This book aims to bring my classroom to you, to empower you with knowledge (as well as outrage) that rests on a firm foundation of objective mathematics and that will give you the confidence to make a difference.

The time is right. There is growing awareness of the faults in our voting systems, and I don't mean fantasies of widespread voter fraud or conspiratorial voting machines. Initiatives to address inequities in representation and to implement something smarter are proliferating. (At the

time of writing, at least ninety U.S. municipalities are trying to enact ranked choice voting.) After the 2016 election, the inadequacies of the Electoral College and its incompatibility with the popular vote have come front and center. As has gerrymandering, especially after the 2020 census and the many legal challenges to redistricting that followed. Politicians are starting to pay attention. More schools are building political quantitative literacy into their curriculum in recognition of its pedagogical appeal and relevance. Now is the time to get on the math and democracy bandwagon and join the movement to restore a functioning democracy.

It would of course be foolish to think that mathematics is the panacea for all of our political dysfunction. The role of politics, religion, community, emotion, greed, and power in democracy is undeniable and apparent to even the most detached of mathematicians. I tend to be even more sensitive to these things as an immigrant from Bosnia-Herzegovina. My life has to a significant extent been determined by that country's terrible war of the early 1990s, a horrific and bloody demise of democracy far removed from anything rational—and hence from anything mathematical.

But this book will intentionally ignore these things. Its scope and its intent are not to stretch into all things democracy. Everything you will read here is grounded in the quantitative. The motivation and the examples will come from a messy reality, but the analysis will proceed in a mathematically impartial way, without political commentary. The political context will be used only to inform the math. My guiding principle is that using the best version of mathematics in democracy is of benefit to everyone, regardless of all those extraneous factors. Using a voting method that best captures the will of the people; electing our officials in a way that respects the basic one person, one vote axiom of democracy; creating conditions so that underrepresented groups have a voice should be universal aspirations, and their implementation should be steered by tools that are equally all inclusive. Mathematics is one of those tools.

On the other hand, democracy is about people, and even the math of democracy is a story of human idealism, shortsightedness, and above all compromise. This means that we'll have to engage with the messiness on occasion. As definitive and unwavering as math is supposed to be, it doesn't do so well when it must proclaim itself the "best," the "most fair," or the "least biased." We'll see these words a lot because they're naturally woven into any discussion of politics and democracy, but they belong to a nonmathematical realm, one occupied by humans, in which opinions, preferences, and interpretations are allowed. For mathematics, these notions turn out to be too elusive. As a result, it will be easy for us to spot bad math (and there will be lots of it), but it will be trickier to find a replacement we can endorse. When considering math in the abstract, a diversity of definitions, theorems, and theories about a single subject can coexist simultaneously and independently (and they can all be equally true and valid), but because we will force them into competition for real-world application to democracy, we will sometimes have to be content with speculative outcomes. But we'll make the best of this. We'll figure out how to embrace the mathematical uncertainty.

This book is also not about the (mis)use of math and statistics in politics. I won't even address, let alone pick apart, the troubling ease with which politicians manipulate numbers, graphs, and charts or the cavalier way with which they bandy about cooked or carefully selected statistics. I have much respect for those who are waging the good fight of educating the public about the exploitation of statistics in politics, but this book is about the mathematics *behind* democratic processes, not in front of them. Of course, the two ends are but two tentacles of the political innumeracy kraken, and those of us who fight it stand shoulder to shoulder, math spears in hand, trying to flank the beast from different angles.

Finally, there is growing recognition that math curricula at all education levels need to be updated in a way that reflects the injustices, discriminations, and intolerances of the world. In this way, the optimistic educator reckons, we might even be able to use mathematics to tackle those issues. Many amazing people are fighting this good fight, writing

and speaking about the archaic way we teach math and producing curricular materials that are relevant and timely.* As worthy and necessary as this effort may be, it is also outside the scope of this book. Our interest here is in the mathematical mechanics of democracy and not how mathematics can be used to explain or analyze specific social justice issues. But that's not to say that the content here has nothing to do with social justice. On the contrary—and as I'll argue repeatedly—implementing better math practices in democracy can lead to more equitable, less discriminatory outcomes.

So what does the math of democracy look like? We'll invest some time in unpacking concrete examples to get a feel for things—what goes right, what goes wrong—and then take on some formalism and abstraction to bring the big picture into focus. With only modest mathematical machinery, we'll be able to synthesize, extrapolate, generalize, and look for patterns in search of a cohesive framework that will support recommendations for better policies and mechanisms of democracy. We'll establish axioms, make definitions, and state theorems. We'll also encounter a surprising number of limitations and trade-offs, which will often manifest in paradoxical behavior, counterintuitive outcomes, and apparent inconsistencies—but we'll celebrate these. Probing strange outcomes can tell us a lot about the system.

On the other hand, the math of democracy is fairly straightforward: basic arithmetic is all you'll need. The focus will be on simple examples. If there is a more complicated or more abstract idea lurking around, I'll mention it in a footnote to avoid interrupting what I hope will be a comfortable, even cozy flow. You won't even notice I've slipped in some legit math!

As we move along, the mathematics will enable you to engage confidently in restoring our democracy by demystifying the systems that

* Examples include *Mathematics for Social Justice* by Gizem Karaali and Lily S. Khadjavi and *Rethinking Mathematics: Teaching Social Justice by the Numbers* by Eric Gutstein and Bob Peterson.

power it and examining how close they come to embodying our ideals. You'll be equipped to reject the prevailing refrain that things are just too complicated. You won't defer to history or tradition. You won't fear that something terrible is lurking in the details of an unfamiliar method that makes it secretly partisan. Math will offer a path to true progress, to tangible improvements and resolutions of impasses. You'll understand how the engine of democracy works, and you'll be ready to make your own judgments and take action.

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