CONTENTS

PREFACE TO THE 2023 EDITION xi

PREFACE AND ACKNOWLEDGMENTS xix

INTRODUCTION xxi

CHAPTER ONE
The Prehistory of the Classical Interpretation of Probability: Expectation and Evidence

1.1 Introduction 3
1.2 Quantitative and Qualitative Probabilities 6
1.3 Expectation, Equity, and Aleatory Contracts 15
1.4 Degrees of Certainty and the Hierarchy of Proofs 33
1.5 Conclusion 47

CHAPTER TWO
Expectation and the Reasonable Man

2.1 Introduction 49
2.2 Expectation as Reasonableness 58
2.3 The Debate over Expectation 68
2.4 Conclusion 108

CHAPTER THREE
The Theory and Practice of Risk

3.1 Introduction 112
3.2 Risk before Probability Theory 116
3.3 The Mathematical Theory of Risk 125
3.4 Risk after Probability Theory 138
3.5 Conclusion 182

CONTENTS

CHAPTER FOUR
Associationism and the Meaning of Probability
4.1 Introduction 188
4.2 Probability, Experience, and Belief 191
4.3 The Dissociation of Objective and Subjective Probabilities 210
4.4 Conclusion 224

CHAPTER FIVE
The Probability of Causes
5.1 Introduction 226
5.2 Bernoulli's Theorem and the Urn Model of Causation 230
5.3 Bayes' Theorem and the Problem of Induction 253
5.4 The Calculus of Induction 267
5.5 Conclusion 293

CHAPTER SIX
Moralizing Mathematics
6.1 Introduction 296
6.2 The Moral Sciences 298
6.3 Testimony and the Probability of Miracles 306
6.4 The Probability of Judgments 342
6.5 Conclusion 368

EPILOGUE
The Decline of the Classical Theory 370

BIBLIOGRAPHY 387

INDEX 413

CHAPTER ONE

The Prehistory of the Classical Interpretation of Probability: Expectation and Evidence

1.1 Introduction

Although the famous correspondence between Blaise Pascal and Pierre Fermat first cast the calculus of probabilities in mathematical form in 1654, many mathematicians would argue that the theory achieved full status as a branch of mathematics only in 1933 with the publication of A. N. Kolmogorov's *Grundbegriffe der Wahrscheinlichkeitsrechnung*. Taking David Hilbert's *Foundations of Geometry* as his model, Kolmogorov advanced an axiomatic formulation of probability based on Lebesgue integrals and measure set theory. Like Hilbert, Kolmogorov insisted that any axiomatic system admitted "an unlimited number of concrete interpretations besides those from which it was derived," and that once the axioms for probability theory had been established, "all further exposition must be based exclusively on these axioms, independent of the usual concrete meaning of these elements and their relations."[1] Although philosophers, probabilists, and statisticians have since vigorously debated the relative merits of subjectivist (or Bayesian), frequentist, and logical interpretations as means of applying probability theory to actual situations, all accept the formal integrity of the axiomatic system as their departure point.[2] The mathematical

[1] Andrei Kolmogorov, *Foundations of the Theory of Probability*, trans. Nathan Morrison (New York: Chelsea Publishing Company, 1950), p. 1.

[2] Ernest Nagel, *Principles of the Theory of Probability*, in *International Encyclopedia of Unified Science*, vol. 1, part 2, Otto Neurath, Charles Morris, Rudolf Carnap, eds. (Chicago: University of Chicago Press, 1955), pp. 368–369.

theory itself preserves full conceptual independence from these interpretations, however successful any or all may prove as descriptions of reality.

This logical schism between the formal axiomatic system and its concrete interpretations is not unique to probability theory: geometry, algebra, and the calculus have also been translated into purely formal systems and explicitly divorced from the contexts from which they emerged historically. For modern mathematicians, the very existence of a discipline of applied mathematics is a continuous miracle—a kind of prearranged harmony between the "free creations of the mind" which constitute pure mathematics and the external world.

Although these are pressing issues for the philosopher of mathematics, they tend to blur historical vision. While innumerable interpretations may logically satisfy the axioms of the mathematical theory of probability, in point of fact the historical development of the theory was dominated almost from its inception until the mid-nineteenth century by a single interpretation, the so-called "classical" viewpoint. Throughout the eighteenth and nineteenth centuries, probabilists understood the classical interpretation and the mathematical formalism underlying it to be inextricable—indeed, to be one and the same entity. If any distinction between the levels of application, interpretation, and formalism existed in the minds of the classical probabilists, the hierarchy in which these levels were arranged reversed the modern order: the mathematical formalisms of probability theory were justified to the extent that they matched the prevailing interpretation and field of application, rather than the interpretation and its ensuing applications being sanctioned to the degree that they satisfied the formal axioms.

Where did the classical interpretation come from? Seventeenth-century texts—literary, religious, philosophical, medical, scientific, legal—abound with references to "probability" of one sort or another, and two recent works have studied these proliferating, mutating usages in fascinating detail.[3] My question about the origins of the classical interpretation cuts at right angles to these concerns: out of the swarm of probabilistic notions abroad at mid-century, which ones supplied the first mathematical probabilists with concepts and problems—and why? Posed in this way, it is a question about quantification. Recasting ideas in mathematical form is a

[3] Ian Hacking, *The Emergence of Probability* (Cambridge, Eng.: Cambridge University Press, 1975), and Barbara J. Shapiro, *Probability and Certainty in Seventeenth-Century England* (Princeton: Princeton University Press, 1983).

selective and not always faithful act of translation. In the seventeenth-century geometrization of mechanics, only local motion survived from that cluster of phenomena Aristotle had called change: a falling body, a growing oak, a wavering mood. Similarly, only some of the ambient seventeenth-century views about what probability meant passed through the filter of the mathematical methods invented by Pascal, Fermat, Christiaan Huygens, and Jakob Bernoulli. Those that did changed their meaning as well as their form. John Wilkins's philosophical certainty, envisioned as three ascending stages of moral, physical, and metaphysical assurance, was not identical to Jakob Bernoulli's full continuum of degrees of certainty ranging from zero to one, any more than Galileo's description of rest as an infinite degree of slowness was identical to scholastic distinctions between the states of rest and motion. Quantification was not neutral translation. This chapter is about how certain qualitative probabilities became quantitative ones in the latter half of the seventeenth century, and created the classical interpretation in the process.

Fitting numbers to the world changes the world—or at least the concepts we use to catch hold of the world. A world of continua spanning rest and motion, certainty and ignorance does not look like a world of sharp either/or oppositions. But the world can change the numbers as well. To be more precise: if we want our mathematics to match a set of phenomena with reasonable accuracy, we may have to alter (or invent) the mathematics to do so. The tandem development of mechanics and the calculus in the seventeenth century is full of examples of new mathematical techniques that mimicked motion: Giles Roberval's velocity method of finding tangents, or Isaac Newton's machinery of fluxions and fluents. The case of classical probability theory is less dramatic in a mathematical sense, for probabilists had few new techniques to call their own until the end of the eighteenth century. Yet this very lack of new mathematical content bound mathematical probability all the more firmly to its applications. Since it belonged wholly to what we would now call applied mathematics, probability theory stood or fell upon its success in modeling the domain of phenomena that the classical interpretation had mapped out for it. Failure threatened not just this or that field of application, but the mathematical standing of the theory itself. Hence classical probabilists bent and hammered their definitions and postulates to fit the contours of the designated phenomena with unusual care. I shall deal at length with examples of their handiwork in Chapter Two; here I only wish to point out that quantification is a two-way street. Neither the original subject matter nor the mathematics emerges entirely unchanged from the encounter.

The classical interpretation of probability was the result of such an encounter between a tangle of qualitative notions about credibility, physical symmetry, indifference, certainty, frequency, belief, evidence, opinion, and authority on the one hand, and algebra and combinatorics on the other. By looking closely at the problems posed by the early mathematical probabilists, and the concepts they used to solve them, we can locate the point of intersection between the quantitative and the qualitative. Of all the then available meanings of probability, which were grist for the mathematicians' mill, and why? Once the mathematicians had made their choice, to what kind of program of applications did it commit them? I shall argue that seventeenth-century legal practices and theories shaped the first expressions of mathematical probability and stamped the classical theory with two of its most distinctive and enduring features: the "epistemic" interpretation of probabilities as degrees of certainty; and the primacy of the concept of expectation. Moreover, legal problems provided the principle applications for the classical theory of probability from the outset. Even the earliest problems concerning games of chance and annuities were framed in legal terms drawn from contract law, and, as will be seen in subsequent chapters, classical probabilists of the eighteenth and nineteenth centuries continued to include other sorts of legal problems, such as the credibility of testimony and the design of tribunals, within the compass of their theory.

1.2 Quantitative and Qualitative Probabilities

No monistic explanation can satisfactorily account for so complex an intellectual phenomenon as the advent of mathematical probability, and I do not intend to put forward any such here. However, I do claim that more than any other single factor, legal doctrines molded the conceptual and practical orientation of the classical theory of probability at the levels of application, specific concepts, and general interpretation. Although some historians have noted in passing the legalistic tone of the writings of the early probabilists, they have tended to regard the more explicitly juridical formulations, such as that of Gottfried Wilhelm Leibniz, as idiosyncratic. Ernest Nagel mentioned the medieval arithmetic of proof in a survey of premodern notions of probability;[4] Alexandre Koyré commented upon the lawyerly approach of the Pascal/Fermat correspondence;[5] Ian Hacking discussed Leibniz's probabilistic proposal to settle conflicting property claims.[6] Ernest Coumet has systematically pursued these allusions in his illuminating discussion of the relationship between Jesuit casuistry, seventeenth-century contract law, and mathematical probability, but only with respect to the Pascal/Fermat correspondence.[7]

Yet the works of the early probabilists are full of legal references. Pascal, in a 1654 address to the Académie de Paris on his current scientific projects, described his research on the "géométrie du hasard" as a means of determining equity: "The uncertainty of fortune is so well ruled by the rigor of the calculus that two players will always each be given exactly what equitably [*en justice*] belongs to him."[8] Huygens and Johann De Witt presented the fundamental propositions of the calculus of probabilities in terms of contracts and equitable exchanges; Part IV of Jakob Bernoulli's *Ars conjectandi* bristled with legal examples; Nicholas Bernoulli wrote an entire dissertation on the applications of mathematical probability to the law. As A. A. Cournot observed in 1843, the early probabilists had for the most part little idea of how their new calculus might be applied to "the economy of natural facts," being primarily concerned with the "rules of equity."[9] The spirit, if not the letter, of Leibniz's views on the close connection between the calculus of probabilities and jurisprudence was widely shared by his contemporaries.

Before going on to argue this claim in detail, however, we must take some account of the alternative theories put forward by historians about the roots of mathematical probability. My survey of this large and growing literature will be necessarily brief, and directed principally toward the adequacy of these explanations for understanding why *mathematical* proba-

[4] Nagel, *Principles*, p. 348.

[5] Alexandre Koyré, "Pascal savant," in *Blaise Pascal, l'homme et l'oeuvre*, Cahiers de Royaumont (Paris: Éditions de Minuit, 1956), p. 291.

[6] Hacking, *Emergence*, chapter 10.

[7] Ernest Coumet, "La théorie du hasard est-elle née par hasard?" *Annales: Économies, Sociétés, Civilisations* 25 (May-June 1970): 574–598.

[8] Blaise Pascal, "Celeberrimas Matheseos Academiae Parisiensi" (1654), in *Oeuvres complètes de Pascal*, Jean Mesnard, ed. (Paris: Bibliothèque Europééne-Desclès de Brouwer, 1970), vol. 1, part 2, p. 1034.

[9] Antoine Augustin Cournot, *Exposition de la théorie des chances et des probabilités* (Paris, 1843), pp. 86–87.

bility emerged when and how it did. I do not contest the value of these accounts for understanding the increasing complexity and importance in the early modern period of probabilistic notions more broadly construed: they are rich in insights that will, I believe, eventually help rewrite the intellectual history of the era. But because I am here interested in which specific kinds of probability passed through the strait and narrow gate of quantification, my criteria for an adequate explanation will be correspondingly narrow and specific.

The prehistory of mathematical probability has excited much interest among historians of mathematics, perhaps because the rise of a mathematical approach to chance in the seventeenth century seems at first glance long overdue. The origins have been sought in astronomy, fine arts, gambling, medicine, alchemy, and the insurance trade. The quest for antecedents has been a frustrating one, uncovering proto-probabilistic thinking everywhere and nowhere. Certain passages of Aristotle, for example, could be construed as an embryonic version of statistical correlation or a scale of subjective probabilities; with an even greater effort of the imagination, Bayes' theorem may be discovered in medieval Talmudic exegesis.[10] However, these philosophical discourses on the nature of chance and rules of thumb for dealing with situations fraught with uncertainty (e.g., an astrological prediction or a medical prognosis) not only fall short of a mathematical treatment of probability considered in and of themselves, but they also manifestly failed to generate such a theory.

More clear-cut elements of mathematical probability, such as an enumeration of all possible outcomes for the throw of several dice, surface as early as the tenth century,[11] but, like the promising hints scattered through the classical and medieval philosophical texts, evidently bore no mathematical fruit. Plausible practical sources of mathematical probability prove equally sterile upon investigation. Despite the popularity of gambling since time immemorial, games of chance apparently did not

[10] O. B. Sheynin, "On the prehistory of the theory of probability," *Archive for History of Exact Sciences* 12 (1974): 97–141, especially pp. 101, 119; Nachum L. Rabinovitch, *Probability and Statistical Inference in Ancient and Medieval Jewish Literature* (Toronto and Buffalo: University of Toronto Press, 1973), pp. 58–60; S. Sambursky, "On the possible and the probable in ancient Greece," *Osiris* 12 (1956): 35–48.

[11] M. G. Kendall, "The beginnings of a probability calculus," *Biometrika* 43 (1956): 1–14, reprinted in *Studies in the History of Statistics and Probability*, E. S. Pearson and M. G. Kendall, eds. (Darien, Conn.: Hafner, 1970), vol. 1, pp. 19–34.

suggest notions of stable statistical frequencies or combinatorial derivations of probabilities until the sixteenth century. The sale of maritime insurance and annuities, both known since ancient times and revived on an impressive scale by fourteenth-century Italian entrepreneurs, also failed to spark a mathematics of chance or even a compilation of statistics, as we will see in Chapter Three. Even the Problem of Points—the division of stakes in an interrupted game of chance which prompted the seminal Pascal/Fermat correspondence of 1654—had been posed in a mathematical context as early as 1494, in Luca Pacioli's *Summa de arithmetica, geometrica, proportioni et proportionalita*.[12]

These attempts to trace the ancestry of mathematical probability usually founder on the issue of timing. Although chance figured in philosophical speculation and practical dealings since ancient times, mathematical probability did not emerge until the middle of the seventeenth century. What was the intellectual seed crystal introduced during this critical period that permitted ambient and often ancient ways of thinking about chance to coalesce in mathematical form?

The catalyst does not appear to have been mathematical. Mathematical prerequisites posed no obstacle. In its original form, probability theory presupposed only elementary combinatorics, and although the work of Pascal, John Wallis, Leibniz, F. van Schooten, and lesser-known figures such as Jean Prestet on this subject kindled mathematical interest during the latter half of the seventeenth century,[13] combinatorial thinking appears to have been more stimulated by nascent probability theory than the reverse. Almost all of the major works on combinatorics were published after the first treatise on mathematical probability, Huygens's *De ratiociniis in aleae ludo* (1657), appeared. Wallis's *Discourse on Combinations, Alternations, and Aliquot Parts* was published as an appendix to the English edition of his *Treatise on Algebra* (1685). Pascal's *Traité du triangle arithmétique* was apparently printed in 1654 (though circulated in 1665),[14] the same year as his exchange with Fermat. However, Pascal's original solution to the Problem of Points reveals that he recognized the relevance of the arith-

[12] Kendall, "Beginnings," p. 27.

[13] Eberhard Knobloch, "Musurgia universalis: Unknown combinatorial studies in the age of Baroque absolutism," *History of Science* 17 (1979): 258–275; also his "The mathematical studies of G. W. Leibniz on combinatorics," *Historia Mathematica* 1 (1974): 409–430.

[14] Pascal, *Oeuvres*, vol. 1, part 2, pp. 33–37.

metic triangle only belatedly, and initially shied away from the combinatorial method it embodied. Leibniz's *Dissertatio de arte combinatoria* was published in 1666; Prestet's *Elemens des mathématiques* in 1675 (Book II discussed combinations and permutations). Van Schooten's comments on combinations appeared in his *Exercitationum mathematicarum* of 1657, to which Huygens's *De ratiociniis* was appended. While van Schooten's work must have influenced his student Huygens's approach to probability, his own remarks on the subject were brief and schematic, serving as the basis for a discussion of prime factorization rather than possible outcomes.

Thus, extended mathematical treatments of combinations postdated the earliest published treatise on probability theory. Indeed, some of the most comprehensive treatments of combinations and permutations appeared as supplements to works on probability, such as Part II of Jakob Bernoulli's *Ars conjectandi* (1713) and Part I of the second edition of Pierre de Montmort's *Essai d'analyse sur les jeux de hazard* (1713). The two subjects developed in tandem.

Nor did any new philosophy of chance develop during this period, although the protracted religious controversies that wracked Europe during the sixteenth and seventeenth centuries did persuade an increasing number of thinkers of the vanity of human pretensions to certainty.[15] Classical probabilists from Jakob Bernoulli through Laplace followed the Thomist line:[16] from the perspective of an omniscient God (or later Laplace's secularized supercalculator), the events of the universe were fully determined. Chance was merely apparent, the figment of human ignorance. Until the nineteenth century, no mathematician, scientist, or philosopher appears to have contemplated the possibility of genuinely random phenomena except to dismiss the idea as nonsensical: causeless events were unthinkable. Indeed, from Hobbes and Spinoza through d'Holbach, the philosophical climate of opinion during the seventeenth and eighteenth centuries grew ever more resolutely deterministic. Let Abraham De Moivre speak for these deterministic probabilists. True chance, he claimed, "imports no determination to any *mode of Existence*; nor indeed to *Existence* itself, more than to non-existence; it can neither be defined nor understood: nor can any Proposition concerning it be either affirmed or denied, excepting this

[15] See Richard Popkin, *The History of Scepticism from Erasmus to Descartes* (Assen, Netherlands: Van Gorcum, 1964), chapter 1; Shapiro, *Probability*, chapters 1–3.

[16] Edmund F. Byrne, *Probability and Opinion* (The Hague: Martinus Nijhoff, 1968), pp. 293–296.

one, 'That it is a mere word.' "[17] The random was simply unintelligible. Although some historians have attributed the tardy development of mathematical probability to the "absence of a notion of chance," the writings of the classical probabilists do not remedy this dearth. On the contrary, they strenuously denied both the subjective and objective existence of real chance. However, failure to articulate a concept of randomness evidently did not hinder the birth and growth of mathematical probability from the seventeenth through the mid-nineteenth centuries.[18]

These and other aspects of the literature on the prehistory of probability have been treated at greater length recently by Ian Hacking in the most sophisticated and stimulating treatment of the subject to date.[19] Concerned with the emergence of a concept of probability in the broadest sense, Hacking ranges over philosophical, practical, and legal, as well as mathematical themes. However, he argues that all of these kinds of probability (1) share a common feature that stamps our understanding of probability to this day, namely a dual aleatory and epistemic aspect; and (2) could not fully emerge in any form before a "mutation" in the concept of evidence prepared the way around the turn of the seventeenth century. According to Hacking, astrologers, physicians, alchemists, and other sixteenth-century practitioners of the nondemonstrative "low" sciences evolved a new concept of diagnosis that linked overt "signs" to hidden properties, and at the same time associated these natural signs with an authoritative text, the "book of nature." Thus the old, epistemic meaning of probability as belief or opinion warranted by authority merged with the new, aleatory idea of observed (if unexplained) correlations between events (e.g., between fever and disease, comets and the death of kings) to create the concept we still recognize as probability.

This, much telescoped, is Hacking's thesis, and it has provoked considerable controversy among historians and philosophers. Like any important and original claim, it is vulnerable to challenges at several levels. Do all significant seventeenth-century (not to mention later) usages of "probability" really reduce to the epistemic ("opinion derived from authority") and aleatory ("natural signs correlated by experience") elements Hacking believes to constitute probability? Using only the English literature of the period, Barbara Shapiro has documented many other shades of probability,

[17] Abraham De Moivre, *Doctrine of Chances*, 3rd edition (London, 1756), p. 253.
[18] Sheynin, "Prehistory," p. 141.
[19] Hacking, *Emergence*, chapters 1–5.

including degrees of certainty or assurance; reasonable doubt; verisimilitude; worthiness to be believed (credibility); epistemological modesty.[20] In fact, seventeenth-century probability had more than Janus's two faces; it was more a group of visages loosely assembled in a family portrait. Conversely, several key seventeenth-century instances of concepts and methods using one or another of Hacking's two constituents do not mention the word "probability": for example, John Graunt's analysis of the London bills of mortality. Only with the benefit of hindsight can we exclude some of the ideas that seventeenth-century writers did call "probability" and include some they did not.

Hacking's dissection of probability, now and then, into two and only two constituents has the great advantages of conceptual clarity and of setting the standards for the solution of a knotty historical problem, namely, when and why did the concept of probability emerge? In essence, Hacking reasons that the concept *X* has components *a, b, c*; if we find *a, b, c*, then *X* has entered the realm of the thinkable. Alas for the clear-thinking historian, the contexts in which *a, b, c* occur may be so disparate from one another, or so alien to current sensibilities, that we can hardly glue these bits and pieces together to form any single notion at all, much less a familiar one.

However, even those who accept Hacking's account of the two constituents of probability might question his explanation of how they came to be fused together just when they did. For Hacking, the aleatory element—probabilities as observed frequencies—derives from the sixteenth-century doctrine of natural signs, which created a new kind of "internal" evidence of things rather than of testimony. This is the "diagnosis," the inference from one particular to another, which Hacking claims achieved "a new conceptualization" in the works of Renaissance practitioners of the low sciences like Paracelsus.[21] A great deal depends on the novelty of the "diagnosis," for the epistemic element of "opinion" had been the standard meaning of "probability" for centuries: a new kind of nondemonstrative knowledge—and a link between new and old—is needed to resolve the problem of timing that bedevils all historians of probability. Daniel Garber and Sandy Zabell have collected instances from the medieval handbooks of law and rhetoric that show that the idea of internal evidence was firmly established in the Latin West after the twelfth century, with a dis-

[20] Shapiro, *Probability*.

[21] Hacking, *Emergence*, pp. 34–37.

tinguished classical pedigree. They also point out that the link between such natural signs and the writ of God—Hacking's bridge between "old" epistemic and "new" aleatory elements—is a venerable one.[22]

My aim here is not to develop yet another account of the rise or emergence of probabilistic notions in the early modern period. In order to settle even the prior question of whether there was indeed such a rise (Hacking and Shapiro claim there was; Garber and Zabell are dubious) would involve a thorough canvassing of most of the classical, medieval, and Renaissance learned corpus to establish a baseline. Probability could surface almost anywhere, and did. Unlike Hacking and the majority of his predecessors and critics, I do not believe that the origins of mathematical probability were identical to those of conceptual probability. But I do maintain that some concepts are more readily quantified than others, and that this was the case within the conceptual field available to the early probabilists. That they had a choice in the matter is evident not only from the several sorts of probability concepts available in the mid-seventeenth century, but also from the way in which the domain of applications for probability theory later shifted. As we shall see in later chapters, eighteenth- and nineteenth-century probabilists sometimes diverged sharply in their views about what kinds of problems their theory could solve. Subjects to which classical probabilists from Jakob Bernoulli through Laplace had devoted much attention—such as the probability of testimony or the probability of causes—were rejected out of hand by their successors: probability theory was no longer "about" those matters.

What was probability theory about in the second half of the seventeenth century? The time-honored answer is: games of chance.[23] This answer has much to be said for it, for the pioneers of mathematical probability—Gerolamo Cardano, Galileo, Pascal, Fermat, Huygens—all solved gambling problems. But it is also incomplete and misleading: incomplete, because it omits the other important applications concerning evidence, demography, and annuities that very soon accreted to the theory in the work of De Witt, Edmund Halley, John Craig, Jakob and Nicholas Bernoulli; misleading, because it suggests that gambling provided the early probabilists with the conceptual framework in which they posed and

[22] Daniel Garber and Sandy Zabell, "On the emergence of probability," *Archive for History of Exact Sciences* 21 (1979): 33–53.

[23] See, for example, Isaac Todhunter, *A History of the Mathematical Theory of Probability from the Time of Pascal to That of Laplace* (London and Cambridge, Eng., 1865).

solved their problems. Upon closer examination, the works of the early probabilists turn out to be more about equity than about chances, and more about expectations than about probabilities. These ideas and the applications they stimulated—for example, to games of chance and annuities—came, as we shall see, largely from the law. The next generation of probabilists owed a second debt to jurists, this time for the interpretation of mathematical probability as a degree of certainty. This new interpretation in turn spawned a new set of applications concerning evidence both in and out of the courtroom. Both these legal borrowings share a common feature: while neither the doctrine of contract equity nor that of evidence were truly quantified, practice in one case and theory in the other had given them a proto-quantitative form that made them seem ripe for a thoroughgoing mathematical treatment. Equally or more familiar seventeenth-century senses of "probability," like verisimilitude, were never so conceived, and hence never made it into the mathematicians' repertoire of applications.

The principal contributions of jurisprudence to early mathematical probability were thus twofold. First, early probabilists like Jakob Bernoulli and Huygens drew upon legal doctrines concerning aleatory contracts—that is, those involving some element of chance, such as games of chance and annuities—as sources not only of problems, but also of fundamental concepts and definitions. Aleatory contracts, like all contract law, centered upon considerations of equity and fair exchange among partners. Classical probabilists quite explicitly translated the legal terms for an equitable contract into mathematical expectation—that is, the value of an uncertain prospect—and made expectation, rather than probability per se, the departure point for the first expositions of mathematical probability. Second, legal theories of evidence supplied probabilists with a model for ordered and even roughly quantified degrees of subjective probability. The hierarchy of proofs within Roman and canon law led mathematicians to conceive of degrees of probability as degrees of certainty along a graduated spectrum of belief, ranging from total ignorance or uncertainty to firm conviction or "moral" certainty.

Thus jurisprudence furnished two striking features of the classical interpretation of probability: the subjective understanding of probability as a "degree of certainty"; and the prominence of the concept of probabilistic expectation. Classical probability theory retained these legal elements, albeit in modified form, throughout its career.

The remainder of this chapter is divided into two parts. Section 1.3 explains how probabilistic expectation derived from the doctrine of aleatory contracts and examines its critical role in the first formulations of mathematical probability. Section 1.4 describes the relationship between the hierarchy of proof in Roman and canon jurisprudence and the subjective or epistemological orientation of classical probability theory.

1.3 Expectation, Equity, and Aleatory Contracts

Between July and October of 1654 the mathematicians Blaise Pascal and Pierre Fermat exchanged a number of letters that tradition recognizes as the beginning of mathematical probability theory. There were of course precursors, chief among them Gerolamo Cardano's manuscript *Liber de ludo aleae* (composed circa 1530, but first published in 1663). Historians have hesitated to count Cardano's work as the origin of probability theory for a number of reasons: only a part of the brief treatise actually deals with the computation of chances, and like most of what Cardano wrote, the treatment now seems odd, peppered as it is with personal anecdotes, philosophical reflections, classical allusions, and much hardheaded advice on cheating, strategies, and the psychology of competition. However, the book is very revealing of some of the early conceptual difficulties facing the mathematical theory, and we shall return to it in Section 1.4 to illuminate later developments. But while Cardano's work was without influence, the Pascal/Fermat correspondence created a research tradition, complete with problems and concepts, that dominated the field for over fifty years. On these grounds alone it deserves its traditional place in the history of mathematical probability, and I shall not break with that tradition.

Apparently at the instigation of the mathematical dabbler and man-about-town, the Chevalier de Méré,[24] Pascal posed the following "Problem of Points" to Fermat: Two players, A and B, each stake thirty-two pistoles on a three-point game. When A has two points and B has one, the game is interrupted. How should the stakes be divided? Fermat's solution, as it can be pieced together from the extant correspondence (particularly Pascal's reply of 24 August 1654), seems to rest upon a full enumeration of all possible outcomes. Pascal's approach, which has been described as "re-

[24] See Hacking, *Emergence*, chapter 7, for the circumstances.

cursive,"[25] rejected Fermat's combinatorial method as unwieldy and potentially liable to error.[26] Pascal's solution, which fortunately survives in full, was based on expectations rather than combinations. With the aid of the definition of expectation formulated by later probabilists, player A's expectation would be

$$(1/2)(32) + (1/2)(64) = 48 \text{ pistoles.}$$

Pascal got the same answer, but his line of reasoning was different. The later definition breaks expectation down into the product of probabilities and their associated outcome values, both of which are assumed to be known a priori. In contrast, Pascal made expectation and equality of condition the primitive concepts of his analysis. Since player A is assured thirty-two pistoles no matter what the outcome of the next round, Pascal contended that it is only the remaining thirty-two pistoles that are at issue. Because "le hasard est égal" for both A and B in the upcoming round, Pascal decided they should halve the remaining thirty-two pistoles. In modern notation, A's expectation would be

$$(1)(32) + (1/2)(32) = 48 \text{ pistoles.}$$

However, the modern notation is misleading in its suggestion that the two conceptualizations of the problem are symmetric, even though they are equivalent. In fact, only one term of Pascal's solution dissected expectation into distinct probability and outcome factors, and even then the terms must be used advisedly: the 1/2 factor derived from the equality of condition between the two players; also, the thirty-two pistoles did *not* represent the outcome value for A's winning the next round of play. Although Pascal clearly knew the outcome values of A's winning or losing the next round, and understood Fermat's combinatorial solution, he chose to analyze the problem in terms of certain gain and a remainder subject to equitable distribution. Only after this fundamental expectation has been established do probabilities of any description enter the argument, and then only to endorse halving the residual amount as fair. Unlike Fermat, Pascal's strategy consisted in eliminating explicit considerations of probability from as much of the problem as possible, substituting certain gain

[25] Kokiti Hara, "Pascal et l'induction mathématique," *Revue d'Histoire des Sciences* 15 (1962): 287–302.

[26] Pascal, *Oeuvres*, vol. 1, pp. 1147–1153.

and equity in their place. Fermat's solution took equiprobable combinations as fundamental; Pascal's approach was built upon expectation. Both mathematicians viewed the problem as one of determining expectations rather than probabilities.

Pascal's distrust of combinations stemmed largely from his belief that they were both cumbersome to manipulate and ambiguous to enumerate (an objection voiced more strongly by Roberval and later by Jean d'Alembert), rather than from any suspicion that Fermat's methods were invalid. Once Pascal realized that combinations (i.e., coefficients of the terms of the binomial expansion) could be systematically read off from the arithmetic triangle, he himself favored this approach to the mathematical analysis of games of chance. (When Pascal claimed that Fermat's method "has nothing in common with my own," he apparently meant that Fermat had suggested no mechanical means of finding combinations such as the arithmetic triangle provided.) Nonetheless, expectation remained fundamental in the treatments of Huygens and De Witt, and continued to play an important role in classical probability theory even after probabilities came to be defined explicitly in terms of ratios of combinations.

Why expectations instead of probabilities? The answer lies in the Problem of Points itself, which had tested mathematical mettle long before the Chevalier de Méré posed it to Pascal,[27] and arose out of a primarily legal context which made equity the paramount consideration. Consider another of the earliest discussions of quantitative probabilities, in the concluding chapter of Antoine Arnauld and Pierre Nicole's famous *La logique, ou l'Art de penser* (1662), better known as the Port Royal *Logique*. The authors[28] criticize those who err on either the side of excessive caution or recklessness in the conduct of their daily affairs. Readers are advised to consider not only "the good and the bad in itself, but also the probability that it will or will not happen, and to consider mathematically [*géometriquement*] the proportion that all of these things have together." In other words, decisions should be based on the expectation of the outcomes.

The example given to illustrate this counsel drove home the association with probabilistic expectation. Ten players each contribute one unit coin to the pot; each has the possibility of losing one or gaining nine, but the

[27] Kendall, "Beginnings," pp. 26–27.

[28] There is some speculation about the authorship of Part IV; see Hacking, *Emergence*, pp. 73–74.

game is so designed that it is "nine times more probable" that any given player will lose one coin rather than gain the other nine: "Thus each hopes for nine écus, has one écu to lose, nine degrees of probability of losing one écu, and only one to win the nine écus: this makes for perfect equality."[29] This then, is what was meant by the injunction to consider the proportion "mathematically": the ratio of "degrees of probability" for gain or loss is inversely proportional to the ratio of the gain or loss itself.

Although this dictum yielded results equivalent to those given by the later definition of expectation as the product of the probability and the outcome value, the conceptual slant differed significantly. The Port Royal version of expectation offered no general means of reckoning probabilities beyond the conventional estimation of odds for equiprobable cases. There was no mention of combinations. Like other early expositions of mathematical probability, the Port Royal *Logique* made expectation rather than probability the central concept, in order to ascertain the conditions that made risk "equitable." These expositions concentrated on problems of rational decision in the face of uncertainty and of the terms of a fair game or just division of stakes, as in the Problem of Points. The Problem of Points antedated the Pascal/Fermat correspondence by at least a century; Pacioli and Nicolo Tartaglia were among the mathematicians who had made unsuccessful attempts to solve it.[30] All of these solutions, including that proposed by Pascal and Fermat which laid the foundations for mathematical probability, grappled with the issue of a "fair" distribution based on a true measure of expectation.

Ernest Coumet has situated the Pascal/Fermat correspondence against the background of late sixteenth- and seventeenth-century legal and theological discussions that debated whether risk taking in trade should be exempted from church prohibitions against gambling and usury.[31] This controversy focused attention on the class of contract law known as aleatory, because it dealt with agreements involving an element of chance: insurance, games of chance, annuities, and so forth. I would like to pursue Coumet's insight beyond the immediate origins of mathematical probability, into the works of the first generation of mathematical probabilists, in order to show how the legal doctrine of aleatory contracts continued to

[29] Antoine Arnauld and Pierre Nicole, *La logique, ou l'Art de penser* (1662), Pierre Clair and François Girbal, eds. (Paris: Presses Universitaires de France, 1965), p. 353.

[30] Kendall, "Beginnings," pp. 26–27.

[31] Coumet, "Théorie," pp. 579–582.

exert a strong influence on the calculus of probabilities at the level of definitions and proofs as well as that of applications. This persistent legal slant guaranteed the concept of expectation a prominent place in the classical theory of probability.

Like all contract law, treatises on aleatory contracts sought to specify conditions of equity and rules for exchanging goods in-hand for the more or less likely prospect of other, more valuable goods. By the sixteenth century, aleatory contracts were an established category in civil—that is, Roman—law, although the types of situations covered by this designation varied from jurist to jurist. Charles Du Moulin, for example, distinguished between the licit practices of purchasing annuities and wheat futures and the reprehensible pastime of gambling, although he admitted that all involved "uncertainty and danger." The prohibition against gambling was primarily a moral one, and did not prevent Du Moulin from treating all such aleatory cases jointly in his *Summaire du livre analytique des contractz usures* . . . (1554). In general, aleatory contracts included any formal agreement in which chance might figure, including not only insurance and games of chance, but also inheritance expectations and even risky business investments. The legal discussions all revolved around the same issue: as contracts, such agreements must assure all parties of maximum "reciprocity" or equality of terms. How should the (certain) price of an uncertain gain be assessed in order to preserve the rule of equity?

Although the answers to this question were largely qualitative, they display attempts to "proportion" risk to gain in a way that provided the prototype for probabilistic expectation. The discussions of risk sharing among business partners are particularly revealing on this point. Many seventeenth-century jurists hoped to override church proscriptions against usury by equating interest reaped on investments in, for example, a merchant-shipping expedition, with the legitimate earnings paid for work done or services rendered. Investors, it was argued, deserved a share of the profit for having shared the risks. "Mixed" partnerships in which some partners supplied capital and others labor dated from Roman times,[32] and by the sixteenth century it had become common practice for one partner to assume the "péril des deniers"—a kind of insurance policy—as their contribution to the venture.

[32] Eli F. Heckshaw, *Mercantilism*, trans. Muriel Shapiro (London: George Allen & Unwin, 1935), vol. 1, p. 332.

Jurists defended the right of such risk-bearing partners, who essentially functioned as insurers, to a share of the profit, known as the "price of the peril."[33] This practice, derived from the Roman *foenus nauticum* (bottomry), had been linked to usury by some medieval canon lawyers. In this type of arrangement, the shipowner need not repay the loan if the goods are lost at sea. By the sixteenth century, however, the risk was widely accepted as a title to profit even in the so-called triple contract, which involved a second, separate insurance contract as well as the original contract of partnership. The third element was a contract "by which an uncertain future gain is sold for a lesser certain gain."[34]

By the early seventeenth century, Hugo Grotius had extended this equation of risk with earnings to exonerate bankers from usury. Dutch financiers were justified in charging merchants a 12 percent interest rate on loans, as opposed to the standard 8 percent rate, "because the hazard was greater. The justice and reasonableness indeed of all these regulations must be measured by the hazard or inconvenience of lending." Already in 1645, the Sacred Congregation of Propaganda, in a reply to a Jesuit request that Chinese converts who lent at interest be granted a dispensation from usury strictures, spoke for many jurists in approving such loans "provided that there is considered the equality and probability of danger, and provided that there is kept a proportion between the danger and what is received."[35]

Rules for translating risk into compensation remained for the most part qualitative, but were guided in spirit by the so-called Rule of Fellowship (i.e., distributive proportion), which specified that the profit of each partner should be proportional to his investment. Every sixteenth-century text on practical arithmetic included a section on the Rule of Fellowship, illustrated with numerous examples and problems. Some also discussed the "double" Rule of Fellowship, which took into account the duration as well as the amount of the investment.[36] Probabilities, or rather expectations,

[33] François Grimaudet, *Paraphrase des droicts des usures pignoratifs* (Paris, 1583), p. 92.

[34] John T. Noonan, Jr., *The Scholastic Analysis of Usury* (Cambridge, Mass.: Harvard University Press, 1957), pp. 137–151, 209.

[35] Noonan, *Usury*, pp. 281–283, 289.

[36] See, for example, Estienne De La Roche, *L'Arismethique* (Lyons, 1520); Thomas Masterson, *His First Books of Arithmeticke* (London, 1652); Pierre Forcadel, *L'Arithmeticque* (Paris, 1557); Simon Stevin, *L'Arithmetique* (Leyden, 1585); P. Taillefer, ed., *Methodiques institutions de la vraye et parfaite arithmetique de Iacques Chauvet* (Paris, 1615); also David Murray, *Chapters in the History of Bookkeeping, Accountancy and Commercial Arithmetic* (Glasgow: Jackson, Wylie, & Co., 1930), pp. 144, 437–445.

were thus familiarly, if qualitatively, conceived in terms of proportions by the late sixteenth century, and this is the format in which early probabilists like Huygens expressed their mathematical versions of expectation.

Jurists and theologians concerned with accommodating usury prohibitions to commercial practices posed the type of questions the probabilists addressed: "What is the price one should offer to those who undergo the perils, and other fortuitous events, to which everything is subject in commerce, and especially money; what is the sum proportioned to the indefinite and uncertain gain which pledges as backing for a Society of Merchants?"[37] Other aleatory contracts posed analogous problems for jurists. Although the jurists who attempted to specify conditions of equity for contracts involving uncertain outcomes, such as insurance policies on seabound cargoes, were no more interested in quantifying risks on a statistical basis than were their clients, they did argue that profits should be scaled according to risk. This precept led to a qualitative conception of expectation as a compound of the magnitude of the risk and the value of the outcome, one very similar to the Port Royal *Logique*'s dictum that both probability and contingent advantage should be considered "in proportion." Like the Port Royal authors, the jurists were primarily concerned with the equality of expectation as a precondition for a fair game, insurance policy, division of profits, price of a lottery ticket, and so on. The determination of the component probabilities that conditioned the outcome values was of secondary interest. It was equal expectations, not equal probabilities, which in most cases guaranteed equitable terms, and it was equity which interested the jurists:

> And these sorts of agreements have their justice in that one prefers a certain and known portion, either of profit or of loss, to the uncertain expectation of events; and the other on the contrary finds it to his advantage to hope for a better condition. Thus there is a kind of equality in their portions, which renders their agreement just.[38]

Contracts were the backbone of the natural law school of jurisprudence of the late sixteenth and seventeenth centuries,[39] since they cemented con-

[37] R.P.E. Bauny, *Somme des pechez qui se commettent en tous les etats* (Lyon, 1646), p. 227.

[38] Jean Domat, *Les loix civiles dans leur ordre naturel* (1689–94), nouvelle édition . . . augmentée des Troisième et Quatrième Livres du Droit Public, par M. de Héricourt (Paris, 1777), p. 30.

[39] See Otto Gierke, *Natural Law and the Theory of Society 1500 to 1800*, trans. with an introduction by Ernest Barker (Cambridge, Eng.: Cambridge University Press, 1934), vol.

senting individuals together to form a society, and even transcended the social bonds: "For the words of agreement between contracting parties are even stronger than those, on which society is founded." Grotius's influential *De jure belli ac pacis* (1625) stipulated that "In all contracts, natural justice requires there should be an equality of terms,"[40] and examined the degrees of equity pertaining to various sorts of contracts, from sales of goods to international treaties, in great detail. Jean Domat, the prominent seventeenth-century French jurist and friend of Pascal, also maintained that "The use of contracts [*conventions*] is a natural consequence of the order of civil society, and of the bonds which God creates among men."[41] Domat summarized the general rules of equity as rendering to all their just expectations, honoring promises and obligations, and taking care to "do hurt to no man." Charles Du Moulin claimed that even the etymology of the word "contract" denoted "mutual attraction and reciprocity."[42]

Jurists considered contracts that worked even implicitly to the disadvantage of one of the parties to be just cause for legal action, and aleatory contracts, including games of chance, were no exception. In his discussion of contracts involving chance, Samuel Pufendorf contended that players' risk of winning or losing must be in "just proportion" to the stake, and that all must share "equally the risk of winning or losing."[43] Domat also made equality of condition the essential guarantee of equality in aleatory agreements. For example, a partnership of as yet childless men might legally arrange to provide their daughters' marriage portions from joint stock, even though only some of the partners might ultimately be able to take advantage of the provision: "The state in which all of them share, with the same uncertainty of the event and with the same right, having rendered their condition equal, also makes their agreement just."[44] Domat's guarantee of equality stemmed from the shared (and therefore equal) subjective

1, pp. 76–78; Leonard Krieger, *The Politics of Discretion: Pufendorf and the Acceptance of Natural Law* (Chicago and London: University of Chicago Press, 1965), pp. 99–118.

[40] Hugo Grotius, *The Rights of War and Peace* (1625), trans. A. C. Campbell (Washington and London: M. Walter Dunne, 1901), p. 147.

[41] Domat, *Loix civiles*, p. 19.

[42] Charles Du Moulin, *Summaire du livre analytique des contractz usures, rentes constituées, interestz & monnoyes* (Paris, 1554), f. 15v.

[43] Samuel Pufendorf, *Le droit de la nature et des gens* (1682), trans. Jean Barbeyrac (London, 1740), vol. 2, pp. 503–504.

[44] Domat, *Loix civiles*, p. 99.

uncertainty of the partners, as well as their equal claims to the dowries. Scholastic theologians even mounted a moral defense of gambling in cases where there was "equality of uncertainty, peril, or chance."[45]

This strict legal insistence upon absolute equality among the parties to risk found its way into the early literature of mathematical probability in a sometimes exaggerated form. Cardano, for example, makes equality of condition—equality of opponents in rank and skill, of bystanders, of money, and of situation, as well as of the dice—his "Principale fundamentum in Alea," and warns that those who deviate from this cardinal rule are "unjust."[46] The Port Royal *Logique* is more narrowly concerned with equality of the chance setup rather than the social status of the players, but even here it is an idea of legal rather than mathematical equality that demands that the situation of all the players be absolutely identical. One could easily invent situations in which the condition of the players was equalized by a balancing of odds and stakes, and we know from Cardano that dice games of the sort were well known.[47] But the jurists held to a more rigid standard as a further guarantee of equity.[48]

The doctrine of aleatory contracts thus furnished the late seventeenth-century probabilists with a set of concepts and problems. Jurists seeking the fair price for an annuity, a lottery ticket, or partnership share thought in terms of expectation, rather than risk per se, and the first mathematical probabilists did as well. This is why Pascal described the results of the new mathematics of chance as rendering to each of the players what was due to him *en justice*. Expectation had the advantage of being already quantified in legal practice, for contracts specified the purchase price of an uncertain gain. If the means for reckoning that price in any given case were nebulous, the price itself was exact.

These influences are palpable in the work of Christiaan Huygens, author of the first published work in mathematical probability, and of Johann De Witt, who applied Huygens's precepts to the problem of pricing annuities. Although Pascal and Fermat invented the calculus of probabilities in their 1654 correspondence, their letters remained unpublished until 1679. Christiaan Huygens, visiting Paris in 1655, heard about the problems addressed in this exchange from Giles Roberval and Claude Mylon,

[45] Du Moulin, *Summaire*, f. 186v.

[46] Hieronymus Cardanus, *Liber de ludo aleae*, in *Opera Omnia* (Lyons, 1663), facsimile reprint (Stuttgart-Bad Cannstatt: Friedrich Fromann Verlag, 1966), vol. 1, p. 263.

[47] Cardanus, *Ludo*, chapter 14.

[48] Pufendorf, *Droit*, p. 504.

a friend of Carcavy, who was the intermediary between Pascal and Fermat. Although Huygens met neither of the principals, he worked out his own solutions to the Problem of Points, and after ascertaining that his answers tallied with those of the French mathematicians, composed a brief treatise, which he sent to his former teacher Frans van Schooten on 20 April 1656.[49] Huygen's *De ratiociniis in aleae ludo* was published as an appendix to van Schooten's *Exercitationum mathematicarum libri quinque* (1657), and was subsequently translated into Dutch, English, and French.[50] It was later reprinted, with commentary, as Book I of Jakob Bernoulli's *Ars conjectandi*.

Huygens's treatise set forth, strictly speaking, a calculus of expectations rather than of probabilities. Huygens posed problems on the fair division of stakes or the "reasonable" price for a player's place in an ongoing game, rather than questions about the probabilities of events themselves. Considered by itself, Huygens's fundamental principle—his definition of expectation—sounds suspiciously circular:

> I begin with the hypothesis that in a game the chance one has to win something has a value such that if one possessed this value, one could procure the same chance in an equitable game [*rechtmatigh spel*], that is in a game which works to no one's disadvantage.[51]

Since later probabilists *defined* an equal or fair game as one in which the players' expectations equaled the price of playing the game (i.e., the stake), Huygens's explanation of expectation in terms of a fair game seems to lead nowhere. However, Huygens here assumed that the notion of an equal game was a self-evident one for his readers. The alternative definition, which gained currency in the eighteenth century, derived expectation and the criterion for a fair game from the definition of probability, expressed as the ratio of the number of combinations favorable to the event to the total number of combinations. This route remained closed to Huygens. Instead, he appealed to an intuitive, or at least nonmathematical, notion of equity: in this case, the equitable exchange of expectations and the conditions of a fair game.

[49] See Henri Brugmans, *Le séjour de Christian Huygens à Paris et ses relations avec les milieux scientifiques français* (Paris: Librairie E. Droz, 1935), p. 40; also "Avertissement," in *Oeuvres complètes de Christian Huygens*, Société Hollandaise des Sciences (The Hague: Martinus Nijhoff, 1920), vol. 14, pp. 1–30.

[50] "Avertissement," in *Oeuvres*, pp. 4–5.

[51] Huygens, *De ratiociniis in ludo aleae*, in *Oeuvres*, vol. 14, p. 60.

Huygens's formulation of expectation was drawn from contemporary doctrines of contract law. Huygens could presume the self-evidence of a fair game or exchange because these were the staples of seventeenth-century legal theory and practice. These legal discussions (recall especially Pufendorf's stipulation that players must all have an equal chance of winning or losing), along with Huygens's definition of equal expectation, may have presumed equiprobability by requiring "equal conditions" among players, but they did so only tacitly. Seventeenth-century jurists did assess trade-offs between various risks and stakes, quantitatively if unmathematically, in the cases of fluctuating insurance premiums, partnerships formed with mixed contributions of capital, labor, and risk bearing, and other probabilistic situations. A well-honed sense of an equitable contract, even one hinging on uncertain outcomes, could be assumed, as could the legal conception of expectation. Hence, Huygens's definition of equal expectations in terms of fair exchange or game, one which worked to the "disadvantage" of no one, would not have struck a contemporary reader as tautologous. The conditions of equity and the legal paradigm of a just contract had been firmly and independently established in legal usage and daily practice. Later probabilists such as Nicholas Bernoulli reversed this order by defining equity in terms of equal expectations,[52] but throughout the eighteenth century probabilists returned to the model of an equitable exchange.

Huygens's propositions and examples made frequent use of this legal device of a fair exchange. In order to prove that the expectation of each of two players in an equal game that awards a sum a to the winner and b to the loser is $(a + b)/2$ Huygens argued in what again appears to be a closed circle. Both players stake an amount x, and agree to offer the loser a consolation prize of a, so that the possible outcome values will be a or $2x - a$. Because "this game is equitable, and thus I have an equal chance" at either outcome,[53] Huygens defines b as equal to $2x - a$, and concludes that he could bet $(a + b)/2$ with another player and make the same arrangement for a consolation prize a. Later probabilists would summarize this argument by asserting that the equation between the stake x and the expectation $(a + b)/2$ guarantees a fair game. Huygens, however, assumed $x = (a + b)/2$ (by setting $b = 2x - a$) *because* the game is, by hypothesis,

[52] Nicholas Bernoulli, *De usu artis conjectandi in iure* (1709), chapter 4, in *Die Werke von Jakob Bernoulli*, Basel Naturforschende Gesellschaft (Basel: Birkhäuser, 1975), vol. 3.

[53] Huygens, *Ludo*, p. 62.

a fair one. He insured that the game would be fair a priori by constructing completely symmetric conditions for all players, and by arranging a series of deals, each certified as self-evidently equitable, among the players to convert one mathematical expression of expectation into another. Returning to his initial hypothesis, Huygens asserted that expectations were equal when they could be fairly traded for one another.

This method of circumventing any explicit statement of equiprobability became more involved as the outcomes (or "chances") proliferated, for Huygens had to posit as many players, bound in as many subcontracts, as there were possible outcomes. Once again, the modern order of reasoning regarding expectations was inverted: instead of the game being fair because the probabilities (and therefore the expectations in a symmetric game) are equal for all players, the probabilities are (implicitly) equal because the game is assumed fair—and the game is fair because the conditions of the players are indistinguishable, as shown by their willingness to exchange expectations in a series of "equitable" subcontracts. For Huygens, expectation represented a mathematical version of equity.

Expectation later came to be defined as a composite notion, the product of the more fundamental components of probability and outcome value:

> In all cases, the Expectation of obtaining any Sum is estimated by multiplying the value of the Sum expected by the Fraction which represents the Probability of obtaining it.[54]

For Huygens and the first generation of probabilists, however, expectation was the irreducible concept from which probability could be in theory derived if the outcome value were known. I have suggested that this order of precedence made sense in the context of legal theories that estimated expectations rather than risks, and that aimed at equalizing these expectations in partnerships and other contracts. Except in extremely simple cases, such as coin tossing and dice throwing, combinatorial arguments were not feasible, and the data required for statistical evaluations were generally unavailable. Although mathematicians like Leibniz, Huygens, and Jakob Bernoulli were quick to perceive the relevance of Graunt's political arithmetic to probability theory, the two disciplines emerged independently of one another. The first attempt to apply probability to annuities made no direct use of statistics and adopted Huygens's methods of expectations.

[54] De Moivre, *Doctrine*, p. 3.

De Witt's *Waerdye van Lyf-Renten* (*Treatise on Life Annuities*), originally written as a series of letters to the Estates-General of the United Provinces of Holland and West Friesland in 1671, was one of the earliest attempts to extend the new mathematics of probability to other sorts of aleatory contracts besides games of chance. Although the sale of annuities dated back to Roman times, there is little evidence that rates were computed on the basis of mortality statistics. That annuities, as well as compound interest, were fixtures of finance and trade by the late sixteenth century can be seen from the tables on annuities and compound interest for various rates and periods regularly appended to late sixteenth- and seventeenth-century treatises of practical arithmetic.[55] De Witt's originality lay in his attempt to estimate the probability of death as a correlate of age, and in his extension of Huygens's calculus of expectations to a new class of problems. De Witt skirted the principal obstacle to such generalizations, *viz.*, the need to deal with what are apparently nonequiprobable outcomes such as age at death, by simply assuming equiprobability for the risk of dying between the ages of three and fifty-three, and assigning proportional probabilities for earlier and later ages on the basis of educated guesswork.

Although De Witt welcomed Johannes Hudde's empirical confirmation of his guesswork with data culled from the records of past holders of Dutch annuities,[56] the initial lack of mortality statistics did not undermine his confidence in his original conclusions, which were no more grounded in statistics than the rules of thumb of the Roman jurists had been. This insouciance is more easily understood within the context of the established practice of gauging the value of the expectation of an insurance policy, an annuity, or other aleatory contract "by eye." De Witt had, after all, been trained in the law. Huygens's mathematics provided him with a more precise method of treating concepts already certified by long use. Although the even greater quantitative precision to be achieved through statistics would have been—and was—immediately appreciated, it was not consid-

[55] See, for example, William Purser, *Compound Interest and Annuities* (London, 1634); John Kersey, ed., *Mr. Wingate's Arithmetick*, 5th edition (London, 1670).

[56] See Société Générale Néerlandaise d'Assurances sur la Vie et de Rentes Viagères, *Mémoires pour servir à l'histoire des assurances sur la vie et des rentes viagères au Pays-Bas* (Amsterdam, 1898), pp. 24–33, for a French translation of the correspondence between Hudde and De Witt on this subject. The original correspondence is preserved in the National Archives in Amsterdam; the AMEV Library in Utrecht holds Hudde's manuscript reckoning sheets, *Stads-finatie geredresfeert in den jaare 1679 . . . Balansenenz: Betreffende de lofen lijfrenten*.

ered a prerequisite for an accurate analysis of the relative financial advantages of redeemable and life annuities.

De Witt's brief treatise is therefore instructive as an early mathematical codification of concepts and practices previously implemented by rules of thumb and seasoned judgment. As in Huygens's treatise, from which De Witt borrowed liberally, games of chance furnished many of the illustrations. However, De Witt hoped to branch out to other sorts of aleatory contracts. De Witt's vocabulary is even more legalistic than Huygens's, rephrasing Huygens's hypothesis explicitly in terms of equitable contracts:

> I presuppose that the real value of certain expectations or chances of objects, of different value, must be estimated by that which we can obtain from equal expectations or chances, dependent on one or several equal contracts.[57]

As in Huygens's definition of equal expectations, only the presumption of an independent notion of "equal contract" rescued De Witt's definition from tautology. Although De Witt's examples of such equal contracts are all fair games with equiprobable outcomes, he did not single out either "equiprobability" or "probability" as distinct concepts requiring definition: these notions were subsumed within the definition of an equal contract, one which balanced the advantages and disadvantages of all parties as precisely as possible. De Witt's demonstration of the proposition (corresponding to Huygens's Proposition III) that the value of several expectations or "chances" is to be computed by summing the value represented by the chances, and by then dividing this sum by the number of chances,[58] relied on an exchange of equal contracts among partners in completely symmetric situations. The symmetry both insured the legality of the contract—in the words of Pufendorf, all run "equal risks"—and obviated the need for explicit discussion of probabilities per se. De Witt could use "expectation" and "chance" as synonyms because the number of outcomes in each example was designed to equal the number of partners, which in turn

[57] De Witt's rare treatise is reprinted in Jakob Bernoulli, *Werke*, vol. 3, pp. 327–350. It was already hard to come by in Bernoulli's time, and he importuned Leibniz in vain for a copy. An English translation by F. Hendriks is reprinted in Robert G. Barnwell, *A Sketch of the Life and Times of John De Witt* (New York, 1856). All quoted passages are taken from this translation; see De Witt, *Waerdye van Lyf-Renten* (1671), in *Werke*, vol. 3, p. 329; Barnwell, *Sketch*, pp. 82–83.

[58] De Witt, *Waerdye*, in *Werke*, vol. 3, pp. 331–332; Barnwell, *Sketch*, p. 86.

equaled the number of chances. Of course, this condition held only if the chances were assumed to be equiprobable, and the chances were equiprobable according to De Witt, because the contracts were fair. By inverting the last claim, as later probabilists would, the proof collapses into circularity. Without equal contracts and the concomitant notion of the symmetric status of the partners, there would be no grounds for asserting equal expectations or (implied) equal probabilities.[59]

Twentieth-century critics of classical probability theory have commented at length on the circular assumption of equiprobable outcomes built into the classical definition of probability, and the Principle of Indifference invoked to defend that assumption. Laplace's definition rests on both:

> The theory of chance consists in reducing all the events of the same kind to a certain number of cases equally possible, that is to say, to such as we may be equally undecided about in regard to their existence.[60]

Ernest Nagel objects that "if 'equipossible' is synonymous with 'equiprobable,' " then the classical definition in terms of a ratio of favorable to total number of equipossible alternatives "is circular, unless 'equiprobable' can be defined independently of 'probable.' "[61] The Principle of Indifference cannot be used to salvage the classical definition, because it does not yield unique probability values.[62]

I have argued that the first formulations of mathematical probability were heavily indebted to seventeenth-century legal notions of contract. These granted late seventeenth-century probabilists a reprieve from the difficult task of justifying the useful assumption of equiprobable outcomes. Equal expectations, rather than equiprobable outcomes, were the departure point for the earliest mathematical treatments. By defining

[59] For further early examples of the various sorts of aleatory contracts treated mathematically, see Jakob Bernoulli, *Meditationes*, nos. 159, 162, 169, in *Werke*, vol. 3, pp. 42–48, 66, 71; and Nicholas Bernoulli, *De usu*, also in *Werke*, vol. 3, pp. 287–326.

[60] Pierre Simon de Laplace, *Essai philosophique sur les probabilités* (1814), in *Oeuvres complètes*, Académie des Sciences (Paris, 1878–1912), vol. 7, p. viii; all cited passages are taken from the English translation of the 6th edition by Frederick Wilson Truscott and Frederick Lincoln Emory, *A Philosophical Essay on Probabilities* (New York: Dover, 1951), p. 6.

[61] Nagel, *Principles*, p. 388.

[62] See John Maynard Keynes, *A Treatise on Probability* (London: Macmillan, 1943), chapter 4.

equal expectations in terms of equitable contracts, which legal usage permitted probabilists to take as self-evident, these expositions could avoid transparent circularity, although the definitions and demonstrations always threatened to close in upon themselves. In games of chance, the physical symmetry of the die or coin lent credence to the legal contention that players enjoyed equal prospects and therefore were not in violation of equity. Hence the somewhat vague allusions to the "equal ease" or "facility" with which certain events could occur: as Huygens phrased it in the original Dutch version of his treatise, "each chance can come about equally easily."[63]

Historians and philosophers of probability theory have viewed the equiprobability assumption from the perspective of the current dichotomy between subjective, epistemological versus objective, frequentist interpretations of probability. From the epistemological standpoint, the a priori assumption of equiprobable outcomes rests on the Principle of Indifference, as in Laplace's formulation. The objectivists argue that the physical symmetry of, for example, a fair coin, validates the assumption that in the long run both sides will turn up with equal frequency.[64] When this opposition is superimposed on the discussions of the classical probabilists, they appear to vacillate, sometimes siding with the subjectivists and sometimes with the objectivists. Hacking has suggested that early probabilists were able to tolerate such ambiguity by taking refuge in the parallel ambiguities in the usage of "possibility," alternately favoring its epistemological and objectivist nuances: "By explaining probability in terms of possibility writers of an earlier period could usefully equivocate."[65] Equity played a similar role in the first expositions of mathematical probability via the fundamental hypothesis concerning equal expectations. As long as probabilists could take equity as an irreducible, undefined concept, the vexing issue of equiprobability could be dodged.

However, in the more complicated situations that involved unequal risks, the problems of ascertaining probabilities were all but insoluble without more extensive statistical information. Roman-canon law offered some rules of thumb, like those of Ulpian, but in general jurists left such uncertain matters to the discretion of an experienced judge, to be arbi-

[63] Huygens, *Ludo*, p. 65. See also Cardanus, *Ludo*, p. 64, for a similar expression.

[64] Hacking, "Jacques Bernoulli's *Art of Conjecturing*," *British Journal for the Philosophy of Science* 22 (1971): 209–229, on p. 210.

[65] Hacking, "Equipossibility theories of probability," *British Journal for the Philosophy of Science* 22 (1971): 339–355, on p. 341.

trated on a case-by-case basis.[66] Like the business of setting insurance premiums, the preferred method brought wide experience and discretion to bear on each individual case, considered on its particular merits.[67] The statistical approach of John Graunt heralded a new way of thinking, but assumptions of equiprobability lingered. De Witt was obliged to assume equiprobable chances of dying in any six-month period between childhood and old age; Graunt also presumed that the same proportion (about 3/8) of the population died every ten years.[68] Until Jakob Bernoulli's limit theorem legitimated the practice of equating statistical frequencies and probabilities in at least some cases, and fact-gathering projects like Edmund Halley's Breslau table of mortality[69] furnished those frequencies, an independent notion of probability applied to anything other than games of chance would have been superfluous. Even in games of chance, the enumeration of the combinations of equiprobable outcomes quickly became unmanageable, as Pascal had complained to Fermat. The problem was further complicated by the mixture of elements of chance and skill in the games analyzed by the early probabilists.[70]

Perhaps this is why Thomas Bayes, whose method of finding a posteriori probabilities seemed to free probabilists from considering only a priori equiprobable cases, chose to return to the expectation-centered approach of Huygens to probability, although Abraham De Moivre's direct estimation of probability as "a Fraction whereof the Numerator be the number of Chances whereby an Event may happen, and the Denominator the number of all Chances whereby it may happen or fail"[71] would have presumably been known to him. Bayes began his posthumous (1763) essay with an

[66] S. P. Scott, trans., *The Civil Law, including the Twelve Tables, the Institutes of Gaius, the Rules of Ulpian, the Opinions of Paulus, the Enactments of Justinian, and the Constitution of Leo* (Cincinnati: Central Trust, 1932). See also Du Moulin, *Summaire*, ff. 187r.-v., for rough methods of estimation. To judge from manuals on annuities, the important temporal variable was interest rather than age; see the tables in William Purser, *Compound Interest.*

[67] See Section 3.4.2.

[68] John Graunt, *Natural and Political Observations Mentioned in a Following Index and Made Upon the Bills of Mortality* (London, 1662).

[69] Edmund Halley, "An estimate on the degrees of mortality of mankind, drawn from curious tables of the birth and funerals at the city of Breslaw; with an attempt to ascertain the price of annuities upon lives," *Philosophical Transactions of the Royal Society of London* 17 (1693): 596–610.

[70] See, for example, Jakob Bernoulli, *Meditationes*, no. 160, in *Werke*, vol. 3, pp. 48–64.

[71] De Moivre, *Doctrine*, p. i.

exposition of the basic principles of the calculus of probabilities, because according to his literary executor and editor Richard Price, Bayes knew of no source which gave a "clear demonstration of them."[72] Price gave no indication of when Bayes composed the memoir, and it is barely possible that at that time Bayes knew neither Montmort's *Essai d'analyse sur les jeux de hazard* (1708, 1713), Jakob Bernoulli's *Ars conjectandi* (1713), nor any of the three editions or supplements of De Moivre's *Doctrine of Chances* (1718, 1730, 1756), although the very title of his essay suggests the contrary. It seems more likely that he found these treatments in some way unsatisfactory.

In any case, Bayes' own exposition of the "general laws of chance" fell squarely within the original expectation-based treatments of probability. Like Huygens, Bayes built his proofs around the reasonable trade of expectations. For example, if the payoff depends on both events A and B happening, Bayes argues that news that B had occurred did not alter the initial expectation:

> For if I have reason to think it less, it would be reasonable for me to give something to be re-instated in my former circumstances, and this over and over again as often as I should be informed that the second event had happened, which is evidently absurd. And the like absurdity plainly follows if you say I ought to set a greater value on my expectation than before, for then it would be reasonable for me to refuse something if offered me upon condition I would relinquish it, and be re-instated in my former circumstances.[73]

If it is "unreasonable" to either buy or sell at a higher price, Bayes concludes that the expectations must therefore be equal.

By 1763 (the third edition of De Moivre's *Doctrine of Chances* had appeared in 1756), Bayes' definition of probability as "the ratio between the value at which an expectation depending on the happening ought to be computed, and the value of the thing expected on its happening" might well have struck knowledgeable readers as outdated. Price felt constrained to explain that his friend had chosen to overlook "the proper sense of the word *probability*" because whatever confusion surrounded the meanings of

[72] Thomas Bayes, "An essay towards solving a problem in the doctrine of chances," *Philosophical Transactions of the Royal Society of London* 53 (1763): 370–418, on p. 375.

[73] Bayes, "Essay," p. 380.

(continued...)

INDEX

Académie des Sciences, Paris, 84–85, 96, 98, 99, 106, 145, 172n, 277, 347, 364–365, 375
Académie des Sciences Politiques et Morales, 366, 378, 383
actuaries, 116, 168–169, 180
Albertus Magnus, 153
aleatory contracts. *See* contracts
d'Alembert, Jean, xxi, xxii, 17, 206, 368; and Beccaria, 344–345; and Condorcet, 78, 96–97, 211–212, 278, 283; criticisms of probabilistic arguments for order, 252–253; exception among classical probabilists, 106, 111; and Laplace, 78, 103–104, 105, 191, 275, 283; on mixed mathematics, 53–55; on risk taking, 81, 88–89; on St. Petersburg problem, 76–81, 88, 95; on smallpox inoculation, 83–89
Allais, M., 109
Allen, Sir Thomas, 170
American School, of economics, 109
Amicable Society, 164, 167, 170–171, 174, 176, 177
analogy, 193–194, 204, 205–206, 208, 220, 229, 244–245, 254, 307, 381, 383
annuities, xix, 14, 19, 23, 50, 112, 166, 167, 173, 174, 176, 178, 348; history of, 116, 121–122, 138–139, 142; mathematically based, 27–30, 55, 69–70, 104, 136, 168–169, 172n; obstacles to application of statistics to, 122, 124–125, 133, 169–170, 172–174; prices, 121–122, 124–125
Appleby, Joyce, 51
applications of probability theory, xx–xxiv, 13, 33, 48, 49, 229–230, 238, 369, 372
applied mathematics, 3–6, 53–54, 354–355, 364
Aquinas, Saint Thomas, 37–38, 151, 153
Arago, François, 380n
Arbuthnot, John, 127, 131, 138, 248, 252, 276, 286
Archimedes, 236, 239
argument from design, 187, 248, 251–253, 267. *See also* natural theology
Aristotle, 5, 8, 38, 51, 61, 151, 153, 154
Arnauld, Antoine, 17, 59
Arrow, Kenneth, 56
assemblies, 215, 343
associationism, xxi, xxiii, 47, 191, 196–225, 228, 255, 261, 290–291, 368, 370–371, 377, 380
astronomy, 35, 192, 216, 228, 271, 364, 379
atomism, 246–249
Augustine, Saint, 151, 153

Babbage, Charles, 181
Bacon, Francis, 51, 65, 127, 228, 241–242, 307
Baker, Keith, xvi, 96, 104, 283
Barbeyrac, Jean, 159
Barrow, Isaac, 59
Bayes, Thomas: assumption of equal prior probabilities, 223, 258–262, 266; expectation approach to probability theory, 31–33, 258, 261–262; and Hartley, 204, 255–256; on inverse probabilities, 208n, 221n, 230, 233,

Bayes, Thomas (*cont.*) 257–262; and Price, 256–257, 258, 262–264
Bayes' theorem, Bayes' version, 257–264; Laplace's version, 268–269
Bayle, Pierre, 159, 304, 309–311, 321, 323
Beaumarchais, Eustache, 148
Beaumont, Charles d'Eon de, 165
Beccaria, Cesare, 304–305, 343–345, 352, 353
Benoiston de Chateauneuf, Louis-François, 360n, 384
Bentham, Jeremy, 297n, 355
Bentley, Richard, 252
Bérenger, Alphonse, 383–384
Bernoulli, Daniel, xxii, 100, 147, 253n
on expectation, 70–77, 93–95, 104, 105, 274; on smallpox inoculation, 53, 83–89, 96
Bernoulli, Jakob, xix, xxi, xxvi, 13, 55, 64, 68, 125, 126, 128, 137, 138; *Ars conjectandi*, 7, 10, 24, 32, 48, 98, 189, 236–237, 296, 316, 318; correspondence with Leibniz, 28n, 129–130, 232–233, 237–240, 286; on degrees of certainty, 5, 33, 34–35, 38; on determinism, 35, 37, 281, 284; on estimation of evidence, 40–44, 306, 311; on frequencies and statistics, 26, 28n, 301; model of causation, 238–241, 246, 249–250, 262, 268, 285; on objective/subjective probabilities, 188, 189–190; and probability of causes, 227–253, 294; rules for conjecture, 49–50
Bernoulli, Nicholas, xxvi, 7, 13, 172, 312, 347; criticism of probabilistic arguments for order, 252–253; on expectation, 25, 48, 69–70, 76; on insurance, 136–137; on life expectancy, 87, 128, 135, 136; on probability of testimony, 46, 193, 306, 311–312, 319, 334
Bernoulli's theorem: formulation of, 231–257; and law of large numbers, 268, 285
Bertrand, Joseph, 375–376, 379, 380n
Bible, 92, 123, 131, 133, 154, 306, 308–309, 313–316
Bicquelley, Charles-François, 339
Bienaymé, I. J., 288n, 379
Binomial expansion, 235, 256, 264, 285, 350
Biot, J. B., 291, 293
Boethius, Anicius Manlius, 151–152, 153
Boltzmann, Ludwig, 246
Boole, George, 223, 373–376
Borda, Jean-Charles, 145
Bossuet, Jacques Bénigne, 321, 323
Bossut, Charles, 278, 352
bottomry, 20, 116–117. *See also* insurance, maritime
Bourchier, Dick, 149
Boyle, Robert: corpuscularism, 241–242, 246–249; on empiricist modesty, 226–227; on evaluation of testimony, 63–307; on kinds of certainty, 38, 56–57, 63–64; and natural theology, 63–65, 205, 267; on reasonableness, 59–66; and skepticism, xix, 283
Boyle lectures, 59, 131
Brissot de Warville, J. P., 167, 179n, 343–344
British Museum, 143
Buffon, George Leclerc, 172n, 344; coin-tossing experiment, 95n, 287; on credibility of testimony, 330–331; on expectation, 76, 90–95, 147; on induction, 206–208; on life expectancy and mortality, 86, 184, 301, 302; on measurement of risk, 91–92, 185, 348
Burnet, Thomas, 310, 313
Butler, Joseph, 62, 204–205, 267

Caillois, Roger, 148
Calas, Jean, 304, 343, 349, 353
calculation, 152, 157–159, 161–163,

173, 187, 215–218, 221, 369, 376, 385–386. *See also* mental calculations; self-interest
Calzabigi, Florentin, 143
Cambridge Platonists, 248
Caminade de Castres, Marc-Alexandre, 145n
Canning, Elizabeth, 343
Canton, John, 256
Carcavy, P., 24
Cardano, Gerolamo, 13, 15, 23, 36–37, 51, 124, 158, 231, 234
Carnap, Rudolf, 189
Cartesianism, xx, 156, 300
Casanova, Giovanni-Jacopo de Seingalt, 143
causes: complexity of, 79–80, 275, 281; hidden, 208, 226, 237, 240–245, 253, 379; in Hume, 199–201, 228, 265–266, 290–291; manifest, 207–208, 226, 241; perturbing, 263, 270–272; reasoning from effects to, 226, 232, 240, 268; regular and irregular, 270–272, 280–281, 286–287, 290, 381, 383; urn model of, 230–231, 237–249, 268–270, 279–280, 294, 375
certainty: degrees of, 14, 33–47, 57, 58–59, 126, 138, 192; moral, 38, 39, 57, 60, 63–64, 91, 205, 206, 207–208, 213, 227, 235, 315, 318, 322, 334n, 344, 348
chance: as absence of cause, 10, 25, 36, 38, 79, 199, 288; and urn model of causation, 237, 245–246. *See also* fortune, randomness
charity, 175, 177, 179
Charleton, Walter, 246–249
Chillingworth, William, 60
Cicero, Marcus Tullius, 38, 153
Civiale, Jean, 367
Clark, Samuel, 136n, 156
classical probability theory, xix–xxvi, 3–6, 52, 67, 116; decline of, 210–211, 223–225, 284, 370–386; description and prescription in, 52–53, 88, 102, 107–108, 198–199, 211–212, 299; good sense interpretation of, xix, xxii, xxvi, 50, 67, 108, 198, 211, 218, 352, 357–359, 360; objective/subjective meanings of probability in, 189–191, 261–262, 370–371, 377; urn model in, 229–231, 238, 379
Clausius, Rudolf, 246
Clavière, Étienne, 166n
Cleirac, Estienne, 50, 118–120
cockfighting, 124, 157, 160, 163
combinatorics, 9, 15–17, 31, 34, 47, 235, 240–241, 245–246
common sense. *See* good sense
Compagnie Royale d'Assurance, 178–179
Comte, Auguste, 298, 354, 366, 375, 377–378, 380, 382
Condillac, Étienne, 93, 110, 191, 198, 208–212, 214, 221, 302
Condorcet, M.J.A.N., 78, 90, 145n, 172n, 198, 206, 318n; and Beccaria, 344–345; educational projects, 98, 216–218; on expectation, 76, 96–103, 349; and Laplace, 104, 268; on meaning of probability, 191, 211–219, 224, 281–282; on measurement, 92–93, 97–98, 216–217; on natural rights, 303, 348–349; praise of printing, 322; on probability of causes, 208, 223, 230, 233, 267, 278–283, 285; on probability of judgments, 107, 215, 296–297, 299, 342–343, 345–352, 354, 356–357, 361–363, 375; on probability of testimony, 323, 324, 329–335, 339–340; on social mathematics, 67, 104, 211, 217–218, 304, 355, 377, 378; and statistics, 302
consensus, 66–67, 108, 193, 197, 217–218, 352, 359
contracts, xxii, 7, 14–15, 21–22, 102, 347; aleatory, 14–15, 18–33, 47–48 69–70, 112, 115, 117, 121, 125, 137–138, 140, 155, 166–167, 169, 172–

contracts (*cont.*)
174, 178; equity in, 19–23, 25–33, 66, 69, 71, 94, 99–100, 136
corpuscles, 242, 244, 246–247, 249. *See also* atomism
Coulomb, Charles-Auguste, 352
Coumet, Ernest, 7, 18
Council of Trent, 126
Counter-Reformation, xix, 308, 320
Cournot, A. A., 7, 189, 190–192, 223–224, 233–234, 284, 359, 370–371, 375
Cousin, Victor, 371, 378
Cox, James, 144
Craig, John, 46, 64, 92, 312–316, 320, 323, 333, 342
Cramer, Gabriel, 95
credibility. *See* testimony

Darwin, Erasmus
decision making, xix, 44, 48, 60, 208, 213, 224, 227
Defoe, Daniel, 140, 147, 164, 165, 166, 170–171
De Moivre, Abraham, xxvi, 268; on annuities and life expectancy, 135, 136n, 137–138, 169; on argument from design, 187, 205–206, 248, 253–257, 267, 288; on Bernoulli's theorem, 198, 204, 228–229, 250, 253–254, 257, 263, 267, 286; on definition of probability, 31–32; on inverse of Bernoulli's theorem, 233, 251–252, 256–257, 262; on mortality statistics, 126; on nonexistence of chance, 10–11, 37, 266
De Morgan, Augustus, 181, 370, 374, 374n
Deparcieux, Antoine, 138, 172n
De Prades, Jean-Martin, 330–331, 337
Derham, William, 131–132, 252
Descartes, René, 49, 50, 93, 226, 241, 243
Destutt de Tracy, Antoine Louis Claude, 354–355, 366, 375
determinism: linked with classical probability theory, 10, 34–37, 51; and moral sciences, 300, 380; opposed to chance, 153, 154–156, 159; and probability of causes, 241, 245, 279; social context of, 113; and statistical regularities, 133, 183
De Witt, Johann, 7, 13, 17, 23, 27–29, 31, 45, 125, 128, 129, 137, 139
Diderot, Denis, 73, 318, 321–322, 344
distributions, 108, 183, 258, 363, 379. *See also* normal curve
Dodson, James, 114, 169, 176, 179
Domat, Jean, 22, 306, 344
doubt, xx, 43, 57, 59, 194, 201, 241
Dover/Calais packetboat (as standard of acceptable risk), 92–93, 348
Du Moulin, Charles, 19, 22, 50
Dupin, Charles, 106, 365–367, 378n
Dupont de Nemours, Pierre, 157
Durkheim, Émile, 298
Duvillard, Étienne, 179n

economics, 109. *See also* political economy
Elizabeth I (queen of England), 141, 143, 322
Ellenborough, Lord, 176n
Ellis, Robert Leslie, 191, 223, 284
empiricism, 226–229, 241–245, 373
Encyclopédie, 54, 78, 97, 318–321, 323, 330, 337, 343
Enlightenment, xxi–xxv, 51, 55, 109–110, 150, 153, 156, 193, 199, 298–299, 302, 321
equiprobability, 23, 29–33, 77, 79, 80, 124, 125, 137, 199, 260, 270, 273, 306, 331. *See also* Principle of Indifference
Equitable Society (est. 1712), 167
Equitable Society for the Assurance of Lives (est. 1762), 169, 174–182
equity, 19, 21–23, 25–26, 28, 53, 57, 67, 69, 76, 100, 110, 133. *See also* contracts

INDEX

error law, 271–272, 383
Erskine, Thomas, 166–167
Euler, Leonhard, 144
evidence, xix, 61; historical, 320–323, 369; and induction, 193–196, 228; legal, xxiii, 43–44, 45–47, 48, 192–196, 228, 306, 356, 369; to persuade reasonable man, 62–63, 202, 300; of signs, 11–12
expectation, xxii, xxiv, 6, 47, 48, 53, 57, 133, 140, 174; applied to probability of testimony, 315; average, 96, 98–99; and contracts, 15–30, 117; mathematical, 57, 58, 70 (definition), 76–78, 81, 104; moral, 57, 58, 70, 71 (definition)–77, 94–95, 102, 104–105, 110; origins of concept, 14–33, 138; and probabilites, 16–18, 24–34, 69, 135; and reasonableness, 58–110, 205, 296

Fermat, Pierre, 3, 5, 7, 9, 13, 15–18, 23, 31
Fielding, Henry, 156
Finetti, Bruno de, 189
foenus nauticum. See bottomry
Fontenelle, Bernard de, 316–317
Fortuna, 151–152
fortune, 36–37, 143, 147, 148, 149, 150, 151–153, 158–163, 186
Francis I (king of France), 126, 141
Frederick II (king of Prussia), 143, 144, 346, 347, 349, 352
free will, 300–304, 317, 342
French Revolution, xxii, 52, 58, 106–107, 147, 211, 217, 298, 342–343, 352–354, 356, 366
frequency. *See* statistics
Fréret, Nicolas, 333–334
friendly societies, 170, 170n

Galiani, Ferdinand, 71, 110
Galilei, Galileo, 5, 13, 37, 124
Galton, Francis, 295
gambling, 8, 13–14, 18, 19, 23, 34, 61, 102, 114, 115–116, 125, 139; and life insurance, 122–123, 135, 137, 140–141, 163–168, 171, 173–176, 182, 184–185; mathematical critique of, 75, 94–95, 104, 105, 147, 156; moral and religious critique of, 104, 117, 123, 146, 147–151, 154–156, 161–163, 173; practices, 124, 157–163; universality of, 123, 141–144, 160–161. *See also* lotteries
Gambling Act, 123, 175
Garber, Daniel, 12–13
Gassendi, Pierre, xix, 56, 59, 243, 246–247
Gataker, Thomas, 155
Gay, John, 203
Gentile, Benedetto, 143
geometry, xx, 3, 4, 54, 226
Glanvill, Joseph, 38, 56, 59, 63, 248, 253, 322
Goffman, Erving, 153, 162
Gonzaga, Louis (duke of Nevers), 155n
good sense: ambiguity in, 110–111, 297; classical probability theory as, xix, xxii, xxvi, 50, 67, 108, 198, 211, 218, 352, 357–359, 360; conflicts with classical probability theory, 52–53, 68, 81, 341, 355, 368–369, 370–371, 376, 377; defined by moral sciences, 106; probability theory altered to fit, 95, 99, 102–103. *See also* St. Petersburg problem
Goodwin, William, 203
Grattan-Guinness, Ivor, 368n
Graunt, John: on bills of mortality, 12, 26, 127–129; influence of, 131, 132, 133, 137, 237; knowledge of probability theory, 128; mortality table, 31, 128, 135, 136, 138
’s Gravesande, Willem, 65, 253n
Gregory IX (pope), 117
Grotius, Hugo, xix, 20, 22, 56, 60, 205
Guerry, André Michel, 287n, 359, 360n, 384

Hacking, Ian, xxiv, 7, 11–13, 30, 34, 188, 189–190, 234
Halley, Edmund, 13, 31, 51, 87, 126, 128, 129, 130, 131, 132, 136, 137, 139, 234, 317
Hartley, David, 191, 198, 203–206, 210, 212, 214, 224, 245, 254–256, 302, 370
Helvetius, Claude Adrien, 302, 344
Henry VIII (king of England), 126
Herodotus, 321
Herschel, John, 370, 374n
Hewit, Beau, 158
Hilbert, David, 3
Hirschman, Albert, 51
history, xxiv; probability of testimony applied to, 304, 313–316, 333–334; written sources in, 310–322
Hobbes, Thomas, 10, 74
Hogarth, William, 165
d'Holbach, P.H.D., 10, 302, 344
homme moyen, 298, 383, 384–385
hommes éclairés, 90, 211, 216, 298, 376, 384–385. *See also* reasonable man
Hooper, George, 315–316, 341
Houdini, Harry, 186
Hudde, Johannes, 27, 125, 145
L'Huillier, Simon, 339
human nature, predictability of, 51–52, 162–163, 302–303
Hume, David: on associationism, 191, 198–203, 210, 212, 214, 222, 224, 293; and Beccaria, 344; and Bishop Butler, 205n; on causation and induction, 228, 264–267, 280, 283, 325–326; on hope and fear under uncertainty, 185–186; on luxury, 73; on miracles, 154, 321, 323, 324–330, 334–335, 340–341
Hutchenson, Francis, 371n
Huygens, Christiaan, 5, 7, 9–10, 13, 17, 21, 23, 45, 97, 114, 126, 261; *De ratiociniis in aleae ludo*, 24–28, 30, 31–32, 34, 128, 134, 158; on life expectancy, 128, 133–135
Huygens, Ludwig, 128, 129, 133–135

idéologues, 354
induction: in Hartley, 204, 205–206; in Hume, 202, 264–267; limitations of, 243, 245; in Locke, 193–195; quantification of, xxiii, 228–229, 237, 262, 267–293, 294–295, 377; reliability of, 254–256
insurance, 8, 19–21, 112, 139–141; fire, 114, 164, 166, 168, 172, 174; life, 114–116, 137, 163–182, 184–185 (*see also* gambling); maritime, 50, 55, 116–120, 136–137, 165, 168, 174; mathematically based, 136–137, 164, 167, 170–171, 175–182; and middle-class values, 174–178, 184–185, 187; obstacles to application of statistics in, 124–125, 133, 172–174, 192; policies, 118–120, 171, 181; premiums, 117–120, 124, 136–137, 171, 177–182, 185
interest rates, 20, 112, 121–122, 169, 179, 181

Jacob, James, 60
Jacob, Margaret, 60
Jeffreys, Harold, 189
Jesuits, 7, 20, 124, 308
jurisprudence: influence on probability theory, xxiii, 6–7, 14–15, 47, 99 (*see also* contracts, proofs, testimony); probability theory applied to, xxiv, 296, 342 (*see also* probability of judgments, probability of testimony); Roman-canon, 15, 19, 41–43, 46, 76, 124, 154n, 353; as model of reasonableness, 57, 63, 66, 106. *See also* expectation
Justinian, 117

Keynes, John Maynard, 38, 189
Kolmogorov, A. N., 3

Koyré, Alexandre, 7
Kramp, Chrétien, 172n

La Barre, Jean-François LeFèvre de, 304, 343, 346, 349
La Condamine, Charles Marie, 84
Lacroix, Silvestre-François, 339–341
Lagrange, Joseph Louis, 278
La Harpe, Jean-François, 354, 366
Lalande, Joseph-Jérôme, 90
Lambert, Johann Heinrich, 338
La Mothe le Vayer, François de, 322
Laplace, Pierre Simon, xix, xxi, xxvi, 253n, 378; on annuities and insurance, 172n, 182; and associationism, 206, 219–222; and Bayes' theorem, 208, 268–270, 274; on connection of natural to moral sciences, 220, 275–277, 299, 381–383; on determinism, 10, 274–275, 284; on expectation, 76, 103–106, 274–275; influence of d'Alembert on, 78, 90, 275, 283; on meaning of probability, 189–191, 210, 218–223, 224, 273–274, 281–282, 371, 374n; and Principle of Indifference, 29–30, 269, 273, 275, 279, 289, 372; on probability of causes, 13, 208, 223, 230, 233, 267–278, 294–295, 299; on probability of judgments, 107, 304, 350, 356–361, 364, 367, 375, 380n; on probability of testimony, 13, 323, 324, 335–338, 339, 340; on probability theory as good sense, 50, 68, 111, 297, 337, 357–359, 360; and statistics, 195, 222, 275–276, 284, 302
latitudinarianism, 59–60
Law, John, 164
law. *See* jurisprudence
law of large numbers, 181, 204, 205, 285–289, 299, 362, 365, 366–367, 381–382
law of succession, 270, 277–279, 295, 334, 370, 372–374, 377, 386
Le Clerc, Jean, 154, 156, 166
legal reform, 298, 304–305, 342–347, 352–354, 356, 360n, 380
Legendre, Adrien Marie, 145n, 172n
Leibniz, Gottfried Wilhelm, xxiii, 6, 7, 26, 126, 188; combinatorics, 9–10, 235; correspondence with Jakob Bernoulli, 28n, 129–130, 232–233, 237–240, 250, 255, 285–286; on legal probabilities, 44–46, 308, 310–311; and mortality data, 128, 132; on probability theory as new logic, 45, 266; on universal characteristic, 66, 385, 386
Leibniz-Clarke correspondence, 252
Leti, Gregorio, 141, 142, 146
life expectancy: methods of computing, 83, 86–87, 126, 128, 133–136, 172; tables, 112, 169, 318
Livy, 323
Lloyds of London, 120, 164, 165, 168
Locke, John: on associationism, 191, 196–199, 200, 201, 203, 208, 210–212, 222, 261; on evidence, 193–196, 291, 323; king of Siam parable, 195, 228, 280, 307, 328; and possessive individualism, 74; on probability, 45, 193–194, 201, 244, 282, 307, 324; and skepticism, xix; theory of knowledge, 55, 227, 241–242
London Assurance, 164, 170–171, 177
lotteries, xix, 78, 88, 91, 144, 141–163, 168, 173; blank, 142–143, 156; Genoan-style, 143; mathematical basis, 144–145, 157; players, 145–163; popularity, 141–144, 148; profits, 144–145, 157
Louis XV (king of France), 165
Louis XVI (king of France), 144
Lucas, Theophilus, 158
luck, 36–37, 124, 152, 153, 156–158. *See also* fortune
luxury, xxii, 72–74, 94–95

Machiavelli, Niccolo, 152–153, 157
MacPherson, C. B., 74

Magens, Nicolas, 166
Malebranche, Nicolas, 310
Malesherbes, Chrétien-Guillaume de Lamoignon de, 345
Mandeville, Bernard, 72–73, 94
Martianus Capella, 151
Marx, Karl, 298
Massé de la Rudelière, 96
mathematical model, xx, 56, 110–111
mathematical theory, xx, 4, 110
Maxwell, James Clerk, 246
measurement: common units of, 97–98; psychological units of, 91–94
Melon, Jean-François, 72–73, 94
mental calculations, 93–95, 98, 108, 197, 201, 305. *See also* associationism
Méré, Antoine Gombault, 15, 17
Merlin de Douai, Philippe Antoine, 352
Mersenne, Marin, xix, 56, 59, 243, 283
Michodière, 302
Mill, James, 203
Mill, John Stuart, 203, 297, 297n, 372–373, 375, 376, 377, 378–379
miracles, 155, 199, 202, 210, 267, 289, 304, 307, 310, 322–327, 329, 332–335, 337–338, 342
Mises, Richard von, 189
mixed mathematics, xxi, 53–56, 79, 88, 110, 369
Molesworth, John, 156
Montesquieu, Charles de Secondat de, 73, 97, 300, 303, 343
Montmort, Pierre de, 10, 32, 69, 253n, 312, 316–318, 341, 342
moral sciences: Enlightenment character of, 52, 55–56, 110, 298–305; influence on probability theory, 102, 106–107, 110–111; and natural sciences, 219, 275, 294, 296, 299–304, 347, 381–383; not subject to mathematical treatment, 354–355, 366, 368, 377–379; probability theory applied to, xxiv, 93, 107, 213, 219, 277, 288, 294, 296–369, 377–379; and social sciences, 106–108, 110, 298–299, 369, 377–382, 384–385. *See also* jurisprudence; political economy; psychology
Morand, 302
Morellet, André, 304, 343–344
Morgan, William, 180–181
Morgenstern, O., 109
Morris, Corbyn, 180
mortality: bills of, 12, 126–127, 192; rates of, 113, 121, 122, 123, 171, 334, 362, 384; regularity of, 127–133, 135, 137, 138, 171, 172, 175, 181, 183–184, 301; tables of, 31, 89, 93, 127–129, 136, 137, 172–173, 176, 180, 181, 185, 301, 317. *See also* statistics, demographic
Mulgrave, Earl of, 149
Mylon, Claude, 23

Nagel, Ernest, 6, 29
Napoleon I, 106, 356
natural laws, 55, 202, 208, 270, 280–281, 299–304, 310, 324–325, 337–338, 379
natural philosophy, 56–57, 59, 227, 240, 242–243, 246–248, 254–255, 267
natural theology, 56–57, 59, 62, 65, 131–132, 137, 203, 205, 253, 267
Navier, Claude, 265, 367, 378n
Neumann, John von, 109
Neumann, Kaspar, 132
Newton, Isaac, 5, 51, 131, 203, 220, 226, 241, 243–244, 250, 255, 299, 300, 313, 333
Nicole, François, 172n
Nicole, Pierre, 17
normal curve, 250, 382, 385

Occam's razor, 250
Oersted, H. C., 291–293

Pacioli, Luca, 9, 18
Paracelsus, Phillipus, 12
parish records, 126–127

Pascal, Blaise, xix, 5, 7, 9, 13, 45, 81, 235; correspondence with Fermat, 3, 15–18, 23, 31; *Lettres provinciales*, 308; *Pensées*, 60–61; wager, 57, 60–63, 67, 99, 221
passions, 51–52, 106, 161–162, 209, 214, 216, 221, 356, 364
Paul V (pope), 126
Payen, Antoine-François, 346
Pearson, Karl, 131
Pepys, Samuel, 160, 161
Petty, William, 127, 128, 171, 232
Peyronnet, Charles-Ignace de, 286, 359–360
physiocrats, 55, 100, 110, 151, 300–301
Pitt, William, 175n
Poinsot, Louis, 106, 365–369, 378n
Poisson, Siméon-Denis, 46, 57, 58, 111, 187, 253, 302, 304; on connection of natural to moral sciences, 287–288, 296, 365–366, 381–382; on expectation, 76, 105–106; on meaning of probability, 191, 219, 222–223, 371; on probability of causes, 227, 267, 284–294, 299; on probability of judgments, 107, 284, 289, 296–297, 304, 343, 350, 359–368, 370, 372, 375, 378n; on probability of testimony, 306, 323, 324, 338–339, 370
political arithmetic, 26, 67, 130, 192, 232
political economy, xxiii, 68, 70–76, 94–95, 99–101, 106, 110–111, 296
Popkin, Richard, xix
popular errors, xxii, 153–154, 321
Port Royal *Logique*, 17, 21, 23, 39–40, 50, 61, 64, 65, 212, 234, 243, 244, 307, 323
Porter, Theodore, xxiv
Pothier, Robert, 172
Povey, Charles, 114
Prestet, Jean, 9–10, 235
Price, Richard: on annuities and insurance, 136n, 169, 179–181; and Bayes' essay, 32–33, 205, 206, 208n, 212n, 230, 256–257, 258, 262–267; criticism of Hume, 203, 228, 248, 264–267, 278, 283, 323, 326–330
Priestly, Joseph, 203
Principle of Indifference, 29–30, 68, 103, 260, 273, 275, 279, 289, 290, 329, 372, 374
probabilism, 308
probability: inverse, 103, 204, 205, 206, 208, 218, 223, 230–231, 236–237, 256–263, 267–293; objective, xxi, 11–13, 30, 37, 103, 188–225, 229, 257, 261–262, 282, 283–284, 289–290, 370, 374n, 377; prehistory of, 3–47, 308; qualitative, 11–12, 14, 38–40, 48, 63, 194–195, 201, 205, 305, 330; quantitative, 7–8, 17–18, 24, 38–40, 194–195, 201, 205, 305; subjective, xxi, 11-13, 14, 30, 33–41, 46, 188–225, 229, 257, 261–262, 282, 289–290, 370, 374n, 377
probability of causes, 13, 229, 318; Bayes' theorem, 253–267, 274, 275, 277, 279, 293; Bernoulli's theorem, 230–253, 274, 275, 282, 285, 288n; critique of, 294–295, 372–375; in moral sciences, 277, 294–295, 296, 299, 302–303, 325, 338–339, 349, 351, 358, 359, 361–362, 365–366; and scientific reasoning, 267–293
probability of judgments, 6, 284, 289; critique and decline of, 106, 107, 229, 297, 305, 354–355, 359, 364, 365–370, 372–373; defended, 215, 358, 364, 367; and legal reform, 298, 304–305, 342–347, 349, 356, 380; political content of, 296, 304–305, 348–349, 357–358, 361–362, 364, 366
probability of testimony, 6; applied to historical sources, 304, 313–316, 333–334; applied to miracles, 323–332, 334–335; critique and decline of, 13, 193, 297, 305, 316–318, 355, 359,

probability of testimony (*cont.*) 370, 372–376; defended, 337; in *Encyclopédie*, 318–320; extrinsic and intrinsic components of, 306, 310, 311, 313, 316, 318–319, 323, 327–329, 331, 335, 339; historical context of, 308–310, 320–323, 341, 380; link with early probability theory, 304–305, 306, 308; and proto-quantification, 133, 305
problem of points, 9, 15–18, 24
proofs, legal, 14–15, 40–47, 192–193, 306, 353–354, 368–369, 376, 380
providence, 131–132, 137, 151, 154–156, 248–253, 286, 288, 310
psychology, 92–93, 109, 191, 199, 202, 203, 209–210, 220–221, 225, 305, 371, 380. *See also* associationism
Pufendorf, Samuel, 22, 28
pure mathematics, 53–54
pyrrhonism. *See* skepticism

quantification, xxiii–xxiv, 4–6, 14, 23, 38, 42, 47–48, 52, 131–133, 305, 355, 369
Quesnay, François, 100, 300
Quetelet, Adolphe, 106–108, 182, 286–287, 294, 298, 359, 369, 370–372, 377, 381–385

randomness, 11, 35, 157n, 240, 248, 251–252, 254. *See also* chance
rationalism, 240–241, 373
rationality, xix, xx, xxii–xxvi, 48, 50, 52, 57, 59, 61, 89, 108–111, 116, 152, 187, 196, 202, 211, 385. *See also* reasonableness
reasonable calculus, xix, 211
reasonable man, xx, xxii, 52–53, 56–111, 187, 197, 222, 293, 322, 342
reasonableness, xix, 48, 50, 53, 59, 90, 211, 262, 267, 369; and associationism, 200, 209, 212; and expectation, 57–82, 93–111; and miracles, 326; nineteenth-century attacks on, 376. *See also* good sense
Reformation, xix, 308, 322
risk, 18–23, 50, 53, 76; averse, 163, 173, 178, 182, 185–186; before probability theory, 116–125; comparison of, 90, 213, 345, 348–349; mathematical theory of, 125–138; perception, 90–95, 103, 112–116, 163, 171, 185–186; quantification of, 90–95, 120–121, 124–133, 173; relation to values and beliefs, 112–116, 140–141, 172–174; seeking, 163, 165, 167–168, 173, 178; of smallpox inoculation, 82–89. *See also* annuities; gambling; insurance; lotteries
Risueño d'Amador, 376–377
Roberval, Giles, 5, 17, 23
Royal Exchange Assurance, 164, 169, 170–171, 177, 181–182
Royal Society of London, 38, 60, 61, 62, 64, 131–132, 176, 196, 205, 248, 256, 265, 315
Royer-Collard, Pierre Paul, 366
rule of fellowship, 20
rules of reasoning, xxiii, 40, 49, 202, 241, 243–244

Saint-Cyran, Paul-Edmé, 301
St. Petersburg problem, xxi, 53, 56, 68–71, 75–82, 88, 95, 97, 99, 101, 103, 104–105, 106, 126n, 198
Schneider, Ivo, xxiv, xxv
scholasticism, xx, 57
Schooten, F. van, 9–10, 24, 235
scientific method, 222, 227, 240, 267, 272, 279–281, 290–293
self-interest, 51, 57, 59, 60, 62, 161, 173, 184, 208–210, 216, 376
Shapiro, Barbara, xxiv, 59
Sheynin, B. O., xxiv, xxv
signs, 11–12, 40–41, 43
Simon, Richard, 309
simplicity in nature, 239–240, 249–252, 281
Simpson, Thomas, 136, 169

Sixtus V (pope), 117n
skepticism: Condorcet's, 215, 283; constructive or moderate, 57–58, 65, 243, 247, 283, 307; historical, 322–323; Hume's, 199, 203, 228, 264, 267; probability theory used against, 228, 253–254, 267, 315, 329; sixteenth-century revival of, xix, 308, 322
smallpox inoculation, xxii, 33, 82–89
Smollet, Tobias, 157
social sciences, 106–108, 110, 298–299, 369, 372, 379–380. *See also* moral sciences
South Sea Bubble, 167
Spencer, Herbert, 298
Spinoza, B. de, 10
statistical inference, 190, 232–234, 295
statistical regularities, 107–108, 130–133, 137, 182–184, 187, 192, 288, 294, 301, 359, 372
statistics: demographic, 27, 91, 126–129, 138, 276–277, 301–302, 372, 384; lack of, 26–27, 30, 67, 83, 85, 120, 126, 136, 286; medical, 367, 376–377, 378n, 384n; relation to probability, 47, 49, 98, 103, 125–126, 135, 138, 230–234; social, 107–108, 286–287, 294, 302, 359, 371–372, 382, 384n; and stable conditions, 120, 124–125, 174, 183–185. *See also* mortality
Stigler, Stephen, xxiv, xxv, 256n
Stone, Lawrence, 184
Struyck, Nicholas, 131
Sun Fire Office, 114
Süssmilch, Johann, 130–131, 301

Talleyrand-Périgord, Charles-de, 146, 156, 157
Tartaglia, Nicolo, 18
testimony, 39–40, 63, 193, 195, 308–310, 320–323, 344, 352, 380. *See also* probability of testimony
Tetens, Johann, 166
Thomas, Keith, 164–165
Thucydides, 323
Tillotson, John, xix, 59, 60, 61–62, 66, 205, 242, 267
time, 115, 161–163, 173, 186
Tonti, Lorenzo, 167n
tontines, 122, 166, 167, 167n, 168, 170n, 178
tribunals, 6, 215, 238, 345, 366, 369. *See also* probability of judgments
Turgot, Ann-Robert Jacques, 96, 97, 100–101, 102, 104, 110, 278, 304, 345, 351

Ulpian, 30, 116, 121, 122
underwriters, 119–120, 165
uniformity of nature, 202, 208, 210, 237, 244, 262–263, 266, 283, 285, 338, 374
Ussher, Henry (bishop of Armagh), 207
usury, 18, 19–21, 115, 117, 121, 124, 160, 167, 169, 173
utility, 70–76, 95, 109. *See also* expectation, moral

value, 71, 74, 76, 77, 93, 99–101, 102, 105, 108, 110. *See also* utility
Vernier, Théodore, 166
Villermé, Louis-René, 384
Voltaire, François Marie Arouet de, 72–73, 83, 85, 94, 144, 304, 321, 323, 334, 343–344, 348, 353–354

Walford, Cornelius, 139
Walker, Joseph, 61
Wallis, John, 9, 235, 249
Walpole, Robert, 165
Westholme, William Dorghtie de, 143
White's of London, 160, 165, 167
widows' funds, 169, 174
Wilkins, John, 5, 38, 56, 59, 60, 62–63, 66–67, 242, 267, 283, 322

Zabell, Sandy, 12–13