

TABLE OF CONTENTS

Introduction	1
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PART I Voting	9
----------------------	----------

1 The Best Way to Choose the Winner	11
<i>Majority, Dictators, and Monarchs</i>	12
<i>The Unreasonable Effectiveness of Mathematics</i>	15
<i>It's as Simple as Simple Majority</i>	17
<i>Quota and Supermajority</i>	22
<i>The Tyranny of Convenient Numbers</i>	25
2 The Worst Way to Choose the Winner	28
<i>Most of Not Many</i>	28
<i>A Splitting Headache</i>	32
<i>Spoils of Plurality</i>	34
<i>A Party of Two</i>	37
<i>Second Time's the Charm</i>	45
<i>Ties and Top Hats</i>	50
3 From Best to Worst	53
<i>Candidate Profiling</i>	53
<i>Successive Elimination</i>	58
<i>Assigning Points</i>	75

viii TABLE OF CONTENTS

<i>Comparing Pairs</i>	86
<i>Everybody (and Nobody) Is a Winner</i>	95
4 The Impossible Democracy	99
<i>Axioms of Democracy</i>	100
<i>Arrow through the Heart of Voting</i>	105
<i>Living with the Impossible</i>	107
5 To Each Their Own	112
<i>They Love You, They Love You Not</i>	113
<i>A Range of Emotions</i>	117
6 Strategy and Manipulation	124
<i>Game of Chairs</i>	124
<i>The Most Manipulable Method</i>	129
<i>A Scheme for Honest Men</i>	132
<i>Deadly Bullet Points</i>	138
7 And the Winner Is . . .	141
PART II Representation	147
8 This Old House	149
<i>A Waiter's Take on Democracy</i>	149
<i>The House Hits the Ceiling</i>	153
<i>One Representative per Seattle</i>	158
<i>The Golden Ratio</i>	164
9 Rather Divisive than Indecisive	173
<i>Divisors and Quotas</i>	173
<i>Hamilton's Paradoxes</i>	176

TABLE OF CONTENTS ix

10	Divisor and Conquer	183
	<i>Jefferson: Going Low</i>	183
	<i>Webster: Taking the Middle Road</i>	187
	<i>Huntington-Hill: A Mean Battle</i>	191
	<i>The Final Impossibility</i>	201
11	A Country Divided	207
	<i>Because America Can Hack Its Own Elections</i>	208
	<i>Mangling Democracy since 1788</i>	212
	<i>He Who Controls Redistricting</i>	217
	<i>A Civic Solution</i>	223
	<i>Carving Up Prisons and Schools</i>	226
12	Math v. Gerrymandering	228
	<i>Axioms of Districting</i>	228
	<i>Symmetry and Bias</i>	233
	<i>Cracking Down on Packing</i>	242
	<i>The Geometry of Districting</i>	249
	<i>Random Brewery Hopping in the Space of Maps</i>	260
	<i>May the Math Please the Court</i>	267
13	Proportional Representation	273
	<i>Sharing Is Caring</i>	274
	<i>Party Loyalty</i>	278
	<i>Candidates before Parties</i>	285
	<i>PR for PR</i>	296
PART III Civic Infrastructure		305
14	The Electoral College	307
	<i>Characters Preeminent for Ability and Virtue</i>	307

X TABLE OF CONTENTS

<i>Spinning Pluralities into Majorities</i>	315
<i>The Bare Minimum</i>	318
<i>One Person, Four Votes</i>	322
<i>Abolish, Amend, Avoid</i>	325
15 The Citizen-Mathematician	336
 An Infrastructure Plan	340
 <i>Acknowledgments</i>	341
<i>Glossary</i>	345
<i>Figure Credits and Sources</i>	353
<i>Notes</i>	355
<i>Index</i>	381

Introduction

ONE DAY MY daughter's fourth grade teacher announced that the following Friday was going to be a movie day. Everyone would come to school in pajamas, bring their favorite stuffed animal, and curl up to watch one of three options: *Bolt*, *Incredibles 2*, or *Coco*. To pick the movie, a vote would take place at the beginning of the day.

That morning, as my daughter was getting ready for school, I asked her to try to remember how the vote turned out. When she returned home, she duly reported that *Bolt* received 7 votes, *Incredibles 2* got 6 votes, and *Coco* got 4 votes. The teacher declared *Bolt* to be the winner and the class settled in for a movie afternoon.

Nothing against *Bolt*, but this was a terrible way to determine the winner. Most of the kids, ten of them, didn't give *Bolt* as their first choice. The will of the minority (7) was imposed on the remaining majority (10).

What could the teacher have done differently? She could, for example, have told the four kids who voted for *Coco* that their movie didn't make it, but they could cast another vote, this time between *Bolt* and *Incredibles 2*. The four new votes would have been added to the existing tallies for those two movies, with the upshot that the winner would now necessarily have majority support. If any two of the four kids who originally voted for *Coco* had voted for *Bolt*, that would have been the winner with at least 9 votes, but if—in a nail-biter twist—three had voted for *Incredibles 2*, that's the movie all seventeen kids would have watched, with *Bolt* dethroned after a 9–8 loss.

We will never know what would have happened. The plurality vote the teacher conducted asked only for the kids' top choice and nothing else. When so little information is asked for, only the coarsest tallying method is possible: count up the votes and the candidate with the most votes wins. The nuance of any preferences beyond the first choice is lost, resulting in a winner who does not necessarily represent the true will of the people.

And yes, this was just a bunch of kids choosing what movie to watch, so what's the big deal? But several months earlier, in the 2018 Democratic primary election in the 3rd District of Massachusetts, a few miles north of my daughter's school, you would have seen the same scenario playing out. Lori Trahan carried the nomination with 21.7% of the vote. Fast forward to the 2020 Republican primary in Florida District 3, far to the south, and you would see Kat Cammack winning with only 25.2% of the vote. Fast forward again, zagging back north to the 2022 Ohio Republican primary in the U.S. Senate race, and you would see J. D. Vance winning with 32.2% of the vote. You get the picture—all around us, people who have earned the support of only a minority of voters represent all of the voters.* This scenario is replicated all over the United States and the world in elections of all magnitudes, at all levels, deciding matters big and small.

What we're seeing is, at its root, a problem in mathematics.

Matters of politics have become mired in personalities and partisanship. Although we recognize that problems exist, we're getting worse at identifying them and increasingly paralyzed when it comes to constructing and assessing solutions. However, democracy is not just a human forum, it is also a system, a piece of civic infrastructure that runs on mathematics. Mathematics powers our basic democratic processes in ways that spread well beyond the seemingly simple matter of voting. Determining the size of representative bodies, distributing legislative seats, districting, and gerrymandering—all of these procedures rest on mathematical foundations.

* In heavily partisan districts, as most of them are, primaries are typically the real contests. All of these victors went on to be elected to office in their general elections.

Just as camera filters and lenses can reproduce an image faithfully or manipulate it intentionally or output a garbled mess, the mathematics of democracy can give the people a voice or silence some and amplify others or lead to results too fragile to trust. And indeed, a closer look at the manifestations of mathematics in our democracy reveals that the ways we use it are flawed, and archaic, and often serve discriminatory intent. They have murky, dubious, or politically motivated origins that few know about and even fewer remember.

The good news—the hopeful news—is that mathematics is also transparent, with no agenda or spin. It lets us see what’s under the hood—we just have to look. If our politics are a screaming toddler and we are a parent incapacitated by the severity of the tantrum, then the math of those politics is the deep breath, a grounding mechanism that helps us understand that the child is just tired or hungry and we actually know how to fix that. Math is a clarifying way of looking at the world. It provides empowering confidence and is accessible to anyone. It is ready to reveal the deficiencies of our current democratic processes and recommend which new or updated ones can work better.

I have proof. For several years, I have witnessed the transformative effect of political numeracy education through teaching a college-level Math and Politics course. Students come to the class intrigued by the odd couple in the course title and hoping to earn a math credit needed for graduation. By the end, they are outraged that no one ever showed them how terrible our voting methods are, how blatantly devious gerrymandering is, how dysfunctional the U.S. Electoral College is. They are fired up about all the inequalities and discriminatory practices built mathematically into our system and are ready to get out there and do something about it. This book aims to bring my classroom to you, to empower you with knowledge (as well as outrage) that rests on a firm foundation of objective mathematics and that will give you the confidence to make a difference.

The time is right. There is growing awareness of the faults in our voting systems, and I don’t mean fantasies of widespread voter fraud or conspiratorial voting machines. Initiatives to address inequities in representation and to implement something smarter are proliferating. (At the

time of writing, at least ninety U.S. municipalities are trying to enact ranked choice voting.) After the 2016 election, the inadequacies of the Electoral College and its incompatibility with the popular vote have come front and center. As has gerrymandering, especially after the 2020 census and the many legal challenges to redistricting that followed. Politicians are starting to pay attention. More schools are building political quantitative literacy into their curriculum in recognition of its pedagogical appeal and relevance. Now is the time to get on the math and democracy bandwagon and join the movement to restore a functioning democracy.

It would of course be foolish to think that mathematics is the panacea for all of our political dysfunction. The role of politics, religion, community, emotion, greed, and power in democracy is undeniable and apparent to even the most detached of mathematicians. I tend to be even more sensitive to these things as an immigrant from Bosnia-Herzegovina. My life has to a significant extent been determined by that country's terrible war of the early 1990s, a horrific and bloody demise of democracy far removed from anything rational—and hence from anything mathematical.

But this book will intentionally ignore these things. Its scope and its intent are not to stretch into all things democracy. Everything you will read here is grounded in the quantitative. The motivation and the examples will come from a messy reality, but the analysis will proceed in a mathematically impartial way, without political commentary. The political context will be used only to inform the math. My guiding principle is that using the best version of mathematics in democracy is of benefit to everyone, regardless of all those extraneous factors. Using a voting method that best captures the will of the people; electing our officials in a way that respects the basic one person, one vote axiom of democracy; creating conditions so that underrepresented groups have a voice should be universal aspirations, and their implementation should be steered by tools that are equally all inclusive. Mathematics is one of those tools.

On the other hand, democracy is about people, and even the math of democracy is a story of human idealism, shortsightedness, and above all compromise. This means that we'll have to engage with the messiness on occasion. As definitive and unwavering as math is supposed to be, it doesn't do so well when it must proclaim itself the "best," the "most fair," or the "least biased." We'll see these words a lot because they're naturally woven into any discussion of politics and democracy, but they belong to a nonmathematical realm, one occupied by humans, in which opinions, preferences, and interpretations are allowed. For mathematics, these notions turn out to be too elusive. As a result, it will be easy for us to spot bad math (and there will be lots of it), but it will be trickier to find a replacement we can endorse. When considering math in the abstract, a diversity of definitions, theorems, and theories about a single subject can coexist simultaneously and independently (and they can all be equally true and valid), but because we will force them into competition for real-world application to democracy, we will sometimes have to be content with speculative outcomes. But we'll make the best of this. We'll figure out how to embrace the mathematical uncertainty.

This book is also not about the (mis)use of math and statistics in politics. I won't even address, let alone pick apart, the troubling ease with which politicians manipulate numbers, graphs, and charts or the cavalier way with which they bandy about cooked or carefully selected statistics. I have much respect for those who are waging the good fight of educating the public about the exploitation of statistics in politics, but this book is about the mathematics *behind* democratic processes, not in front of them. Of course, the two ends are but two tentacles of the political innumeracy kraken, and those of us who fight it stand shoulder to shoulder, math spears in hand, trying to flank the beast from different angles.

Finally, there is growing recognition that math curricula at all education levels need to be updated in a way that reflects the injustices, discriminations, and intolerances of the world. In this way, the optimistic educator reckons, we might even be able to use mathematics to tackle those issues. Many amazing people are fighting this good fight, writing

and speaking about the archaic way we teach math and producing curricular materials that are relevant and timely.* As worthy and necessary as this effort may be, it is also outside the scope of this book. Our interest here is in the mathematical mechanics of democracy and not how mathematics can be used to explain or analyze specific social justice issues. But that's not to say that the content here has nothing to do with social justice. On the contrary—and as I'll argue repeatedly—implementing better math practices in democracy can lead to more equitable, less discriminatory outcomes.

So what does the math of democracy look like? We'll invest some time in unpacking concrete examples to get a feel for things—what goes right, what goes wrong—and then take on some formalism and abstraction to bring the big picture into focus. With only modest mathematical machinery, we'll be able to synthesize, extrapolate, generalize, and look for patterns in search of a cohesive framework that will support recommendations for better policies and mechanisms of democracy. We'll establish axioms, make definitions, and state theorems. We'll also encounter a surprising number of limitations and trade-offs, which will often manifest in paradoxical behavior, counterintuitive outcomes, and apparent inconsistencies—but we'll celebrate these. Probing strange outcomes can tell us a lot about the system.

On the other hand, the math of democracy is fairly straightforward: basic arithmetic is all you'll need. The focus will be on simple examples. If there is a more complicated or more abstract idea lurking around, I'll mention it in a footnote to avoid interrupting what I hope will be a comfortable, even cozy flow. You won't even notice I've slipped in some legit math!

As we move along, the mathematics will enable you to engage confidently in restoring our democracy by demystifying the systems that

* Examples include *Mathematics for Social Justice* by Gizem Karaali and Lily S. Khadjavi and *Rethinking Mathematics: Teaching Social Justice by the Numbers* by Eric Gutstein and Bob Peterson.

power it and examining how close they come to embodying our ideals. You'll be equipped to reject the prevailing refrain that things are just too complicated. You won't defer to history or tradition. You won't fear that something terrible is lurking in the details of an unfamiliar method that makes it secretly partisan. Math will offer a path to true progress, to tangible improvements and resolutions of impasses. You'll understand how the engine of democracy works, and you'll be ready to make your own judgments and take action.

INDEX

Page numbers in *italics* refer to figures.

- Academy Awards, 295–96
 Adams, Eric, 68, 135–36
 Adams, John, 100, 311
 Adams, John Quincy, 187, 190, 311, 320
 Adams's method, 187, 189, 198, 203
 Alabama, 58, 215, 216, 271
 Alabama paradox, 179–80, 186, 190, 201, 345
 Alaska, 46, 73, 160, 315; instant runoffs in, 58, 64–65, 110, 134, 317, 318; top-four primaries in, 29
 American Mathematical Society, 116
 anonymity criterion, 18–19, 101, 104, 115, 345
 Apportionment Act (1792), 156
 Apportionment Act (1842), 156, 232
 Apportionment Act (1941), 200
 apportionment problem, 153, 345
 approval voting, 112–14, 134, 138, 345; bloc, 289; multimember, 289, 349
Approval Voting (Brams and Fishburn), 113
 Argentina, 279, 313
 arithmetic mean, 194–96, 345
 Arizona, 157, 191, 223, 230, 288, 319–20, 334
 Arkansas, 58, 200, 317
 Arrow, Kenneth, 104–7, 111–12, 137
 Arrow impossibility theorem, 106–8, 115, 202, 345
 Australia, 42, 224, 297, 300–1
 authoritarianism, 31, 42, 143
 axioms, 99, 345
Baker v. Carr (1962), 215
 Balinski, Michel, 122, 201–2, 203
 Balinski-Young impossibility theorem, 202, 345
 ballot exhaustion, 59, 62n, 89n
 Barros, John, 47
 baseball awards, 78
 batch elimination, 68n
 Bayesian regret, 120
 Begich, Nick, 64, 73
 Belgium, 281
 Benedict XVI, Pope, 24
 Berlin Philharmonic, 24n
 Beyer, Don, 302
 Bézout, Étienne, 86
 Biden, Joe, 29, 32, 129; lack of enthusiasm for, 115; in 2020 election, 220, 313, 317–20, 323, 332
 Bill of Rights, 212
 binary choices, 12
 Black, Duncan, 108–9
 bloc approval voting, 289
 bloc voting, 286–89, 294–95, 345
 Bloomberg, Michael, 39
 Borda, Jean-Charles de, 75–76, 86–87, 133
 Borda count, 57, 97, 98, 109, 117; benefits of, 77–78; defined, 345; institutional use of, 78–80; manipulability of, 125, 126–27, 136; origins of, 76; shortcomings of, 81–86, 102–3, 133

- Bosnia-Herzegovina, 26–27, 184n, 273, 275,
336–37; proportional representation in,
274, 278–79, 285; strategic voting in, 130
- Boston, 31, 47
- Bouk, Dan, 158, 173n
- Brams, Steven, 73, 113, 115
- Brazil, 46, 279, 313
- Buchanan, James, 190
- Buchanan, Pat, 63
- bullet voting, 119, 122–23, 139, 345
- Burr, Aaron, 51, 311
- Burton, Phil, 221
- burying, in ranked voting, 132–33, 138,
140, 345
- Bush, George H. W., 36, 37, 127, 326
- Bush, George W., 34–36, 63, 161, 315, 317,
320, 332
- Bush v. Gore* (2000), 313
- Buttigieg, Pete, 115
- Byrne, David, 303n
- California, 162n, 300, 315–16, 324; compact-
ness rules in, 230, 259; independent
districting commission in, 221, 223, 225;
instant runoffs in, 66, 110, 144; recall elec-
tions in, 34, 38; top-two primaries in, 46
- Campbell, Andrea, 47
- Cambridge, Mass., 278–79, 297, 299
- Cammack, Kat, 2
- Canada, 31, 278; independent districting
commission in, 224, 254; multiparty sys-
tem in, 40, 41; representation ratio in,
169, 170, 171; strategic voting in, 131
- candidate-centered proportional represen-
tation, 286, 345
- candidate cloning, 136, 345
- cardinal voting, 112, 138, 345
- Carroll, Lewis, 91n, 127
- Cass, Lewis, 36
- center-squeeze effect, 43, 69, 78, 345
- Chicago, 47, 229–30
- Cicciolina (actress), 42
- Chen, Jowei, 260–61, 268, 271
- Chile, 46, 297, 313
- choice voting (generalized Hare method;
proportional ranked choice voting; single
transferable vote), 289–96, 300–2, 340, 351
- Citizens United v. Federal Election Commis-
sion* (2010), 40
- Civil Rights Act (1957), 23, 215
- Civil Rights Act (1964), 216
- Clay, Henry, 36
- Cleese, John, 276
- Cleveland, Grover, 36, 320
- Clinton, Bill, 36, 37, 123, 127, 317
- Clinton, Hillary, 32, 36, 39, 319, 320n, 329,
331; faithless electors and, 314; Russian
meddling and, 129; Stein's candidacy and,
37, 317–18
- closed list proportional representation, 279,
346
- cloture, 23
- coalition building, 22
- Cohen, Steve, 326
- Coleman, Norm, 33–34
- collective preferences, 9, 17, 100
- Colorado, 116, 216–17, 223, 230
- combined voting methods, 123
- committee size paradox, 293n
- Common Cause v. Lewis* (2019), 271
- communities of interest, 216–17, 228, 346
- compactness, 217, 230–31, 249–56, 346
- compensatory seats, 280, 346
- competitive district, 163, 211, 224, 225,
330, 346
- conclave, 24, 128
- Condorcet, Marie-Jean-Antoine-Nicolas,
Marquis de, 86–87
- Condorcet criterion, 102–3, 104, 109–10
- Condorcet cycle, 90–92, 101, 346
- Condorcet loser, 88–89, 110–11
- Condorcet method, 57, 86–92, 97, 101,
136, 346
- Condorcet winner, 88–89, 95, 97, 102,
109–10
- Conservative Party (UK), 276

- Considerations on Representative Government* (Mill), 296
- constituency, 346
- constitutional amendment, 22–23
- Constitutional Convention, 154, 307–8
- contiguity, 217, 232, 346
- continuity, 246
- convex hull score, 257, 258, 346
- convexity, 346
- Copeland, A. H., 93
- Copeland's method, 101, 102, 112, 132, 133;
majority requirement met by, 102; in
round robin sports tournaments, 93;
shortcomings of, 95, 104, 136
- Correia, Jasiel, 37–38
- cracking, in gerrymandering, 210, 246–47, 346
- Croatia, 48, 49, 233, 279, 336–37
- Cruz, Ted, 32–33, 63, 114
- cube root law, 170–72, 346
- cumulative voting, 122–23, 140, 289, 297n, 346
- Cutler, Eliot, 39
- Cusanus, Nicolaus, 76
- cut edge score, 260n
- Daley, David, 218
- Dasgupta, Partha, 109
- Davis, Gray, 34
- Davis v. Bandemer* (1986), 269
- Dayton Accords (1995), 337
- Dean, James, 187
- Dean's method, 187–90, 198, 346
- Declaration of Independence, 99–100
- DeFord, Daryl, 263, 271–72
- degressive proportionality, 277, 346
- Delaware, 160, 167, 179, 186, 315, 323
- Democratic-Republican Party, 213
- Denmark, 285
- D'Hondt, Victor, 281
- D'Hondt method, 277, 283–84, 285, 346
- dictatorship, as voting method, 13, 18, 19,
106, 346
- discovery vs. invention, 15
- dispersion, in district shapes, 255, 257, 347
- district, defined, 347
- District of Columbia, 312, 315, 317
- divisor methods, 184–88, 194, 195, 198, 201,
202, 347
- double bunking (incumbent pairing), 212,
347
- Douglas, Stephen, 78n
- Droop, Henry Richmond, 289–91
- Droop quota, 290–91, 294
- Duchin, Moon, 263, 271–72
- Duke, David, 48
- Duverger, Maurice, 39, 41
- Duverger's law, 39, 40–41, 42, 78, 132, 291
- educational gerrymandering, 226–27, 347
- Edwards, Edwin, 47–48
- efficiency gap (EG), 225, 242–49, 261, 266,
267–68, 347
- Eisenhart, Luther, 200
- Electoral College, 34–35, 153, 340; alternatives
to, 325–35; apportionment flaws embed-
ded in, 322–24, 325; deadlocks in, 312,
315–16, 332; faithless electors in, 313–14,
334, 347; origins of, 306–12; popular vote
vs., 318–22, 326, 329–30; unrepresenta-
tiveness of, 161, 315–18, 322–24
- electors, 347
- elimination paradox, 277n
- Ellenberg, Jordan, 270n, 308
- England, 225
- ensemble sampling, 265–66, 266n, 268,
270, 271
- equal proportions method (Huntington-
Hill method), 191–200, 202–5, 277, 323,
331, 347–48
- equal protection, 221
- Essay on the Applicability of Analysis to the
Probability of Majority Decisions* (Con-
dorcet), 86
- Euclid, 99, 100
- Euler's theorem, 16, 260
- European Parliament, 277
- Eurovision, 79–80, 128

- eventual majority, 62
- Eves, Mark, 44
- exhausted ballot, 59, 62n, 89n

- Fahey, Katie, 225
- Fair Representation* (Balinski and Young), 201
- faithless electors, 313–14, 334, 347
- Fall River, Mass., 37–38
- Fargo, N.D., 116, 139
- favorite betrayal, 39, 48, 131, 134, 138, 347
- Federalist Papers, 154, 156, 310, 328, 347
- Federalist Party, 213
- FIFA World Cup, 93, 94
- Fifteenth Amendment, 213
- figure skating scores, 84–85
- filibuster, 23
- Finland, 46
- first past the post. *See* plurality voting
- Fishburn, Peter, 73, 113
- flip, in Markov chain algorithm, 347
- Florida, 34–35, 63, 88, 162n, 222, 317, 334
- football polls, 81
- Ford, Gerald R., 326
- Formula One rankings, 78
- Fourteenth Amendment, 177n, 221
- France, 46, 48, 49, 163

- game theory, 105–7
- Gaming the Vote* (Poundstone), 120, 129
- Garcia, Kathryn, 44, 135
- generalized Hare method (choice voting; proportional ranked choice voting; single transferable vote), 289–96, 300–2, 340, 351
- geometric mean, 194–95, 347
- George, Annisssa Essaibi, 46
- Georgia (U.S.), 45, 58, 187, 215, 319–20
- Germany, 163, 280–81, 285, 300–1, 313
- Gerry, Elbridge, 213
- gerrymandering, 3, 4, 207–9; alternatives to, 222–25; axioms of, 228–33; competitive districts eliminated by, 211, 330; cracking in, 210, 246–47, 346; defined, 347; definitional elusiveness of, 267–72; educational, 225–26; “eyeball test” for, 270; incumbents entrenched by, 300; minorities disenfranchised by, 213–16, 221, 226; origins of, 212–13; packing in, 209–10, 215, 224, 242–49, 349; partisan bias in, 233–42; prisons and, 226; proportional representation as threat to, 299; random maps and, 260–66; Republican mastery of, 217–20, 224, 276; secretiveness of, 221; shape of districts in, 249–61; Supreme Court’s acquiescence in, 222–23; technological aids in, 217
- Gibbard, Allan, 137
- Gibbard-Satterthwaite impossibility theorem, 137–38, 202, 347
- Giffords, Gabby, 129
- Gill v. Whitford* (2018), 222, 249, 260, 267–68, 271
- Gingrich, Newt, 326
- Ginsburg, Ruth Bader, 288
- Gödel’s incompleteness theorem, 108
- Goldwater, Barry, 43
- Gomillion v. Lightfoot* (1960), 215
- Gore, Al, 34, 88, 161, 315, 317, 320, 332; Nader’s candidacy and, 35–36, 39, 63, 129
- Gorsuch, Neil, 269
- grade point average, 119
- Graham-Squire, Adam, 111, 293n
- graph theory, 88n
- greatest divisors method. *See* Jefferson apportionment
- Green, Angela, 130, 132
- Green, Roger, 207
- Greenback Party, 41n
- Groover, Denmark, 45n
- Gross, Al, 134–35
- Guinier, Lani, 123

- Ham, Gracie, 15
- Hamilton, Alexander, 177, 310
- Hamilton apportionment method (Vinton method), 177–82, 186, 190–91, 202, 277, 331, 347

- Harding, Warren, 156
Hare, Thomas, 58, 296
Hare-Niemeyer method, 191n
Hare's method. *See* instant runoff method
harmonic mean, 198n
Harrison, Benjamin, 36, 320
Hawaii, 232
Hawkins, Howie, 317
Hayes, Rutherford, 320
Heisman Trophy, 78
Henry, Patrick, 212
Herschlag, Gregory, 271
Hill, Joseph, 192–93, 197
Hofeller, Thomas, 221–22
Holy Roman Empire, 313
Hong Kong, 233
Hoover, Herbert, 157
house monotonicity, 201, 202, 347
House of Representatives, 22–23, 26; de-
 gressive proportionality in, 277; insuf-
 ficient size of, 152–72, 340
Humphrey, Skip, 33–34
Huntington, Edward, 197–98
Huntington-Hill method (equal propor-
 tions method), 191–200, 202–5, 277, 323,
 331, 347–48
Iceland, 78, 279
Idaho, 230–31
Illinois, 187–88, 221, 222, 249, 250, 298n
immigration, 153, 157–58, 192, 215
impossibility theorems, 106–8, 115, 137–38,
 202
incumbency, 300
incumbent pairing (double bunking), 212,
 347
independence of irrelevant alternatives (IIA),
 84–85, 95, 103–4, 106, 107, 109, 115, 348
independent districting commissions,
 223–26, 244, 254, 302, 340
India, 31
insincere voting (strategic voting; tactical
 voting), 71, 120, 122, 123, 127–41, 351
instant runoff method (Hare's method;
 sequential runoff method), 46, 60–63,
 96–97; algorithm for, 58–59; benefits
 of, 50, 63–66, 143–45; candidate diver-
 sity linked to, 144; defined, 348; grow-
 ing use of, 145; objections to, 66–69;
 monotonicity failed by, 69, 72, 73, 102,
 104, 133–34; for presidential elections,
 328–29, 340; research on, 109, 111,
 143–44; shortcomings of, 69–75, 82,
 102, 103, 136; single transferable vote
 linked to, 290, 295
Institute for Mathematics and Democracy,
 338
invariance under scaling, 253, 348
invention vs. discovery, 15
Iowa, 226, 231, 249
Iran, 232–33
Ireland, 42, 297, 298
isoperimetric inequality theorem, 252–53,
 260, 348
Israel, 279, 281, 297
Italy, 171, 302
Jackson, Andrew, 311, 320
Janey, Kim, 47
Jankowski, Chris, 217
January 6 insurrection, 32
Japan, 279, 302
Jefferson, Thomas, 51, 86, 100, 178–79, 182–83,
 205, 311
Jefferson apportionment (greatest divisors
 method), 183–85, 194, 198, 202, 203; adop-
 tion of, 179; defined, 348; D'Hondt
 method likened to, 184n, 277, 283–84;
 large states favored by, 185–86, 187, 189, 203,
 204n; quota rule violated by, 186, 191,
 Webster's method compared with, 189
Jeffries, Hakeem, 207
John Paul II, Pope, 24
Johnson, Gary, 37, 129, 315, 317, 331
Johnson, Lyndon B., 43
Jorgensen, Jo, 317

- jungle primaries. *See* top-two primaries
- juries, 12; size of, 26; unanimous verdicts of, 24
- Kagan, Elena, 223
- Kasich, John, 32–33, 63, 314
- Kavanaugh, Brett, 223
- Kemeny’s method, 91n
- Kennedy, Anthony, 223, 267–68
- Kentucky, 187
- Kerry, John, 320
- Kim, Jane, 44
- Kim Jong Un, 128
- Kiss, Bob, 68
- Klobuchar, Amy, 115
- Koza, John, 333
- Labor Party (U.S.), 41n
- Labour Party (UK), 224–25, 276
- Lagrange, Joseph-Louis, 86
- Laplace, Pierre-Simon, 133
- Laraki, Rida, 122
- largest remainders method, 177
- League of United Latin American Citizens (LULAC) v. Perry* (2006), 267
- length-width score, 258, 259, 348
- Leno, Mark, 44
- LePage, Paul, 39
- Le Pen, Marine, 46, 48
- Liberal Democrat Party (UK), 276
- Liberia, 128
- Lincoln, Abraham, 78n
- literacy tests, 214
- Llull, Ramon, 76n, 86n
- Locke, John, 154
- Loeffler, Kelly, 45
- Louisiana, 45, 47–48, 58, 250
- lower quota (minimum quota), 176, 177, 348
- Macron, Emmanuel, 46, 48
- Madison, James, 154, 155–56, 158, 212, 308, 309, 312
- Maine, 39, 181, 182, 191, 226; compactness rules in, 259; district system in, 35, 312, 317, 329; instant runoffs in, 29, 53, 55, 58, 318
- major fractions method (Webster apportionment), 194–205, 277, 285, 331, 340; adoption of (1910), 191–92; bias absent from, 203–4, 284; Dean’s method compared with, 189–90; defined, 351–52; Jefferson’s method compared with, 189
- majority criterion, 25, 101, 102, 104, 122, 348
- majority judgment (majority grading), 122, 348
- majority-minority districts, 213n, 216, 267n
- malapportionment, 162, 348
- Malta, 42, 297
- Maptitude (districting software), 217
- March Madness, 93–94
- Markov chain Monte Carlo (MCMC) algorithm, 263–64, 270, 348
- Maryland, 175, 288, 300; gerrymandering in, 220–21, 222, 231, 244, 250, 251, 254, 258
- Maskin, Eric, 109, 144
- Massachusetts, 178, 187, 234, 258, 300, 310, 332
- mass incarceration, 226
- math education, 5–6, 340
- Mathematical Association of America, 116
- Mattingly, Jonathan, 263–65, 270–71
- May, Kenneth, 20
- mayoral elections, 31, 37; instant runoffs in, 68; runoffs in, 45, 47
- May’s theorem, 20–21, 25, 28, 89, 100, 348
- McCain, John, 329
- McCune, David, 110, 111, 293n
- McCune, Lori, 110
- McGhee, Eric, 242, 248
- McGuire, Thomas, 51
- McMullin, Evan, 317
- means vs. medians, in range voting, 122
- Mélenchon, Jean-Luc, 48
- Merrill v. Milligan* (2023), 234n, 271
- Mexico, 169, 301
- Michaud, Mike, 39

- Michigan, 200, 219, 220, 334; compactness rules in, 230, 256; independent districting commission in, 223, 225; in 2016 election, 37, 317–18, 320n
- Mill, John Stuart, 296
- Miller v. Johnson* (1995), 216
- minimum quotas (lower quotas), 176, 177, 348
- Minneapolis, 58, 297
- Minnesota, 29, 334
- Minnesota Farmer-Labor Party, 41n
- Mississippi, 58
- Missouri, 224, 230
- mixed electoral system, 280, 348
- mixed-member proportional representation, 280, 300, 348
- modified divisor (MD), 184–85, 196, 349
- monarchy, as voting method, 13, 18, 19, 349
- monotonicity, 18, 19, 69, 72, 73, 102, 115; defined, 349; house monotonicity, 201, 202; impossibility theorem and, 106, 107; population monotonicity, 201, 202, 349; types of elections vulnerable to, 111
- Monroe, James, 212
- Montana, 160, 167, 224, 230, 323
- Morgan of Glamorgan, King, 26
- Morgenbesser, Sidney, 103n
- Morgenstern, Oskar, 105
- Morse, Marston, 200n
- Most Valuable Player Award, 78
- multimember approval voting, 289, 349
- multimember districts, 233, 274, 278, 286–88, 297, 301–3, 340
- multiwinner approval voting, 289, 290, 349
- Murkowski, Lisa, 134–35
- Nader, Ralph, 35–36, 39, 63, 127, 129, 315, 317, 332
- Napoleon Bonaparte, emperor of the French, 76
- NASCAR rankings, 78
- National Academy of Sciences, 116, 198, 200
- National Popular Vote Interstate Compact (NPVIC), 333–34
- Native Americans, 177n
- NATO (North Atlantic Treaty Organization), 22
- NCAA, 78, 93–94
- Nebraska, 35, 46, 317, 329
- negative campaigning: approval voting as antidote to, 114; Borda count as antidote to, 78, 82; effectiveness of, 43–44; instant runoff as antidote to, 66, 144, 329; proportional representation as antidote to, 300; single transferable vote as antidote to, 293
- Nepal, 285
- Netherlands, 297
- neutrality, in voting methods, 18, 19, 101, 104, 115, 235, 349
- Nevada, 46n, 174
- New Hampshire, 132, 187, 288
- New Jersey, 224, 288
- New Mexico, 157, 191, 224
- Newsom, Gavin, 38, 56n
- new states paradox, 181–82, 186, 191, 201–2, 349
- New York City, 58, 68, 135–36, 296
- New York state, 182–83, 185–86, 187, 222, 224, 226
- New Zealand, 277, 280, 285, 302
- Nigeria, 46
- Nixon, Richard M., 326
- Noah, Trevor, 209
- North Carolina, 177, 212, 219, 220, 221–22, 268, 300
- North Dakota, 288, 315
- Northern Ireland, 224–25
- North Korea, 127–28
- Norway, 279, 285
- no-show paradox, 73, 122, 136, 349
- Novoselic, Krist, 302
- NP-hardness, 262n
- Oakland, 58, 66, 144
- Obama, Barack, 329–30

- O'Connor, Sandra Day, 270
- Ohio, 219, 220, 222, 224, 231, 241, 300, 320, 334
- Oklahoma, 182
- Oliver, John, 210
- Olympic Games: medal counts at, 79; scoring at, 85; site selection for, 62, 68n
- One Nation Party, 42
- One Person, One Vote* (Seabrook), 221
- one person, one vote doctrine, 4, 168, 215, 328; Electoral College incompatible with, 323, 327, 335; interstate malapportionment incompatible with, 162; ranked choice voting compatible with, 67
- open list proportional representation, 279, 349
- open primaries, 132
- optimization, 5, 14, 97–98, 105
- Oregon, 116, 123, 221, 222
- packing, in gerrymandering, 209–10, 215, 224, 242–49, 349
- pairwise (sequential) voting, 93–95, 136
- Palin, Sarah, 64–65, 73, 110
- papal succession, 24, 128
- paradox of positive association, 71
- Paraguay, 313
- Pareto, Vilfredo, 106n
- Pareto criterion, 106n
- parity, as voting method, 13, 19, 349
- partisan bias, 225, 238, 239–40, 266, 349
- partisan symmetry, 235–37, 240, 340
- party list proportional representation, 278–81, 285–86, 349;
- Paul, Ron, 314
- Pegden, Wesley, 270–71
- Pelosi, Nancy, 225n
- Peltola, Mary, 64–65, 73
- Pennsylvania, 116, 309, 334; gerrymandering in, 219, 222, 231, 250, 251, 259, 263, 267, 271; in 2016 election, 37, 318, 320n
- Perdue, David, 45
- Perot, Ross, 36–37, 117, 127, 318
- Perron-Fröbenius theorem, 16, 260
- Perry, Rick, 129
- plurality voting (first-past the post; relative majority; winner take all), 28, 96, 276, 277; defined, 349; manipulability of, 129; shortcomings of, 29–31, 34–38, 42, 44, 143, 204
- plurality bloc voting, 286–89, 349
- +2 effect, 324, 330, 349
- Political Parties* (Duverger), 39
- Polk, James, K., 36, 187
- poll taxes, 214
- Polsby-Popper compactness score, 253–54, 258–60, 270, 349
- popular vote, 349
- population monotonicity, 201, 202, 349
- population paradox, 181, 186, 190, 201, 202, 349
- population polygon score, 257, 259, 350
- Populist Party, 41n
- Portugal, 46, 279
- positional voting, 78–79
- positive political theory, 107
- Poundstone, William, 120, 129
- Powell, Colin, 314
- precinct, 350
- preference order (preference ballot), 53
- preferential method (ranked choice method), 53, 132–33, 350; growing use of, 4, 58. *See also* instant runoff method
- primaries: in one-party districts, 2n; open, 132; presidential, 276; spoiler effect in, 37, 132; top-two, 29, 46, 351
- Princeton Gerrymandering Project, 258
- prison gerrymandering, 226, 350
- probability distribution function, 264, 350
- profile, in ranked choice voting, 54–56, 350
- Progressive Party, 41n
- Prohibition Party, 41n
- proportional ranked choice voting (choice voting; generalized Hare method; single transferable vote), 289–96, 300–2, 340, 351
- proportional representation, 208, 234–35; benefits of, 41–42, 278, 298–303; bloc

- voting, 286–89, 294–95; in Bosnia, 273–74, 275, 278–79; candidate-centered, 286, 345; closed list, 279, 285; defined, 350; D'Hondt, 277, 281–84, 285, 346; efficiency gap and, 247–48; mixed-member, 280; open list, 279, 349; party list, 278–81, 285–86, 349; prevalence of, 277–78, 279, 281; quotas for, 280–81; Saint-Laguë, 277, 284–86, 350; shortcomings of, 297–98; single transferable vote, 289–96, 300–2, 340, 351; Supreme Court's preoccupation with, 268; in U.S., 276–77, 296
- Quan, Jean, 66
- Quinn, Jameson, 123n
- quota: defined, 350; Droop's, 290–91, 294; for proportional representation, 280–81; for supermajorities, 22, 25
- quota rule, 186, 191, 201, 202, 284
- quotients, in proportional representation, 281–82, 350
- random walk, 263
- range voting (score voting), 112–13, 117–22, 134, 139–40, 350
- ranked choice method (preferential method), 53, 132–33, 350; growing use of, 4, 58. *See also* instant runoff method
- Ratfu**ed* (Daley), 218
- reality shows, 80n
- Reapportionment Act (1929), 157, 158–59, 162, 165, 192, 196, 216n
- recall elections, 38
- recombination, 263, 350
- REDMAP (Redistricting Majority Project), 217–21, 267
- relative majority. *See* plurality voting
- Reock score, 256, 258–59, 270, 350
- representation ratio (standard divisor; SD), 173–84, 188, 189, 193, 195, 285; defined, 155, 351; disparities in, 160–62, 167, 186; growth of, 158–59; proposals for, 164–72; worldwide, 167–69, 171
- representative population, 177n
- responsiveness, of seats-votes curve, 240–42, 350
- Reynolds v. Sims* (1964), 215, 323
- Rhode Island, 160, 177
- Richie, Rob, 143–44
- Riker, William, 107
- Roberts, John G., 268
- Rodden, Jonathan, 260, 271
- Roemer, Buddy, 47–48
- Roman Empire, 76n
- Romney, Mitt, 329–30
- Roosevelt, Franklin D., 200
- Roosevelt, Theodore, 36, 318
- rotten boroughs, 212
- rounding, 23, 151–52; in Adams's vs. Jefferson's models, 203; of standard quotas, 175–76, 186, 189, 194, 195, 324
- round robin tournaments, 93
- Rove, Karl, 220
- RSA encryption, 16
- Rubio, Marco, 32–33, 63
- Rucho v. Common Cause* (2018), 222–23, 268, 270–71
- runoff elections, 45–50, 350. *See also* instant runoff method
- Saari, Donald, 81, 109, 115, 117, 127
- safe district, 350
- Saint-Laguë method, 277, 284–86, 350
- St. Louis, 116, 139
- St. Paul, 58
- Sanders, Bernie, 115, 314
- San Francisco, 58
- Santa Fe, 58
- Santorum, Rick, 129
- Satterthwaite, Mark, 137
- scale invariance, 250–51, 348
- school secession, 226–27
- Schwartzberg score, 255, 256, 259, 270, 350
- Schwarzenegger, Arnold, 34, 221
- score voting (range voting), 112–13, 117–22, 134, 139–40, 350

- Scotland, 224–25, 276, 297
- Seabrook, Nick, 221
- seats bonus (winner's bonus), 235, 351
- seats-votes curve, 237–42, 351
- secret ballot, 25n
- seeding, in sports tournaments, 95n
- self-sorting, 217, 224, 242, 245
- Sen, Amartya, 106–7, 109, 137
- Senate, 22–23, 154; filibusters in, 23
- sequential runoff method (Hare's method; instant runoff method), 46, 60–63, 96–97
- sequential (pairwise) voting, 93–95, 136
- Serbia, 336
- Shape* (Ellenberg), 308
- shareholder voting, 289
- Shaw v. Reno* (1993), 216
- Sherman, Roger, 308n
- sieve method, 20
- Silver Party, 41n
- Simonds, Shelley, 51
- simple majority, 12–14, 22–24, 351
- Simpson's paradox, 75, 110
- Sinema, Kyrsten, 130
- single mixed vote, 280, 351
- single transferable vote (choice voting; generalized Hare method; proportional ranked choice voting), 289–96, 300–2, 340, 351
- Slay the Dragon* (documentary film), 218, 222
- Sliwa, Curtis, 135
- Slovenia, 78, 131, 233, 336
- Smith, Warren, 120
- Social Choice and Individual Values* (Arrow), 105
- social choice function, 101n
- social choice theory, 13n, 14, 17, 92, 100, 108, 351
- Socialist Party (U.S.), 41n
- social welfare function, 101n
- Solomon, Justin, 263, 272
- South Carolina, 58, 132, 212, 214, 312
- South Dakota, 160, 288, 315
- South Korea, 170
- Southwest Territory, 177
- Soviet Union, 116
- Spain, 131
- Sparta, 120
- spoiler effect, 31, 36–37, 63, 64, 78, 89, 103–4; Condorcet criterion linked to, 110; defined, 351; IIA linked to, 104, 110; in instant voting, 72–73, 83, 132, 135, 339; manipulative use of, 129; multiparty coalitions vs., 130
- Spotted Eagle, Faith, 314
- stacking, in gerrymandering, 212n
- Stalin, Joseph, 128n
- standard divisor (SD). *See* representation ratio
- standard quota, 174, 177, 189, 195, 285, 351
- STAR (Score-Then-Automatic-Runoff) voting, 123
- Stein, Jill, 37, 39, 129, 317–18, 331
- Steiner, Jakob, 252, 260
- Steingart, Alma, 205
- Stephanopoulos, Nicholas, 242, 248
- Stewart, Potter, 270
- Stoppard, Tom, 128
- strategic voting (insincere voting; tactical voting), 71, 120, 122, 123, 127–41, 351
- supermajority, 21–24
- Sweden, 285, 301
- Sweet, Betsy, 44
- Switzerland, 191n
- symmetry score, 238n
- tactical voting (insincere voting; strategic voting), 71, 120, 122, 123, 127–41, 351
- Taagepera, Rein, 170
- Tabarrok, Alex, 127
- Taft, William Howard, 318
- Taylor, Zachary, 36
- tennis rankings, 78
- Texas, 116, 132, 173, 300, 315; gerrymandering in, 222, 231, 240, 241, 242, 250
- theorems, 14
- Theory of Committees and Elections, The* (Black), 108

- Theory of Games and Economic Behavior*
(von Neumann and Morgenstern),
105
- Thornburg v. Gingles* (1986), 216
- Three-Fifths Compromise, 309, 351
- Thurmond, Strom, 23, 326
- ties, 21, 50–51, 101; in approval voting, 114; in
instant runoffs, 59; in parity method, 13;
in simple majority method, 12, 20
- Tilden, Samuel J., 191, 320
- top-two primaries, 29, 46, 351
- Tour de France, 78
- Trahan, Lori, 2
- transitive preferences, 90, 101, 109
- Treatise on Election of Representatives, A*
(Hare), 58
- Trenk, Adam, 51
- Trudeau, Justin, 254
- Trump, Donald: Electoral College assailed
by, 326; faithless electors and, 314; in 2016
election, 31, 32–33, 36, 108, 114, 318–19, 37,
39, 88–89, 314, 315, 331, 332; in 2020 elec-
tion, 32, 115, 220, 317–20, 332
- Tshibaka, Kelly, 134–35
- two-candidate elections, 19–21, 25, 28
- Turkey, 46, 279, 281
- turnout, 44, 247n, 248, 299, 324n, 327
- Twelfth Amendment, 311
- Twenty-Third Amendment, 312
- unanimity, 22, 25, 351; in jury trials, 24
- Uniform Congressional District Act (1967),
288, 302
- uniform partisan swing, 351
- United Kingdom, 41, 163, 313; districting
bias in, 224–25; plurality voting in, 31, 40,
277; strategic voting in, 131; votes vs. seats
in, 276
- United Nations, 115
- universal domain, 108, 109
- unreasonable effectiveness of mathematics,
16–17
- UN Security Council, 26
- upvotes and downvotes, 119
- Urban VI, Pope, 24
- Utah, 224
- Vance, J. D., 2
- Vandenberg, Arthur, 192
- van Newenhizen, Jill, 115
- variance, 351
- Vaughn, Christy, 263
- Ventura, Jesse, 33–34
- Vermont, 160, 226, 288, 315
- veto power, 23
- Vieth v. Jubilire* (2004), 267
- Vinton, Samuel F., 179, 190
- Vinton Act (1850), 190
- Vinton method (Hamilton apportionment
method), 44, 247n, 248, 299, 324n, 327
- Virginia, 178, 181, 185, 212, 224, 309
- von Neumann, John, 105, 200
- voter satisfaction efficiency, 123n
- voter suppression, 248
- voter turnout, 44, 247n, 248, 299, 324n, 327
- vote splitting: approval voting resistant to,
114; Borda count resistant to, 77–78;
Condorcet method resistant to, 89; de-
fined, 351; fringe beneficiaries of, 33, 34,
42; instant runoffs resistant to, 63, 64, 132,
329; multiparty coalitions vs., 130; plural-
ity voting vulnerable to, 31, 42
- voting, defined, 351
- Voting Rights Act (1965), 216, 228–29, 266,
301, 302
- Wales, 224–25, 297
- Walker, Herschel, 45
- Wang, Sam, 271–72
- Ware, William, 58n
- Warren, Elizabeth, 115
- Washington, D.C., 312, 315, 317
- Washington, George, 155, 156, 177–79,
182–83
- Washington state, 46, 221
- wasted votes, 143, 242–48, 280, 281, 351

- Webster, Daniel, 187
- Webster apportionment. *See* major fractions method
- Wesberry v. Sanders* (1964), 162, 215
- West, Kanye, 129, 317
- Whig Party, 213
- Wikimedia Foundation, 90
- Willcox, Walter, 191–92, 197, 198
- Wilson, Chris, 167–68
- Wilson, Jennifer, 110
- Wilson, Erica, 227
- Wilson, Woodrow, 36, 167–68
- winner’s bonus (seats bonus), 235, 351
- winner take all. *See* plurality voting
- Wisconsin, 175; gerrymandering in, 219, 221, 222, 224, 248, 260, 264, 265, 267; in 2016 election, 37, 318, 320n; in 2020 election, 319–20
- Wolf, Tom, 271
- Wright, Kurt, 68
- Wu, Michelle, 47
- Wyoming, 160, 315–16, 324
- Wyoming Rule, 166–168, 352
- Yancey, David, 51
- Yang, Andrew, 44
- Young, Danny, 226
- Young, Peyton, 201–2, 203
- Yugoslavia, 336