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I.1
CHAPTER

Matter and the forces that move it

The prologue to the book gave you a preview of our quest, something like the video a tour agency might show you. Now we embark on the actual trip.

Where do forces come from?

In just about any physics course, the professor would be talking about forces, the force of gravity, the electric force, so on and so forth. I am here to tell you that, until quantum field theory was invented, physicists did not really know where these forces came from. Sure, they could describe the forces, but that was about it.

So, that was a fairly big deal: quantum field theory could explain how forces arise.

Matter

First, I have to remind you that matter consists of molecules, and molecules are built out of atoms. An atom consists of electrons whirling around a nucleus, which in turn consists of protons and neutrons, collectively known as nucleons. The nucleons are made of quarks. That's what we know.[1]

The universe also contains dark matter and dark energy. Indeed, by mass, the composition of the universe is 27% dark matter, 68% dark energy, and only 5% ordinary matter. To first approximation, the universe may be regarded as one epic cosmic struggle between dark matter and dark energy.[2] The matter we know and love and of which we are made hardly matters. Unhappily, at present we know little about the dark side. Nevertheless, essentially all reputable speculations about the dark side are based on quantum field theory.

Forces

We know of four fundamental forces between these particles. When particles come into the vicinity of each other, they interact, that is, influence each other. Here is a handy summary of the four forces, known as gravity, electromagnetism, the strong interaction, and the weak interaction:

G: Gravity keeps you from flying up[3] to bang your head on the ceiling or from floating off like a space cadet.
E: Electromagnetism prevents you from falling through the floor and dropping in on your neighbors if you live in an apartment.*
S: The strong interaction causes the sun to provide us light and energy free of charge.
W: The weak interaction stops the sun from blowing up in our faces.

While we all have to come to terms with gravity, we know electromagnetism best, as our entire lifestyle is based on enslaving electrons.

Only four forces!

The world appears to be full of mysterious forces and interactions. Only four?

As you toddled, you banged your head against a hard object. What is the theory behind that? Well, the theory of solids can get pretty complicated, given the large variety of solids. But a simple cartoon picture suffices here: the nuclei of the atoms comprising the solid are locked in a regular lattice, while the electrons cruise between them as a quantum cloud. A collective society in which all individuality is lost! The atoms no longer exist as separate entities. The arrangement is highly favorable energetically; that is jargon for saying that enormous energy is required to disturb that arrangement. Revolution is costly. It takes quite a tough guy to crack a rock into halves.

So, the myriad interactions we witness in the world, such as solid banging on solid, could all be reduced to electromagnetism. What we see in everyday life is by and large due to some residual effect of the electromagnetic force: since common everyday objects are all electrically neutral, consisting of equal numbers of protons and electrons, the electromagnetic force between these objects almost all cancel out. Even the steel blade of a jackhammer smashing into rock is but a pale shadow of the real strength of the electromagnetic force.[4]

When you first emerged into this world, you might have thought that there must be thousands, if not millions, of forces in the world. Thus, to be able to state that there are only four fundamental forces is totally awesome, a feat summarizing centuries of painstaking investigations. For example, realizing that light is due to electromagnetism stands as a towering achievement.

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*Plus a lot of other good deeds. Electromagnetism holds atoms together, governs the propagation of light and radio waves, causes chemical reactions, and last but not least, stops us from walking through walls.

No contact necessary

Our common everyday understanding of force involves contact: we can exert a force on an object only if we are in contact with it. In a contact sport such as American football, without tackling the ball carrier, a linebacker could hardly exert anything on him. And in the movies, a slap is not a slap until the leading lady's palm makes contact with the leading cad's cheek. At the supermarket, you can push the shopping cart only if you grip the handle. If you could just hold out your hands and command the shopping cart to move, a crowd would gather and honor you as a wizard.

Everyday forces, except for gravity, are short ranged, indeed zero ranged on the length scales of common experience. These forces are but pale vestiges of the electromagnetic force, as I've just said. The palm molecules have to be practically on top of the cheek molecules before the latter could acquire any carnal knowledge of the former.

Gravity is the glaring exception. When the earth pulls Newton's apple down, no hand comes out of the earth grabbing the apple as in a horror movie. Gravity is invisible, thus all the more horrifying as we age.

Just about the only commonplace example of a force acting without contact is the refrigerator magnet: You can feel the refrigerator pulling on the magnet before the magnet makes contact with the refrigerator. This shows that the electromagnetic interaction, like gravity, is also long ranged.

Hence, in quantum physics, the word "interaction" is preferred rather than the word "force." No contact is necessary for particles to interact with each other. Indeed, the very concept of "contact" is problematical in the quantum world.

The universe as a finely choreographed dance

While the proverbial guy and gal on the street are plenty acquainted with gravity and electromagnetism, they have no personal experience with the strong and the weak interactions. But in fact, the physical universe is a finely choreographed dance starring all four interactions.

Consider a typical star, starting out in life as a gas of protons and electrons. Gravity gradually kneads this nebulous mass into a spherical blob, in which the strong and the electromagnetic forces stage a mighty contest.

The electric force causes like charges to repel each other. Thus, the protons are kept apart from each other by their mutual electric repulsion. In contrast, the strong force, also known as nuclear attraction, between the protons tries to bring them together. In this struggle the electric force has a slight edge, a fact of prime importance to us.[5] If the nuclear attraction between protons were a tiny bit stronger, two protons could get stuck together, thus releasing energy. Nuclear reactions would then occur very rapidly, burning out the nuclear fuel of stars in a short time, thereby making steady stellar evolution, let alone civilization, impossible.

In fact, the nuclear force is barely strong enough to glue a proton and a neutron together, but not strong enough to glue two protons together. Roughly speaking, before a proton can interact with another proton, it first has to transform itself into a neutron. This transformation necessitates the intervention of the weak interaction. Processes effected by the weak interaction occur extremely slowly, as the term "weak" suggests. As a result, nuclear burning in a typical star like the sun occurs at a stately pace, bathing us in a steady, warm glow.

Short and long ranged

The reason that the proverbial guy and gal in the street do not feel the strong and the weak interactions is because these two interactions are short ranged. The strong attraction between two protons falls abruptly to zero as soon as they move away from each other. The weak interaction operates over an even shorter range. Thus, the strong and weak interactions do not support propagating waves.

In contrast, the gravitational force between two masses and the electric force between two charges both fall off with the separation r between the two objects like $1/r^2$, the famous inverse square law of Newton. Gravity and electromagnetism are long ranged, as was mentioned earlier, and thus can and do support propagating waves. We will see how quantum field theory could explain this curious state of affairs in chapter III.2.

For r large, these forces still go to zero, but slowly enough that we can feel the tug of the sun, literally an astronomical distance away.[6] For that matter, our entire galaxy, the Milky Way, is falling toward our neighbor, the Andromeda galaxy.

Thus, in the contest between the four interactions, brute strength is not the only thing that counts: many phenomena depend on an interplay between range and strength. A case in point is fusion versus fission in nuclear physics. When two small nuclei get together, each consisting of a few protons and some neutrons, the strong attraction easily overwhelms the electric repulsion and they want to fuse. In contrast, in a large atomic nucleus, famously, the uranium nucleus, the electric repulsion wins over the strong attraction. Each proton only feels the strong attraction of the protons or neutrons right next to it, but each proton feels the electric repulsion from all the other protons in the nucleus. The nucleus wants to split into two smaller pieces, accompanied by the release of energy.

Notes

[1] Whether or not quarks and electrons are tiny bitty strings is an intriguing, but at the moment purely speculative, possibility.

[2] See *GNut,* chapter VIII.2.

[3] You know how fast the earth is spinning to cover about 24,000 miles in 24 hours. Anybody who has studied some physics could calculate what the centrifugal acceleration would be.

[4] Just about the only time the true fury of electromagnetism shakes us is when thunder and lightning fill the sky. While we modern dudes have totally enslaved electromagnetism, all ancient people attribute its occasional bursts of temper to the gods. We still devote one day a week to electromagnetism: Thursday is Thor's day.

[5] Quantum mechanics enters crucially here. The protons are not energetic enough to climb over the repulsive barrier set up by the electric force but have to tunnel through. See the discussion about Gamow tunneling in my book *Fly by Night Physics* to be abbreviated henceforth as *FbN*. See the bibliography.

[6] Of course, the feebleness of gravity compared to the other three interactions is also compensated for by the enormous number of particles contained in the sun and in the earth.

I.2
CHAPTER

The rise of the classical field

Bizarre physics in the time of Newton

"So great an absurdity!" All right, class, who said that?

School children learn that the moon is attracted to the earth across the vastness of empty space. In contrast to their experience of pushing and shoving on the playground, no contact is necessary for a force to act. The earth is incessantly moving around the sun, as they know, and any change in the position of the earth is instantaneously communicated to the moon. In Newtonian gravity, the moon is slavishly yoked to the earth. In turn, the earth is yoked to the sun, and the entire galaxy moves as a collective entity.

That this sounds bizarre was already apparent to Newton, who complained in a letter to his friend Richard Bentley: "That . . . one body may act upon another at a distance through a vacuum without the mediation of anything else by and through which their action or force may be conveyed from one to another is to me so great an absurdity that I believe no man who has in philosophical matters any competent faculty of thinking can ever fall into it."

Do you recall when you first learned about Newtonian gravity? Did you wonder[1] how a moon could know instantly that its planet had moved? Were you lacking in "competent faculty of thinking"? Ooh oh.

Faraday and our mother's milk

Look up at the night sky and admire the serenity of the moon. It is impossible to imagine, let alone to feel, the earth pulling on the moon, trying to bring that giant rock in the sky down to earth. But hold a tourist souvenir magnet close to your refrigerator, and you can feel the the magnetic force reaching across

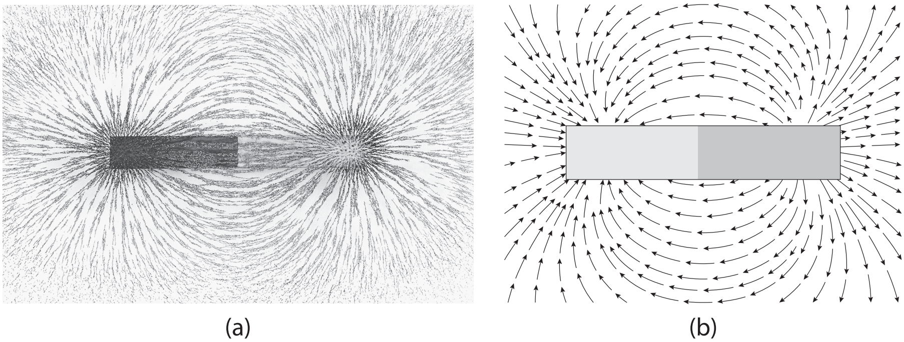

Figure 1. (a) Iron filings around a bar magnet. (b) The magnetic field of force around a bar magnet.

space. The great 19th century experimentalist Michael Faraday[2] introduced the term "field of force," or field[3] for short, in his study of magnetism.

Place a piece of cardboard on a magnet. Sprinkle iron filings (as if you usually have that around) on the cardboard. The iron filings eagerly, almost magically, line up to form a characteristic pattern. See figure 1. Faraday visualized a field of force around the magnet. When iron filings are introduced into this field, they are acted upon by the field.

Instead of the magnet acting directly on the filings, physicists think of the magnet creating a magnetic field around it, which in turn acts on the filings. The key point is that, even in the absence of the filings, the magnetic field still exists, just sitting around shooting the breeze, so to speak.

Analogously, physicists say that an electric field surrounds a charged sphere, with the field of force pointing radially outward like the spines of a sea urchin, outward because a test charge* with the same sign (that is, positive or negative) of charge as that on the sphere would be repelled, feeling a force in the direction of the arrow. See figure 2. In contrast, a test charge with the opposite sign of charge as that on the sphere would be attracted toward the sphere, and the arrows would point inward. Like and like repel, like and unlike attract, as was mentioned in chapter I.1 and as the reader surely already knows, if not in studying electrostatics, then perhaps in other contexts.

Gravity famously does not know about yin and yang, in sharp contrast to the electric force, as was also mentioned in chapter I.1. All masses attract each other, a fact responsible for some of the most salient features of the universe. The gravitational field looks just like the electric field around a charged ball, except that the force field is always pointing radially inward.

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*A test charge is simply an infinitesimal charge imagined by physicists to test or measure an electric field, infinitesimal in order not to disturb or add significantly to the electric field.

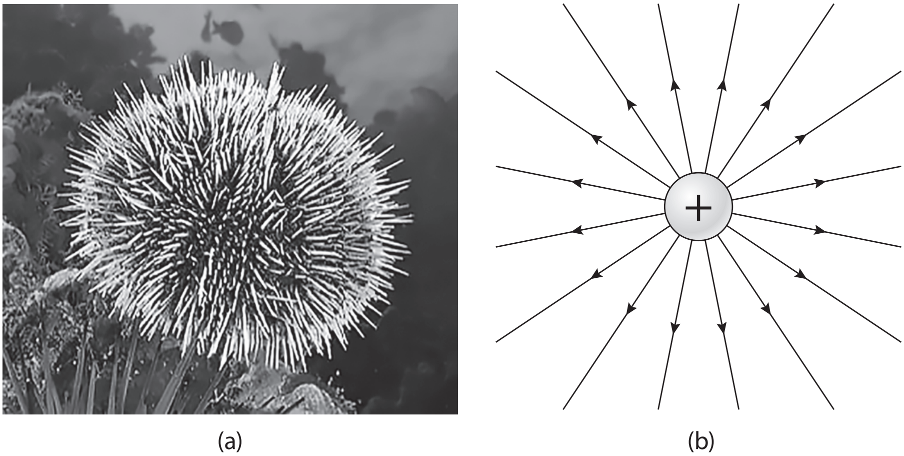

Figure 2. (a) A sea urchin, a leading California export to Asia. (b) The electric field of force around a positive charge.

That Einstein sure had a way with words. Listen to him:[4] "For us, who took in Faraday's ideas so to speak with our mother's milk,[5] it is hard to appreciate their greatness and audacity." Yes, some physicists have indeed forgotten[6] how audacious this concept of a field truly is.

Take-home message: Physicists need fields to make physics local. Following Newton, they run screaming away from the horror of action at a distance!

Able to leave home

Then physicists discovered that a moving magnet generates an electric field. At least in hindsight, the next question almost suggests itself: what does a moving charge generate? Moving charges are manifested most conveniently in the form of an electric current in a wire. Surprise! Electric currents do generate magnetic fields.

The important conclusion for physics is that an electric field changing in time could generate a magnetic field. Dualistically, a magnetic field changing in time could generate an electric field. This suggests that the electric field and the magnetic field would henceforth lose their separate identities, merging into one entity known as the electromagnetic field.

In Faraday's work, the electric field and the magnetic field served mostly as descriptive devices. But later, James Clerk Maxwell had the fantastic insight that an electromagnetic field changing in time could generate itself, moving across space as an electromagnetic wave. And thus, the electromagnetic field was able to leave home and take on a life of its own, bidding farewell to the charges, magnets, and currents that begot it in the first place.

Lo, our telecommunicating civilization was born!

As real as a rhino

These days, physicists visualize the earth creating a gravitational field, which in turn acts on the moon. The gravitational field is the mediator Newton longed for, to get around the absurd concept of action at a distance. And we are literally swimming in a sea of electromagnetic fields.

Dear reader, these are not mere words. The crucial, and meaningful statement, is that the field[7] as a physical entity is entirely real. As real as a rhino, according to the Indian American physicist Anupam Garg.[8] And so on the back cover of his textbook on electromagnetism, I blurbed that quantum fields are as real as quantum rhinos.

Cartesian coordinates

René Descartes, watching a fly while lying in bed, taught us that we could locate a point in the 3-dimensional space we were born into by 3 numbers $(x,\ y,\ z)$, which we will write for short as $\vec{x}$. Consider the electric field we just talked about. At any instant in time, call it t, and at any point $\vec{x}$ in space, the electric field is specified by 3 numbers, $(E_x,\ E_y,\ E_z)$, namely, the component of the electric force pointing in the x-direction, the component pointing in the y-direction, and the component pointing in the z-direction, respectively. In other words, $E_x(t,\ x,\ y,\ z)$ is a function of 4 variables: t and $(x,\ y,\ z)$. It varies in time and in space. Similarly for E_y and E_z. Again, we could write all this for short as $\vec{E}(t,\vec{x})$. Similarly, physicists write the magnetic field as $\vec{B}(t,\vec{x})$.

Incidentally, Maxwell, working before some clever fellows thought of putting little arrows on top of vector quantities such as the $\vec{E}(t,\vec{x})$ and of using subscripts to distinguish the different components of $\vec{E}$, actually wrote out all six components $(E_x,\ E_y,\ E_z,\ B_x,\ B_y,\ B_z)$ of the electromagnetic field and all four spacetime coordinates (t,x,y,z). Thus, his treatise is almost impossible for contemporary physicists to read. That he managed to see the electromagnetic wave though this morass is almost a miracle.

In theoretical physics, a good notation is often said to be half the battle.[9]

A notational confusion

At this point, I must mention a notational confusion that has confounded and brought grief to generations of beginning students of quantum field theory.

In Newtonian physics, the motion of material objects is first abstracted to the motion of point particles. The position of the point particle being studied is denoted by (x, y, z), as Descartes taught us. The three numbers (x, y, z) change with time: that is what it means to say that the particle is moving around. The goal of Newtonian mechanics is to determine the three functions $x(t), y(t)$, and $z(t)$, each a function of the time t. These three functions, written more compactly as $\vec{x}(t)$, are the dynamical variables of Newtonian mechanics.

In contrast, the electric field is denoted by $\vec{E}(t,\ x,\ y,\ z)$. As you can see, (x, y, z) here label an arbitrary location in space. You specify (x, y, z) and $\vec{E}(t,\ x,\ y,\ z)$ tells you what the electric field is at that location in space at time t. Clearly, when discussing fields, we regard $\vec{E}$ as our dynamical variable, not (x, y, z).

These two conceptually distinct uses of the letters (x, y, z) do not pose a problem in introductory physics courses, but obviously would wreak havoc when we are fooling around with both particles and fields. In that case, it would be mandatory to specify the position of the particle by something other than (x, y, z). One standard choice is $(q_x,\ q_y,\ q_z)$, or even better, $(q_1,\ q_2,\ q_3)$ packaged as $\vec{q}$, or more precisely, $\vec{q}(t)$. In fact, I have already used this "more advanced" notation ($\vec{q}$ instead of $\vec{x}$ for the position of a particle) when I discussed the Heisenberg uncertainty principle back in the prologue.

You actually know what a field is, you just don't know that you know

In the popular imagination, the word "field" conveys a certain mysterious, perhaps even mystic, air. But physicists actually use the word rather broadly and loosely, even with abandon. Essentially, almost any physical quantity that varies in space and time, namely $(t, \vec{x})$, may be called a field.

For instance, suppose you are studying the temperature $T(t, \vec{x})$ of the earth's atmosphere, or the air's flow velocity $\vec{v}(t, \vec{x})$. The former is a scalar* field, the latter a vector field. Evidently, their dynamics (that is, behavior) can be described by classical physics, and hence these are known as classical fields. In everyday circumstances, the electromagnetic field is also a classical field.

Sound furnishes another everyday example of a classical field, being a density wave in air. Humans can hear sound waves with frequencies between 20 Hz and 20,000 Hz.† Using the Greek letter ρ ("rho"), let us denote by $\rho(t, \vec{x})$ the deviation of the actual density of air from the quiescent density (that is, the density in the absence of sound). In other words, in the denser regions, the density fluctuation ρ is positive, while in the less dense regions, ρ is negative. In the absence of sound, $\rho = 0$. For ease of writing, I will henceforth often drop the extra words "deviation" and "fluctuation."

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*This simply means that temperature is a number and does not have a direction in space.

†A hertz, denoted by Hz, is defined as 1 cycle per second

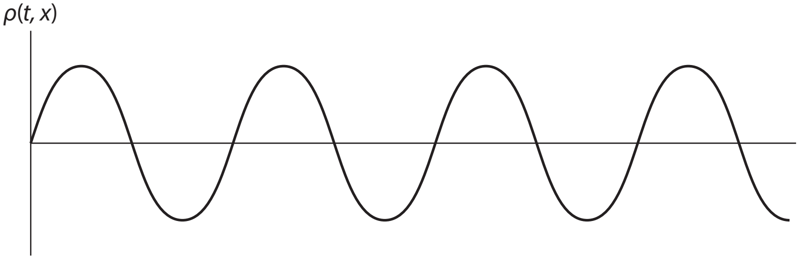

Figure 3. A sound wave characterized by a single frequency or equivalently, a single wavelength. Such waves are called "monochromatic," evidently a term originating in the study of light waves.

Sound normally propagates in 3-dimensional space, but to keep the discussion as focused and as simple as possible, consider sound traveling down a long tube, so that space is effectively 1-dimensional, and we can write $\rho(t, x)$, with x measuring the distance along the (infinitely) long tube and t the time.

I plot in figure 3 the density fluctuation in a sound wave. I have intentionally not labeled the horizontal axis. You might have naturally interpreted the coordinate along the horizontal axis as x. Then the figure represents a snapshot of the density fluctuation at an instant in time. The figure is literally a picture of the density fluctuation in the tube, positive here, and negative there. An instant later, the figure would be different. It changes with time.

One figure could be interpreted in two different ways

Interestingly, you could have equally well taken the coordinate on the horizontal axis in figure 3 to be time t, so that the figure represents the density fluctuation at a fixed location in the tube. To the ear of an observer at that location, the density goes up and down, periodically denser and then less dense. The density changes, positive now, negative later, and then positive again yet later. Now the figure shows the entire history of the density fluctuation at one particular location.

That one figure could be interpreted in two different ways is an important point which we will exploit later, in chapter III.2, when we explain the origin of forces in quantum field theory. Keep that in mind!

Frequency and wave number

A wave is characterized by its frequency and by its wavelength (namely, the distance from crest to crest), in other words, by its variations in time and in

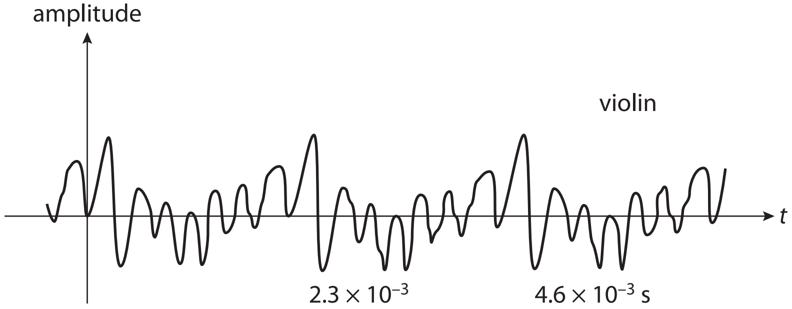

Figure 4. The note A, played on the violin, is formed by superposing a fundamental and a few harmonic waves. The pattern repeats itself after 2.3 milliseconds.
Modified from H. C. Ohanian and J. T. Markert, *Physics for Engineers and Scientists*, Norton, 2007.

space. Physicists denote[10] frequency by ω, but instead of wavelength, prefer to use the inverse of the wavelength, known as the wave number and written as k. The relationship between frequency and wave number, that is, the function $\omega(k)$, is characteristic of the wave.

If the only sound wave you could produce is that shown in figure 3, people would shun you as a rather monotonous person. That sound wave consists of one single frequency. Interesting sound waves are composed by superposing many different frequencies, as would be familiar to those readers who are musicians. See figure 4. Indeed, a chord consists of sound waves of several different frequencies that are in agreement, or accord, with each other.

Physicists sometimes call the sound wave shown in the figure a wave train. Of course, pleasing music cannot consist of a single note, but rather consists of a sequence of notes arranged cleverly to follow one upon another. Similarly, speech or song. It is almost miraculous, that the human vocal cord could exercise such fine muscular control, capable of rapidly producing one syllable after another.[11] No other life form on earth has mastered this "trick."

Beating between two waves with slightly different frequency and wave number

The wave train in figure 4 exhibits periodically a wave of higher amplitude than the others. You might have observed this same phenomenon at the seashore.[12] Waves come in sets, with a large wave followed by smaller waves and then larger waves, and then the cycle repeats, as shown in figure 5.

This pattern can be understood by picturing the interference of two waves with the same amplitude, but with slightly different frequency and wave number. Imagine a moment in time when the crest of one wave is matched with

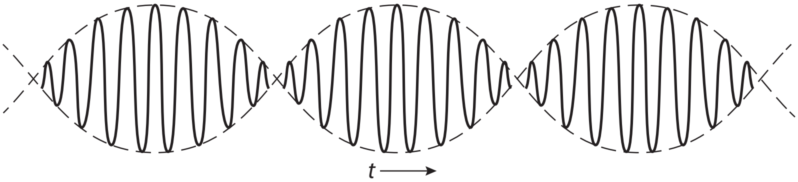

Figure 5. Beating between two waves.
Redrawn from A. Zee, *Fly by Night Physics*, Princeton University Press, 2020.

the crest of the other wave, say, with slightly lower frequency. The two waves add, resulting in an exceptionally large amplitude; we have constructive interference.

Higher frequency means less time between crests. Thus, the next time the crest of the higher frequency wave arrives, the crest of the slightly lower frequency wave is not quite there yet. After each cycle, the lag grows a bit larger. Eventually, the two waves are totally out of phase, leading to destructive interference, thus explaining the pattern shown in the figure.

How long would that take? Well, after each cycle, the time lag is given by $1/\Delta\omega$, where, as explained in the prologue, $\Delta\omega$ denotes the difference between the two frequencies.[13] So, the time it takes for the phase lag to build up to π is given by $\pi/\Delta\omega$.

One key result to remember from this: The time between two big waves equals twice the time period worked out above, and thus $\Delta T \simeq 2\pi/\Delta\omega$.

Incidentally, waves on the beach commonly originate from storms at sea. By counting the number of waves between two large ones (as shown in figure 5) you could actually deduce how wide an area over which the storm occurred if you knew from the weather report where the storm was.[14]

Fourier and the frequency spectrum

We just illustrated the musical phenomena of beating by adding two waves with the same amplitude but different frequencies* We are certainly allowed to add waves with different amplitudes. For that matter, we could add a third wave with yet a different amplitude and frequency. Then, how about a fourth? Carrying this to its logical conclusion, Joseph Fourier showed in 1822 that, by adding many (possibly infinitely many) waves with different frequencies, we can construct a sound wave with any shape we like. Fourier, son of a tailor, was orphaned at a young age. Due to his lowly birth, he was excluded

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*Note that frequency and wave number (or wavelength) are not independent variables. The relation $\omega(k)$ is determined by the physics relevant to the wave; for instance, for sound, it depends on how compressible air is.

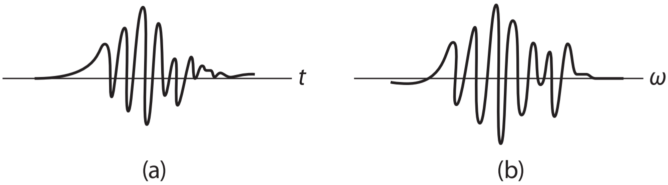

Figure 6. (a) A wave packet consisting of the superposition of waves with many frequencies. (b) The same wave packet as in (a) analyzed into its frequency components.

from being an officer in the artillery corps, and instead was assigned to teach mathematics.[15]

Thus, a sound wave may be characterized by its frequency spectrum. The sound wave shown in figure 3 consists of a single frequency. The wave train in figure 4 is composed by adding several waves with different frequencies that are multiples of each other.

Consider a pulse of sound (known as a wave packet in physics since it is composed of many waves), each with a definite frequency. At a given location, the density profile of air as a function of time t may look like what is shown in figure 6a. Before the pulse arrives, the density fluctuation ρ of air is zero by definition. Then it rapidly oscillates, varying between positive and negative values.

A pulse, such as that in figure 6a, may be analyzed into its frequency components, as shown in figure 6b. This important idea, universally used in the physical sciences and in engineering, is known as Fourier analysis.[16] Switching back and forth between figures 6a and 6b is known as a Fourier transform. Since in quantum physics, particles are revealed to be waves (much more on this later), the Fourier transform is an essential mathematical tool in quantum mechanics and in quantum field theory.

Or, as before, you could regard the figure as a snapshot of the density fluctuation as a function of space x. Then you would be talking about wavelength, or better yet, wave number k, instead of frequency. Simply change the labels on the horizontal axis in figures 6a and 6b to x and k, respectively. To summarize, Fourier transform allows us to hop back and forth between (t, x) and (ω, k).

The result we obtained earlier can now be extended. Let ΔT denote the duration of the sound pulse and $\Delta\omega$ the spread in its component waves in frequency. Then

$$\Delta T \Delta\omega \simeq 2\pi$$

Does this remind you of anything you have seen before? Yes: the uncertainty principle.[17]

And yes, the "other" uncertainty principle, relating the uncertainties in position and momentum, is also a manifestation of the Fourier transform.

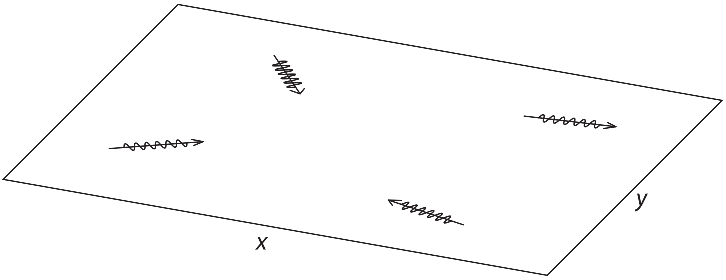

Figure 7. Wave packets in a 2-dimensional universe moving about and colliding with each other, possibly merging or producing other wave packets.

Wave packets could collide and scatter off each other

We could readily extend this discussion of wave packets, in 1-dimensional space to 2-dimensional space. Consider an elastic membrane, such as the surface of the drum.* By banging appropriately, we could create vibrational waves on the membrane, even construct wave packets.

Any point on this elastic membrane could be identified by two numbers, namely, (x, y), as Descartes taught us. Denote the deviation of the membrane from its equilibrium position, that is, its position in the absence of a wave, by $\varphi(t, x, y)$. Again, we could characterize the wave by its frequency ω and wave number. The discussion proceeds just as before, except that the wave number k has to be generalized to $\vec{k}$, a wave number vector or wave vector for short, with the vector pointing in the direction of propagation of the wave. (By the way, you see why the wave vector is a more useful concept than wavelength: it tells us about the direction of the wave also.)

Once again, we can form wave packets zinging around on the elastic membrane. The new concept that comes in when we move from 1-dimensional space to 2-dimensional space is "direction." Picture wave packets moving around in this two-dimensional universe in different directions, scurrying here and there, and occasionally even colliding with each other (figure 7).

By now, it takes no effort to move up to 3-dimensional space. Picture space filled with an elastic medium, perhaps a jello-like substance in which we could set up density waves. Simply write $\varphi(t, x, y, z)$, or more compactly, $\varphi(t, \vec{x})$ to denote density fluctuations. Once again, we could have wave packets moving around, but now in the 3-dimensional space we live in.

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*A word of caution and clarification here. At the mention of a drum, a "normal person" thinks of an everyday drum, surrounded by air, so that the vibration of the drum surface produces sound. A theoretical physicist, in contrast, immediately thinks of a drum of infinite extent, surrounded by nothing, existing by itself as a 2-dimensional universe. I merely said "drum" to help you fix in mind what I meant by "elastic membrane." We are thinking about the rippling waves on the surface of the drum, not the sound wave in the air enveloping the drum.

The power of free association

What do you think these wave packets remind physicists of? If you say elementary particles such as electrons, you may have what it take to be a theoretical physicist. The power of free association! These wave packets can move around in space, collide with each other, scatter and move off in different directions. They walk and talk like particles. Keep that thought in mind! We will come back to it.

Notes

[1] A lay reader to whom I sent the manuscript said that he was kept sleepless, not so much by this, but by the gravitational force between two bodies blowing up to infinity as they approach each other. In fact, many great physicists shared his worry. However, in classical physics, bodies have finite sizes, and a point particle is merely a convenient idealization. In quantum mechanics, this problem is obviated by quantum fluctuations. However, it is in some sense the origin of a notorious difficulty in quantum field theory involving the somewhat obsolete concept of "renormalization," a difficulty that has long been overcome, in spite of what you might have read elsewhere. Some voices on the web are decades behind the times.

[2] For a brief biography, see *Fearful,* pages 58–62. Rising from poverty, Faraday managed to find a job (without which we almost certainly would have never heard of him) working for the famed chemist Humphrey Davy. "Sir Humphrey's wife found Faraday physically awkward, and even irritating. He was small and stocky—not more than five foot four—with a large head that always seems slightly too big for his body. He spoke all his life with a flat London accent and had difficulty pronouncing his 'r's, so that as he himself said, he was always destined to introduce himself as Michael *Fawaday*." See R. Holmes, *The Age of Wonder,* page 352, Pantheon Books, 2008.

[3] In fact, I already snuck the word "field" past you in the prologue.

[4] A. Einstein, *Out of My Later Years,* Philosophical Library, 2015.

[5] Those of you who were bottle fed may be excused.

[6] Perhaps they were bottle fed.

[7] Read about how the earth's magnetic field preserved a memory of when the Babylonians torched Jerusalem in 586 BCE. https://www.timesofisrael.com/burnt-remains-of-586-bce-destruction-of-jerusalem-help-map-physics-holy-grail/.

[8] A. Garg, *Classical Electromagnetism in a Nutshell,* Princeton University Press, 2012.

[9] I could hardly believe it, but it is true. Einstein in his 1905 paper on special relativity still wrote out all 4 components of x explicitly.

[10] The frequency f (and its reciprocal, the period T) in everyday usage differs from ω by a factor of 2π: $\omega = 2\pi f = 2\pi/T$. We are certainly not going to quibble about such details here. Similarly, the wave number is defined by $k = 2\pi/\lambda$, with λ the wavelength.

[11] For a detailed analysis of an American politician's voice, see https://www.youtube.com/watch?v=waeXBCUkuL8.

[12] See *FbN,* chapter VII.2.

[13] For readers who remember some high school trigonometry, start with the identity

$\cos A' + \cos A = 2\cos\frac{A'+A}{2}\cos\frac{A'-A}{2}$. Set $A = \omega(k)t - kx$ and $A' = \omega(k')t - k'x$, with $k' = k + \Delta k$ and $\omega(k') = \omega(k) + \frac{\Delta\omega}{\Delta k}\Delta k$. Thus

$$\begin{aligned}&\cos(\omega(k')t - k'x) + \cos(\omega(k)t - kx)\\ &\quad\simeq 2\cos(\omega(k)t - kx)\cos\left(\frac{1}{2}\left[x - \frac{\Delta\omega}{\Delta k}t\right]\Delta k\right)\end{aligned}$$

We obtain a rapidly varying wave $\cos(\omega(k)t - kx)$ with large wave number k, with a correspondingly short wavelength, modulated by a slowly varying envelope

$$\cos\left(\frac{1}{2}\left[x - \frac{\Delta\omega}{\Delta k}t\right]\Delta k\right)$$

with small wave number Δk, and hence a long wavelength, as shown in figure 5.

[14] The point is that of the two interfering waves, one comes from the edge of the storm closer to us, the other from the edge farther from us. Noting that they arrive at the same time and knowing how the speed of ocean waves depends on frequency, we could estimate the difference between their frequencies. See *FbN,* page 280.

[15] Let me ask you: in the 21st century, how many remember Fourier and how many remember the commander of France's artillery corps at the end of the 18th century? Incidentally, Fourier was among the first to show that the earth would be much colder than it actually is given its distance from the sun and that the atmosphere, acting as a greenhouse, is crucial.

[16] For more, see https://en.wikipedia.org/wiki/Fouriertransform.

[17] In quantum physics, the energy of a particle is related to its de Broglie frequency by $E = \hbar\omega$ (see chapter I.5) with $\hbar$ being Planck's constant. Thus, multiplying the equation in the text by $\hbar$, we obtain $\Delta T \Delta E \sim \hbar$.

I.3
CHAPTER

Time unified with space

In previewing our quest, I already mentioned Einstein's 1905 theory of special relativity, which, in total defiance of common sense, unified space and time into spacetime. Before heading toward the promised land of quantum field theory, we need to explore a bit this fabled region that we must cross. Ladies and Gents, I give you special relativity, in three ways! But first, some commonsense relativity.

Galilean, or commonsense, relativity

I remind you (see chapter I.2) that Descartes taught physicists to locate "events" in time and space by using coordinates (t, x, y, z), the when and where of happenings in our universe. But another observer is free to use a different set of coordinates (t', x', y', z'). How one set of coordinates is related to another is known as "relativity" to physicists.

One misconception is that relativity started with Einstein, but in fact, Galileo was the first, to quantify what we might call commonsense or everyday relativity (see figure 1).

Commonsense relativity states that the temporal and spatial coordinates of the two observers gliding by each other are related by the Galilean transformation:

$$t' = t, \qquad x' = x + ut, \qquad y' = y, \qquad z' = z$$

In particular, the point assigned coordinates $x = 0,\ y = 0,\ z = 0$ by one observer would be assigned coordinates $x' = ut,\ y' = 0,\ z' = 0$ by another. The equation $x' = ut$ asserts that the two coordinate frames are moving with velocity u relative to each other along the x-direction.

To be a bit more concrete, go back to the duo from the prologue, with Ms. Unprime riding on a train smoothly gliding through a station, and Mr. Prime,

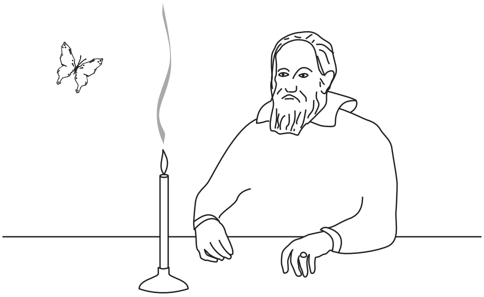

Figure 1. Galileo observing a butterfly flying normally and the smoke rising vertically from a candle on a smoothly moving ship.
Reproduced from A. Zee, *Group Theory in a Nutshell for Physicists*, Princeton University Press, 2016.

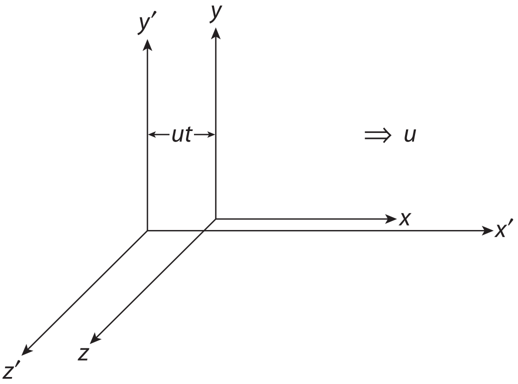

Figure 2. Two coordinate frames moving with velocity u relative to each other along the x-direction.
Redrawn from A. Zee, *Group Theory in a Nutshell for Physicists*, Princeton University Press, 2016.

the station master, standing on the platform. The point underlying relativity is that, while Mr. Prime could say that Ms. Unprime is moving, Ms. Unprime could equally well say that Mr. Prime is moving in the opposite direction.

Of the four equations displayed, the first, $t' = t$, states that time is universal: when one second has passed for Ms. Unprime, one second has also passed for Mr. Prime. Pure common sense.

The second equation defines the velocity u. The point designated by Ms. Unprime as $x = 0$ is seen by Mr. Prime as moving according to $x' = 0 + ut = ut = ut'$. See figure 2.

The third and fourth equations say that the two coordinates perpendicular to the direction of motion are not affected. This follows from a foundational principle of physics, that only relative motion could be defined, not absolute motion.[1]

Relativity simply states that the laws of physics cannot distinguish between coordinate frames moving smoothly relative to each other. On a jet moving along with no turbulence in sight, you could pour yourself a drink just as if you were at rest[2] at home!

The everyday rule for adding velocities, which we invoked in the prologue, is easily derived from the Galilean transformation. (Notational alert: Physicists use the Greek letter delta Δ to mean different things. In the prologue, it means "uncertainty;" here, "the change in." Even with the Greek alphabet included, there are only so many letters.) Suppose that in the time interval $\Delta t' = \Delta t$, Ms. Unprime sees an object moving through Δx and hence velocity $v = \frac{\Delta x}{\Delta t}$, Mr. Prime would see it moving with velocity

$$v' = \frac{\Delta x'}{\Delta t'} = \frac{\Delta(x+ut)}{\Delta t} = \frac{\Delta x + u\Delta t}{\Delta t} = \frac{\Delta x}{\Delta t} + u = v + u$$

Incidentally, in my experience, American physics students might be more familiar with moving sidewalks in airports than smoothly moving trains. In that case, Mr. Prime would be the guy in the souvenir shop watching people go by, and Ms. Unprime would be standing on the moving belt. (Another traveler walking on the belt would be the object moving with velocity v relative to Ms. Unprime. Mr. Prime sees this traveler moving with velocity $v + u$, as indicated by the calculation we just did.)

Does the universe have a speed limit?

I promised you three ways to special relativity, which I will now discuss in turn.

First way: Should we sit in on some physics crazed sophomores arguing in a beer soaked late night bull session, or listen with an air of feigned reverence to some chaired philosophers from America's most elite universities at a symposium to discuss deep truth? Your choice.[3] The topic: Is there a speed limit in the universe?

One philosopher intones, "There cannot possibly be a speed limit."

Proof by contradiction. Suppose nothing could go faster than the speed c. Suddenly, we see a spaceship zoom by with speed almost equal to c. Consider an observer moving by with speed u in the opposite direction. To this observer, the spaceship is receding in his rearview mirror with speed $c + u$. But this violates the assumed speed limit. "No speed limit! Quod erat demonstrandum," the philosopher crows.

Another philosopher, stunned by this argument, gravely nods in agreement, but could not resist muttering: "But what would Aristotle and Kant say?" Appeal to authority, yeah! A third philosopher muses. "With a speed limit, the notion of simultaneity becomes suspect. Communication between distant points necessarily takes time. We would have to conclude that absolute time does not exist, which is manifestly absurd! What kind of universe would that be if you can't tell what time it is?"

The three philosophers concluded by pure thought that physicists would never find a speed limit, but physics is not philosophy. Proof is by experiments. Well, physicists did find that nothing[4] could go faster than light. Hence, no universal time.

Dueling thinkers: the fall of simultaneity

Time is in the eyes of the beholder. The notion of simultaneity crashes and burns.

The second way: To see why simultaneity fails, let's watch[5] Professor Vicious and Dr. Nasty.[6] They have been at each other's throats for decades. Theoretical physicists are forever fighting over "who did what when." They are constantly bickering, telling each other (as the joke goes), "Nyah, nyah, what you did is trivial and wrong, and I did it first!"

Of course, the fight for credit goes on in every field, but in theoretical physics it is almost a way of life, since ideas are by nature ethereal. And the stakes are high: the victor gets to go to Stockholm, while the loser is consigned to the dustbin of history, a history largely written by the victor with the help of an army of idolaters and science journalists.

We are finally going to settle matters between Vicious and Nasty once and for all. The two of them are seated at the two ends of a long hall, Vicious at $x = 0$ and Nasty at $x = L$.

We now tell Vicious and Nasty to solve the basic mystery of why the material world comes in three copies.[7] As soon as they figure it out, they are to push a button in front of them. When the button is pushed, a pulse of light is flashed to the middle of the room where, at $x = L/2$, our experimental colleague, an electronics wiz, has set up a screen. When the screen detects the arrival of a light pulse, all kinds of bells and whistles are rigged to go off. In particular, if, and only if, two light pulses arrive at the screen at precisely the same instant, a huge imperial Chinese gong will be bonged.

"Fair is fair, any and all priority claims will be settled," we told Vicious and Nasty. "Now go to work and explain why quarks and leptons come in three sets." The dueling duo immediately assume the Rodinesque pose of the deep thinker and lock themselves in a "think to the death."

Meanwhile, you are sitting on a smooth train, moving relative to the dueling thinkers. Denote the time and space coordinates in your rest frame by t' and x'. In the Newtonian universe, time is absolute, and so we have $t' = t$. In your frame, you are sitting at $x' = 0$, but Vicious and Nasty are moving by according to $x' = ut'$ and $x' = L + ut'$ respectively. Of course, in the duelists' frame, with

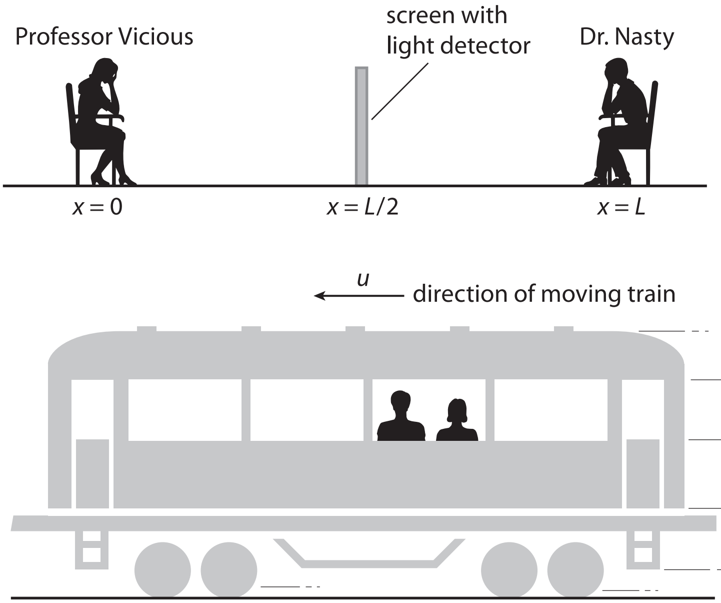

Figure 3. Professor Vicious and Dr. Nasty locked in a “think to the death." Modified from A. Zee, *Group Theory in a Nutshell for Physicists*, Princeton University Press, 2016.

time and space denoted by t and x, you are the one who appears to be moving, gliding by at $x = -ut$. See figure 3.

Some time passes, and all of a sudden we hear a loud bong of the gong. “The best possible outcome, you solved the problem simultaneously!" we exclaim joyously with much relief. “You guys are equally smart and you could go to Stockholm together!"

The arrangement is fool proved electronically. We won’t have either of them gloating, “I did it first!" Peace shall reign on earth.

But guess what? A Swede is sitting next to you. He too heard the gong. That’s the whole point of the gong: you either heard it or you didn’t. It’s all admissible in a court of law. Now, not only is the Swede on the Nobel Committee, but he also happens to be an intelligent Swede. He reasons as follows.

Professor Vicious is gliding by as described by $x' = ut'$. When Professor Vicious pushed the button, she sent forth a multitude of photons surging toward the screen at the speed of light c. But the screen was also moving forward, away from the surging photons. Of course, light moves at the maximum allowed speed in the universe, and it soon catches up with the screen. The opposite is true for Dr. Nasty. The screen is moving toward the photons he sent forth. Thus, to reach the screen, Nasty’s photons have less distance to cover than Vicious's photons. (Draw some photons moving toward the screen. It will clarify what you just read.)

Hence, reasons the intelligent Swede, for the two bunches of photons to reach the screen at the same time and so cause the gong to bong, the photons sent out by Vicious must have gotten going earlier. Thus, Vicious solved the problem first. With malicious glee, the Swede solemnly intones, "After Professor Vicious is awarded the Nobel Prize, she will kindly help us stuff Dr. Nasty into the dustbin of history!"

As Vicious enjoys her fleeting immortality, we bemoan or toast, as our taste might be, the fall of simultaneity. Nasty, trying to climb out of the dustbin, insists that he and Vicious had been sitting still, thinking hard, and it was the Swede that was moving. Since the gong had bonged, Nasty is absolutely sure that he and Vicious hit their buttons at the same instant and is entitled to half the prize, while the Swede is equally sure that Vicious hit her button before Nasty hit his.

The very notion of simultaneity depends on the observer!

Meanwhile, another Swede, also on the Committee, also intelligent, is moving by on another train described in the duelists' frame by $x = ut$. You can fill in the rest. He solemnly announced Nasty's destiny in Stockholm and Vicious's fate in the dustbin. Do you see why?

Young Einstein has bent the stately flow of time out of shape. Albert himself thought up this Gedanken experiment—I have merely added a few dramatic details—showing that the constancy of the speed of light necessarily has to alter our commonsense notion of simultaneity.

In Maxwell's electromagnetic wave, a varying electric field generates a magnetic field, and a varying magnetic field generates an electric field, with the cycle repeating indefinitely, moving the wave along. The rate at which a varying electromagnetic field generates a varying electromagnetic field has nothing to do with observers.

In theoretical physics we say, "Mind boggler in, mind boggler out!" We feed the mind-boggling fact that the speed of light does not depend on the observer into the wondrous machinery of logic and out pops another mind-boggling fact, namely, that simultaneity is, alas, no more.

The patent clerk uses a high tech clock to discover a secret about spacetime

Next, the promised third way: I sketch for you how Einstein deduced special relativity. Remarkably, of all the developments in theoretical physics since Newton, this requires the least amount of mathematical knowledge, only a tiny bit of high school algebra. Let's follow Einstein and consider a clock consisting of two mirrors separated by distance L, between which a light beam

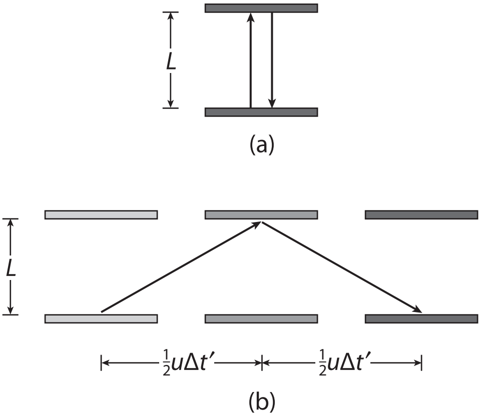

Figure 4. Einstein's clock (a) in its rest frame and (b) in a moving frame. Reproduced from A. Zee, *Einstein Gravity in a Nutshell*, Princeton University Press, 2013.

bounces back and forth. See figure 4. Einstein was, after all, a patent examiner living in a time of technological innovations[8] of all sorts, including ever-better chronometers.[9]

Ms. Unprime, sitting on a smoothly moving train, has one of these high-tech clocks with her.[10] For each tick-tock, three events occur: $A =$ light leaves the lower mirror, $B =$ light bounces off the top mirror, and $C =$ light arrives back at the lower mirror.

Let us write down the separation between events A and C in space and time. Denote these separations in space and time by $\Delta x,\ \Delta y,\ \Delta z,\ \Delta t$. (A reminder: Δ means "difference" or "the change in.") Since the pulse of light gets back to where it started, clearly $\Delta x = 0,\ \Delta y = 0,\ \Delta z = 0$, that is, no change in the spatial coordinates. By construction, $\Delta t = 2L/c$, namely the distance $2L$ traveled by light divided by its speed c.

Mr. Prime watches the train with Ms. Unprime on it moving by with speed u in the x' direction and sees a pulse of light bouncing up and down in the y' direction. What is the separation between A and C as seen by Mr. Prime?

Let's figure that out in the coordinate system he uses. Since he sees the clock moving along the x-axis, he notes that $\Delta y' = 0,\ \Delta z' = 0$, (that's what "moving along the x-axis" means), But $\Delta x'$, unlike $\Delta x = 0$, is nonzero and given by $\Delta x' = u\Delta t'$ (that's what "moving with speed u" means). In the duration $\Delta t'$, the train has traveled the distance $\Delta x'$.

But how do we determine $\Delta x'$ and $\Delta t'$ separately?

Use the fabulously astonishing equation $c = c$!

The distance traveled by the light pulse equals $c\Delta t'$. But what is $\Delta t'$? Ask Mr. Pythagoras for help! We have two right angled triangles back to back, each with right sides (figure 4(b)) with length $\frac{1}{2}u\Delta t'$ and L, and so the hypotenuse equals[11] $\sqrt{\left(\frac{1}{2}u\Delta t'\right)^2 + L^2}$. So, from tick to tock, light travels twice this distance, and hence

$$c\Delta t' = 2\sqrt{\left(\frac{1}{2}u\Delta t'\right)^2 + L^2}$$

Anybody who got a passing grade in high school algebra could solve this equation to determine $\Delta t'$ (hint: square both sides). Instead, let us follow Einstein. Noting that $\Delta x' = u\Delta t'$, we write the right hand side of this equation as $2\sqrt{\left(\frac{1}{2}\Delta x'\right)^2 + L^2}$. Squaring both sides, we obtain $(c\Delta t')^2 = 4\left[\frac{1}{4}(\Delta x')^2 + L^2\right] = (\Delta x')^2 + 4L^2$. Hence,

$$(c\Delta t')^2 - (\Delta x')^2 = 4L^2$$

But, remembering that $\Delta t = 2L/c$, we also have $(c\Delta t)^2 - (\Delta x)^2 = (c\Delta t)^2 = 4L^2$ since $\Delta x = 0$. Thus, no need to solve for $\Delta t'$. We can already see that

$$(c\Delta t')^2 - (\Delta x')^2 = (c\Delta t)^2 - (\Delta x)^2$$

even though $\Delta t' \neq \Delta t$ and $\Delta x' \neq \Delta x$. Since $\Delta y' = \Delta y$ and $\Delta z' = \Delta z$, we could also write this as $(\Delta x')^2 + (\Delta y')^2 + (\Delta z')^2 - (c\Delta t')^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - (c\Delta t)^2$.

Since this equality does not depend on u, the relative velocity between Mr. Prime and Ms. Unprime, we could imagine yet another observer named Double Prime, moving relative to Ms. Unprime with some other velocity along the x-axis. By the same reasoning, $(\Delta x'')^2 + (\Delta y'')^2 - (\Delta z'')^2 + (c\Delta t'')^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - (c\Delta t)^2$.

Conclusion: Even though different observers in uniform motion relative to each other observe different values for Δx and Δt, they all see the same value for the combination $(\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - (c\Delta t)^2$. This combination of Δx, Δy, Δz, and Δt is the same for all observers. We have discovered an invariant of relative motion, namely, a quantity that is the same for all observers in relative motion!

By this clever thought experiment, Einstein used the Pythagoras theorem for space to obtain a sort of generalized Pythagoras theorem for spacetime.

Distinction between a very good physicist and a great physicist! A very good physicist knows math (high school algebra in our case) and can solve equations (solve for $\Delta t'$ in our example) till the cows come home, but a great physicist listens to what the equations are telling him or her (that Nature likes Pythagoras theorem so much that She wants to generalize it!)

Lorentz transformation

> He meant more than all the others I have met on life's journey.
> Einstein speaking of Lorentz

Let us now find a transformation from the spacetime coordinates (t, x, y, z) of one observer to that (t', x', y', z') of another observer, such that $(\Delta x')^2 + (\Delta y')^2 + (\Delta z')^2 - (c\Delta t')^2 = (\Delta x)^2 + (\Delta y)^2 + (\Delta z)^2 - (c\Delta t)^2$.

Two trivial comments to start with. Since $\Delta y' = \Delta y$ and $\Delta z' = \Delta z$, we can forget about them and simply insist that $(\Delta x)^2 - (c\Delta t)^2$ remains unchanged under the transformation. Furthermore, in studying the separation between two events in space and time, we could take one of the events to occur at the origin, that is, at $t = 0,\ x = 0,\ y = 0,\ z = 0$. Thus, $\Delta x = x - 0 = x$, etc., and we could stop writing Δ and lessen clutter.

Here is an algebra homework problem for a bright high school student: Find the relations between (t', x') and (t, x) such that such that $x'^2 - (ct')^2 = x^2 - (ct)^2$.

Even a dull high school student could already see by eyeball that the Galilean transformation $t' = t,\ x' = x + ut$ given earlier ain't gonna cut it: $x'^2 - (ct')^2 = (x + ut)^2 - (ct)^2$, which is most certainly not equal to $x^2 - (ct)^2$. We could already see that t' cannot possibly be equal to t: universal time does not exist!

Meanwhile, the bright kid[12] turns in the answer:

$$ct' = \frac{ct + \frac{u}{c}x}{\sqrt{1 - \frac{u^2}{c^2}}} \quad \text{and} \quad x' = \frac{x + ut}{\sqrt{1 - \frac{u^2}{c^2}}}$$

(plus $y' = y,\ z' = z$, of course.) This is the celebrated Lorentz transformation.[13] You could verify[14] that this indeed satisfies $x'^2 - (ct')^2 = x^2 - (ct)^2$.

You certainly do not have to study this Lorentz transformation in detail; this is not a textbook. I merely ask you to note three points:

- In the domain of everyday experience, namely, when u is much much less than c, $\frac{u}{c}$ is approximately 0, so that $\sqrt{1 - \frac{u^2}{c^2}}$ is almost equal to 1. Hence, $ct' = ct$ and $x' = x + ut$. The Lorentz transformation reduces to the Galilean transformation given earlier, as it must.
- Since $\sqrt{1 - \frac{u^2}{c^2}}$ becomes imaginary for $u > c$, we have learned that a universal speed limit c exists. The train cannot go faster than the speed of light without all of our equations breaking down.
- Surprise, t and t' are definitely not equal! The commonsense fallacy was that we thought for sure that when one second passed for us, one second had passed for everybody else.[15]

Take home message: There is no universal clock in the universe ticking off the same universal time for everyone.

As I said earlier, Einstein's special relativity is the subject in physics that requires the least amount of mathematics to understand, nothing beyond high school algebra.

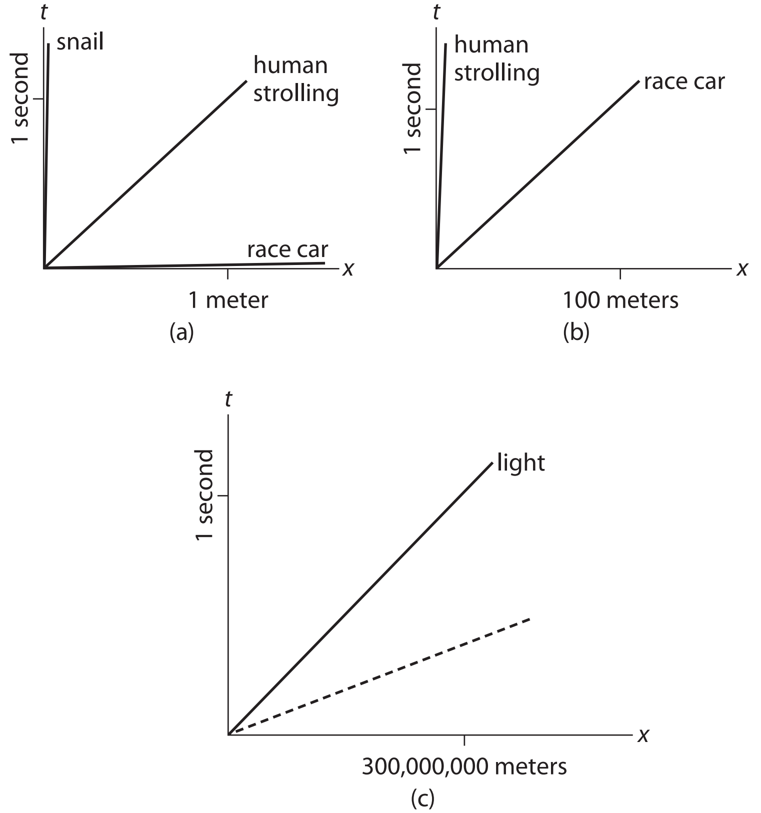
t
snail
1 second
human
strolling
race car
x
1 meter
(a)
t
human
strolling
1 second
race car
x
100 meters
(b)
t
1 second
light
x
300,000,000 meters
(c)

Figure 5. Three different spacetime diagrams, differing by the units on the horizontal axis.

Spacetime diagrams: using units with $c = 1$

One of my childhood memories was being told that in the time it takes me to say "tick tock," light would have traveled a distance equal to going around the earth seven times. For years, I tried to imagine how fast that would be. (Seven turns out to be about right: the earth's circumference is $\simeq$40,000 km, while the speed of light c is $\simeq$ 300,000 km/sec.)

Suppose we make a plot of where we are in our daily lives as time goes on. For simplicity, confine our movement to one dimension. With everyday units, meter and second, a plot might look like figure 5(a).

A human walking along at 1 m/sec corresponds to the 45° line. (In discussing special relativity, physicists plot time along the vertical axis, contrary to the everyday practice of putting time on the horizontal axis.) Note that

1 m/sec = 3,600 m/hour = 3.6 km/hour is a very leisurely stroll indeed. In this plot, an extremely fast car moving at 360 km/hour would be shown as an almost horizontal line at approximately $45°/100 = 0.45°$ from the x-axis.

Physicists call the line traced by a moving object in a spacetime diagram such as figure 5(a) a "worldline." Thus, if the worldline of a strolling human is the 45° line, then the worldline of the experimental car would be almost indistinguishable from the horizontal axis. At the other extreme, the worldline of a snail would be very close to the vertical axis. Note that the worldline of an object at rest, which is physics talk for "not moving," is just a vertical line.

Next, consider figure 5(b). For unit of distance, we now use 10^2 m. The worldline of the race car is now the 45° line, and the worldline of the strolling human is barely distinguishable from the vertical axis. (Incidentally, straight lines are drawn merely for simplicity; the worldline of an object with a varying speed would be curved.)

The expressions thus far in this chapter show that keeping c around merely adds to the clutter. Clearly, physicists living in the relativistic world would be wise to use, for the distance unit, the light second, that is, the distance light travels in one second, so that $c = 1$. Light would now be moving along the 45° line and the worldline of the race car is indistinguishable from the vertical axis, let alone lumbering enormities such as humans.

That nothing could move faster than light translates into the statement that no worldline could make an angle of less than 45° from the x-axis, as shown by the dotted line in (c).

Preview of an exciting development to come in part III. In the quantum world, worldlines of particles could have a slope of less than 45°! (In truth, yes, but not really. Stay tuned!)

Setting $c = 1$

> "An inch of time is worth an inch of gold,
> An inch of gold cannot buy an inch of time."
> Chinese adage[16]

Clearly, to describe the relativistic world, it pays to measure space and time using the same unit, that is, to set $c = 1$. The Lorentz transformation displayed earlier in this chapter simplifies to the more eye-pleasing form

$$t' = \frac{t + ux}{\sqrt{1 - u^2}}, \qquad x' = \frac{x + ut}{\sqrt{1 - u^2}}$$

The square root implies that the relative velocity u between the two observers is limited by $u^2 \leq 1$.

Compare this with the Galilean or commonsense transformation given at the beginning of this chapter. The key difference is that t' now depends on both t and x, rather than being simply equal to t. Absolute time is dead. Long live relative time!

As the Chinese adage shows, human languages tend to measure time figuratively in terms of space. In English, we say that while our past is now behind us, the future is still ahead of us, as in a queue of people. For us humans, time is more difficult to visualize than space.

Notes

[1] To see this, have the two observers each build a fence out of sticks of a specified length, pointing in the y-direction, say, and such that the two fences are of the same height. Relativity states that the two observers are not able to tell who is moving and who is at rest. Thus, the top of the two fences must continue to coincide as they glide by each other; otherwise, if one fence is seen to be taller, the other shorter, the two observers could be distinguished from each other.

[2] Hardly at rest! You may not be aware of it, so smooth is the motion, but the entire galaxy you are in is hurtling toward a neighboring galaxy at high speed.

[3] The joke is that the two discussions are the same.

[4] You know the famous riddle due to R. Smullyan? One of my favorites. “What is greater than God, and if you eat it, you die?”

[5] This section is taken from *GNut,* p. 7.

[6] I love making up names for the characters recurring in my books. Readers have told me that their favorite is Confusio and his struggle to earn tenure. Not long after writing this chapter, I was astonished to read in the *New York Times* about a certain Professor Vile. A Google search shows that he actually exists.

[7] I am referring to the fact that quarks and leptons come in three families. See chapter V.4.

[8] According to the literary scholar Dame Gillian Beer, around 1865, when Lewis Carroll, an early practitioner of photography, wrote *Alice in Wonderland*, photography “froze or made portable a moment and a place.” To me, that could have easily led to the concept of events in spacetime. Carroll was notoriously concerned with the notion of time, for example, with the white rabbit constantly consulting his pocket watch, an affectation and necessity when railways, with timetables and Einstein’s trains, came into common use. To a physicist like myself, the two Alice books are full of allusions to concepts from physics: gravity, scale transformation, and mirror reflection, to name a few.

[9] P. Galison, *Einstein’s Clocks and Poincare’s Maps: Empires of Time,* Norton, 2004.

[10] This story is adapted from chapter III.2 of *GNut*.

[11] Actually also known in several other ancient civilizations, Babylonian, Chinese, Egyptian, and so on.

[12] An even brighter kid might see that the best approach would be to recognize that we could write $x^2 - (ct)^2$ as $(x - ct)(x + ct)$.Then clearly, the desired transformation is simply to multiply $(x - ct)$ and divide $(x + ct)$ by the same arbitrary number. The Lorentz transformation derived in one line!

[13] As is often the case, the history is a bit convoluted. In 1887, when Einstein was 8 years old, the German physicist W. Voigt proposed an erroneous version of this transformation. Not knowing Voigt’s work, Lorentz derived in 1895 the transformation in a better form than Voigt’s, but still not quite in the form we now know. Then J. Larmor found the correct form in 1900. Not knowing Larmor’s work, Lorentz reproduced it in 1904. In 1905, H. Poincaré, knowing only of Lorentz’s work, developed the transformation further and named it the Lorentz transformation. As for Einstein, he only knew the 1895 version of the Lorentz transformation. The term “Lorentz transformation” is an example of the Matthew principle in theoretical physics: Whoever has will be given more . . . . Whoever does not have, even what he has will be taken from him (Matthew 13:12).

[14] Note, by elementary algebra,

$$x'^2 - (ct')^2$$

$$= \left((x + ut)^2 - \left(ct + \frac{u}{c}x\right)^2\right) / \left(1 - \frac{u^2}{c^2}\right)$$

$$= \left((x^2 + 2uxt + u^2t^2) - (c^2t^2 + 2uxt + \frac{u^2}{c^2}x^2)\right)/$$

$$\left(1-\frac{u^2}{c^2}\right)=\left(1-\frac{u^2}{c^2}\right)(x^2-c^2t^2)/\left(1-\frac{u^2}{c^2}\right)$$

$$=x^2-(ct)^2$$

[15] Thus, the commonsense addition of velocities we mentioned in the text is modified, on the application of the Lorentz transformation, to

$$v'=\frac{\Delta x'}{\Delta t'}=\frac{\Delta x+u\Delta t}{\Delta t+\frac{u}{c^2}\Delta x}=\frac{v+u}{1+\frac{uv}{c^2}}$$

The third equality follows by dividing through with Δt. For u and v much less than c, the denominator is very close to 1, and this reduces to the commonsense $v'=v+u$. But for $v=c$, we see that this gives $v'=(c+u)/(1+\frac{u}{c})=c$ also. Thus, $c=c$ regardless of observer! This is the strange addition law for velocities alluded to in the prologue.

[16] Nicole Mones, *Night in Shanghai,* 2014.

(continued...)

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