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CHAPTER 1

Introduction

Flying has become so common that we tend to take many details of flight for granted. Nevertheless, flight is a complex process, involving the equilibrium, stability, and control of a machine that is both intricate and elegant in its design. All aircraft are governed by the same rules of physics, but the details of their motions can be quite different, depending not only on the shapes, weights, and propulsion of the craft but on their structures, control systems, speed, and atmospheric environment. This book presents the flight dynamics of aircraft, with attention given to mathematical models and techniques for analysis, simulation, evaluation of flying qualities, and control system design.

Here, we introduce the basic components of configuration that are common to most aircraft (Section 1.1) and provide illustrative examples through descriptions of contemporary aircraft (Section 1.2). Notation that is used throughout the book is presented in Section 1.3, with an introductory example based on the flight of a paper airplane. Section 1.4 presents example syllabuses for first and second university courses based upon the book.

1.1 Elements of the Airplane

Aircraft configurations are designed to satisfy operational mission and functional requirements, with considerable consideration given to cost, manufacturing, reliability, and safety. All aircraft have structures that generate aerodynamic lift, mechanisms for effecting control, and internal spaces for carrying payloads. Most aircraft also have propulsion systems and associated propellant tanks, undercarriage for takeoff and/or landing, and navigation systems. The layout and interactions of all these components have major influence on an aircraft's motions (e.g., [B-1, H-1, K-1, K-3, N-1, R-1, S-1 to S-4, T-1, T-2, W-1]).

The components and propulsion systems must be adequate to perform all necessary phases of flight. In most cases, this includes taxiing at an airport, takeoff from a runway, climbout to ascent path, ascent to cruising altitude, steady flight at essentially constant or slightly increasing altitude, "top of descent," descent to landing approach, lining up with a runway, landing, rollout on a runway, and taxiing to a parking location (Fig. 1.1-1). The configuration of *fixed-wing* or *conventional-takeoff-and-landing* (CTOL) aircraft normally is unchanging throughout these phases, subject only to control-surface deflections, thrust variation, gradual loss of weight as fuel is burned, or discrete weight loss if payload is dropped. A principal exception would be an aircraft with variable-sweep wing to optimize performance in sub- and supersonic flight such as the Grumman *F-14 Tomcat*. *Vertical- or short-takeoff-and-landing* (V/STOL) aircraft may have morphing configurations (e.g., *V-22 Osprey*) to augment aerodynamic lift during initial and final flight phases.

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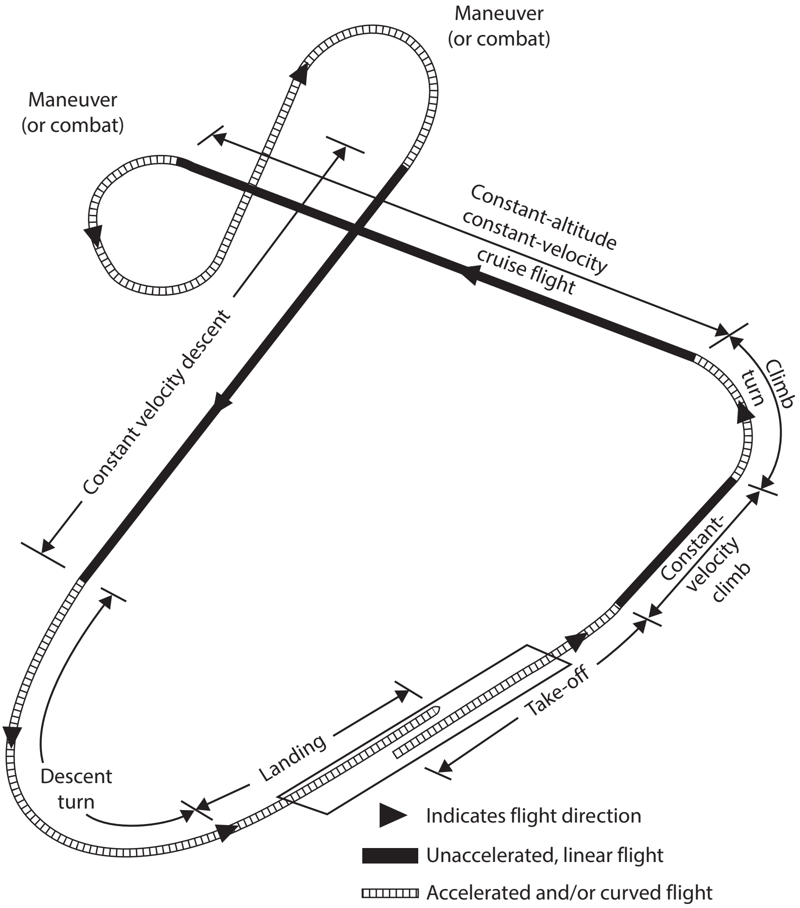

FIGURE 1.1-1. Phases of flight [T-1]. (courtesy of NASA)

Airframe Components

Consider the Cirrus *SR20* aircraft shown in Fig. 1.1-2. A single-propeller-driven fixed-wing aircraft, the *SR20* has all the elements of a conventional airplane: wing, fuselage, horizontal tail, vertical tail, and control surfaces, as well as cockpit/cabin, engine, and landing gear. The *wing* provides the aircraft's largest aerodynamic force, used principally for supporting the vehicle's weight and changing the direction of flight. The wing's fore-aft location is near the vehicle's center of mass for good balance, and the wing itself is shaped to provide a large amount of lift with as little energy-absorbing drag as possible. The *fuselage* is the aircraft's principal structure for containing payload and systems. Its slender shape provides a usable volume for the payload and a mounting base for engine and tail components, with minimum drag and weight penalty.

Collectively, the tail surfaces are called the *empennage*. Just like a weathervane, the *vertical tail* produces directional (yawing) stability about an aircraft-relative vertical axis; it gives the aircraft a tendency to nose into the *relative wind* that results from forward motion through an atmospheric wind field. Similarly, the *horizontal tail* provides pitching stability for rotations about an axis parallel to the wingspan.

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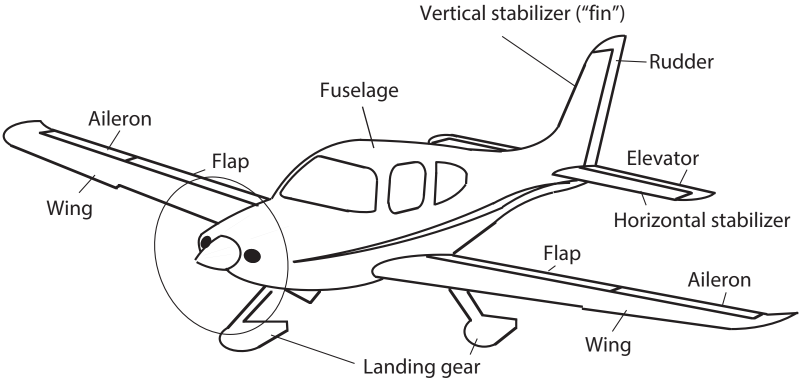

FIGURE 1.1-2. Principal components of an aircraft.

Aerodynamically, the vertical and horizontal tails are small wings whose lift acts through *moment arms* (i.e., the distance between aerodynamic centers and the center of mass) to generate restoring moments (or *torques*) that are proportional to angular perturbations. Stability about the third (rolling) axis is a complex effect, dominated by the wing's vertical location on the fuselage and the upward inclination of the wing tips relative to the wing roots (the *dihedral angle*).

The *SR20* has conventional control effectors: ailerons, elevator, rudder, flaps, and thrust setting. The *ailerons* are movable surfaces located near the wing tips that produce large rolling torques; the two surfaces are linked so that the trailing edge of one moves up when the trailing edge of the other moves down. The *elevator* is a movable surface that extends across the trailing edge of the horizontal tail for angular control. In addition to regulating the aircraft's angle of attack,[1] which, in turn, governs the amount of lift generated by the wing, the elevator deflection controls the pitch angle[2] when the wings are level, an important function during takeoff and landing. The *rudder* has similar effect for yawing motions, controlling the sideslip angle[3] for turn coordination and crosswind takeoffs and landings. It is a movable surface mounted on the trailing edge of the vertical tail.

The wing's trailing-edge *flaps* are movable surfaces mounted inboard of the ailerons for direct control of lift and drag during takeoff and landing. The left and right surfaces act in unison, deflecting downward from their cruising-flight (stowed) positions. Whereas the pilot exerts continuous control of the ailerons, elevator, and rudder, the flaps normally are adjusted to discrete settings that depend on the flight phase. Engine *thrust setting* is regulated only occasionally to achieve takeoff acceleration, climb, desired cruising conditions, descent, and landing sink rate.

As explored in later sections, other subsystems have aerodynamic and control effects. They include retractable landing gear, trim tabs, leading-edge flaps, spoilers, speed brakes, thrust reversers, engine inlet shape and bleed air, jet exhaust deflection,

---

[1] The *angle of attack* is the aircraft-relative vertical angle between the centerline and the relative wind. The *relative wind* is the velocity of the aircraft relative to the air mass through which it flies. (See Fig. 1.3-1)

[2] The *pitch angle* is the angle of the aircraft's centerline relative to the horizon. (See Fig. 1.3-1)

[3] The *sideslip angle* is the aircraft-relative horizontal angle between the centerline and the relative wind. (See Fig. 1.3-2)

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and external stores. Functions may be combined or distributed, as for tailless aircraft or those possessing redundant control surfaces.

If each of the aircraft's elements performed only the functions described above and performed them perfectly, aircraft dynamics and control could be simple topics; however, there are numerous complicating factors. The greatest is that important physical phenomena are inherently *nonlinear*. The significance of nonlinearity is revealed in the remainder of the book, but the general notion is that doubling a cause does not always double the effect. For example, the wing's lift is linearly proportional to angle of attack up to a point; then, the wing stalls and no greater lift can be achieved, even if the angle continues to increase.

A particular element (e.g., the wing) produces a primary effect (lift), but it also may produce secondary effects (drag and side forces, as well as pitching, yawing, and rolling moments). Because the components are in close proximity, the aerodynamic forces and moments that they generate are interrelated. Hence, the airstream downwash and vorticity induced by the lifting wing have major effects on tail aerodynamics, thrust changes may upset pitch or yaw equilibrium, and so on. A good deal of useful analysis can be accomplished with single-input/single-output (scalar or SISO) models, but the actual dynamic system has multiple inputs and outputs, requiring a more comprehensive multi-input/multi-output (vector or MIMO) approach.

There are challenging structural and inertial effects as well. The aircraft's structure must be lightweight for good overall performance, but it is very flexible as a result. Consequently, the wing, fuselage, and tail deflect under air loads, and the changes in shape have both static and dynamic effects on equilibrium and motion. In modern aircraft, natural frequencies of significant vibrational modes may be low enough to be excited by and to interfere with aircraft maneuvers, leading to major problems to be solved in control system design. They also contribute to poor ride quality for the passengers and crew and to reduced fatigue life of the airframe. Although most aircraft possess mirror symmetry about a body-fixed vertical plane, inertial and aerodynamic coupling of motions can occur in asymmetric maneuvers, such as steady turns and crosswind landing. Rotating components of propulsion systems produce gyroscopic torques that couple motions about the aircraft's axes. The loading of asymmetric wing-mounted stores on combat aircraft clearly presents a dynamic coupling effect in otherwise symmetric flight. Nevertheless, considerable insight can be gained by studying simplified mathematical models that ignore these complicating factors.

Propulsion Systems

All operational aircraft other than unpowered gliders and rocket-powered aerospace planes produce thrust by moving air and exhaust gasses backwards with power from combustion engines or electric motors. The principal elements of propulsion systems are *energy storage*, *energy conversion*, *power conditioning*, and *thrust generation* (Fig. 1.1-3). *Reciprocating engines* burn fuel and air cyclically in hollow cylinders; the linear motion of pistons is transformed to rotary motion by rods and crankshafts that drive propellers. *Turbine engines* produce rotary motion by the continuous action of combustion gasses flowing through finned wheels (compressors and turbines near the inlet and exit). Turbine engines may produce thrust directly from the high-velocity exhaust gasses, or they may spin propellers or fans that move large masses of air at low speed rather than small masses of air at high speed. *Electric motors* use energy gathered by solar cells, stored in batteries, or generated by fuel cells to drive

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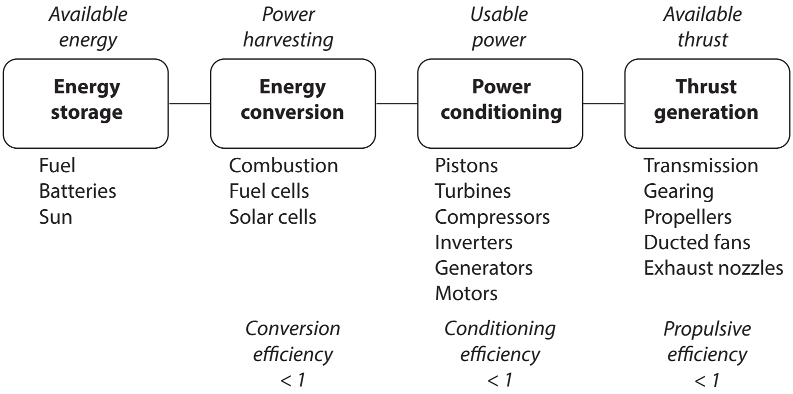

FIGURE 1.1-3. Principal elements of aircraft propulsion.

propellers or fans. Hybrid-electric powerplants convert energy from combustion engines to drive electric motors.

As airspeed increases, propeller tip speeds approach or exceed the speed of sound, absorbing unacceptable amounts of power. The *turbofan* engine overcomes this problem by enclosing a smaller-diameter fan within a streamlined duct, allowing higher aircraft speeds but reducing power conversion efficiency. The proportion of air that is accelerated by the fan compared to that which flows through the engine core is called the *bypass ratio*. A *turbojet* engine has no bypass air—all of the thrust-producing air mass passes through the combustion chambers and turbines. Turbojets were used in early supersonic[4] aircraft, where the drag losses of the fan outweighed the fan's conversion efficiency. Advances in fan design have led to supersonic aircraft with turbofan engines. *Afterburning* produces increased thrust by burning additional fuel in the oxygen-rich exhaust of a turbojet engine at the expense of reduced combustion efficiency.

The *ramjet* has neither compressor nor turbine; its inlet provides the necessary compression by forward motion through the atmosphere. A supersonic-combustion ramjet (*scramjet*) operates on the same principle with accelerated flow through the combustion chamber. The *pulsejet* relies on a tuned flapper valve to periodically open and close the forward end of the combustion chamber. Air and fuel are sucked into the chamber between resonant deflagration pulses. Flow through the pulsejet is subsonic, and significant static thrust can be generated.

The *rocket* motor can produce thrust at any Mach number and in a complete vacuum. Moreover, its structural weight per unit of thrust is less than that of air-breathing engines. The principal penalty is that the oxidizer as well as the fuel must be carried onboard the aircraft, increasing the size and weight of the vehicle. Rocket motors are useful for launch to orbit, but they have restricted roles to play in atmospheric flight, such as in short-term thrust augmentation for takeoff, maneuvering, or braking.

---

[4] *Mach number*, M, is the ratio of airspeed V to the speed of sound a in the surrounding air. The flow is *supersonic* when M is greater than one, and it is *subsonic* when M is less than one. Air moves faster or slower than the *freestream airspeed* over different parts of the aircraft. The *transonic region* begins at the freestream Mach number for which the flow reaches sonic speed over some part of the aircraft and ends when the entire flow is supersonic, typically for $0.7 < M < 1.4$.

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1.2 Representative Aircraft

Attributes of several aircraft types are presented in preparation for the more general study of flight dynamics and control. The principal objectives are to indicate the effect that flight conditions and missions have on aircraft configurations and, conversely, to suggest the latitude for motions and control afforded by existing designs. The order of presentation implies increasing performance, as represented by operating speed, altitude, and maneuverability. The descriptions are approximate; more comprehensive descriptions for most aircraft types can be found on the internet (e.g., *Wikipedia*, https://en.wikipedia.org/) and, of course, in the manufacturers' proprietary documentation.

Light General Aviation Aircraft

The Cirrus *SR20* (Fig. 1.2-1) is a 4-place aircraft with a 149-kW (200-hp) reciprocating engine. The wingspan is 10.8 m, the length is 7.9 m, and the wing area is 12.6 m^2. The empty and maximum-takeoff masses are 885 and 1,315 kg; hence, the maximum wing loading[5] is 105 kg/m^2. The cruising airspeed[6] is 296 km/hr [184 statute miles per hour (mph) or 160 nautical miles per hour (*knots*, kt)] at 75-percent power and 1,980-m (6,500-ft) altitude, and its range is 1,480 km (920 mi). Maximum Mach number is 0.24. The *SR20*'s straight wing has a moderate aspect ratio[7] (9.1) for efficient subsonic cruise, and it is mounted low on the fuselage with 5-deg dihedral angle. The horizontal tail also is mounted low, and the vertical tail is swept, with a full-length rudder whose deflection is unrestricted by the horizontal tail location. The principal airframe material is a structural composite, and the structure uses semi-monocoque construction (i.e., the skin carries a significant portion of the loads), with surfaces bonded to spars and ribs. A recovery parachute that can bring the entire aircraft safely to the ground is standard equipment.

Uninhabited Air Vehicle

The *Zip 2 drone* (Fig. 1.2-2) provides much needed medical-supply delivery in central Africa. As shown, the 21-kg plane flies low over the rural destination and drops its 1.75-kg payload at a local clinic; the payload has a small parachute to slow the speed for a safe landing. The uninhabited air vehicle (UAV) then returns to its base. The drone has a straight wing inboard with a single taper angle on the outboard and a broad fuselage that is connected to the V tail by a slender beam. It is powered by two coaxial 1.74-hp electric motors mounted above the fuselage. The wingspan is 3.3 m, and the wing area is 0.775 m^2, giving it an aspect ratio of 14.04. The *Zip 2*'s maximum speed is 126 km/hr (68 kt), and it can fly a round trip of 160 km.

---

[5] *Wing loading* is defined in Système International (SI) Units as aircraft mass (kg) divided by wing area (m^2). Using U.S. Customary Units, it is defined as weight (lb) divided by wing area (ft^2).

[6] *Airspeed* is the magnitude of the aircraft's velocity relative to the surrounding atmosphere, i.e., of the *relative wind*. If the wind is blowing, airspeed and ground speed are not the same.

[7] Wing *aspect ratio* is defined as the square of the span divided by the wing area.

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FIGURE 1.2-1. Cirrus *SR20* Airplane. (courtesy of Cirrus Aircraft Co.)

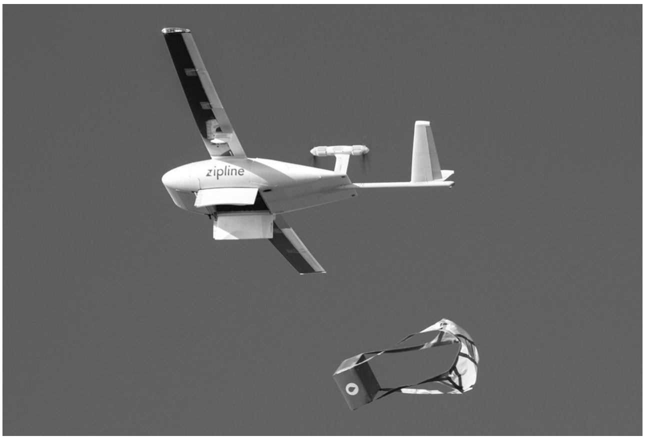

FIGURE 1.2-2. Zipline *Zip 2* Drone. (courtesy of Zipline)

Variable-Stability Research Aircraft

The Princeton *Variable-Response Research Aircraft* (*VRA*) is a highly modified single-engine North American *Navion A* airplane designed for research on flight dynamics, flying qualities, and control (Fig. 1.2-3). The most distinguishing feature of the *VRA* is the pair of vertical side-force-generating surfaces mounted midway between wing roots and tips, and it has fast-acting wing flaps that produce positive and negative lift. The *VRA* is the first general aviation aircraft to be equipped with a *digital fly-by-wire* (DFBW) control system. The system parallels the standard *Navion*'s mechanical control system. In operation, a safety pilot/test conductor has direct control of the aircraft through the mechanical system, while the test subject controls the aircraft through the experimental electronic system.

Although limited to airspeeds below 200 km/hr (105 kt), the *VRA* can simulate the perturbational motions of other aircraft types through independent, closed-loop control of all the forces and moments acting on the airplane. Feedback control of

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FIGURE 1.2-3. Princeton *Variable-Response Research Aircraft* (*VRA*). (courtesy of Princeton University)

motion variables to the control surfaces allows the natural frequencies, damping ratios, and time constants of the *Navion* airframe to be shifted to values representative of other airplanes, while the direct-force surfaces (side force and lift) provide realistic lateral and longitudinal accelerations in the cockpit. The *VRA* and a second *Navion*, the *Avionics Research Aircraft* (without side-force panels), are currently owned and operated by Flight Level Engineering in cooperation with Embry-Riddle Aeronautical University.

Sailplane

The Schleicher *ASG 32* (Fig. 1.2-4) is a two-place glider with a wingspan of 20 m, an empty mass of 545 kg, and a maximum takeoff mass of 850 kg. With an aspect ratio of 25.5, a wing area of 15.7 m^2, and winglets, the aircraft's maximum glide ratio is 52; its maximum speed is 270 km/hr. The airplane can be equipped with a retractable 55-hp Wankel engine and propeller for self-launching (i.e., without the need of a tow plane). Alternatively, it can be equipped with a 34-hp electric motor and propeller that can extend flight range following a towed launch. The *ASG 32* primary structure is carbon-fiber-reinforced plastic. The mechanical flight control system is conventional, with all-moving horizontal tail, ailerons, rudder, flaps, and speed brakes (or "spoilers"). An *ASG 32* won the 2018 World Gliding Championships in the 20-m class.

Business Jet Aircraft

The Honda *HA-420 HondaJet* is representative of many small executive jet transports (Fig. 1.2-5). It has a "T" tail, two 9.1-kN (2,050-lb)-thrust turbofan engines in separate nacelles mounted on pylons above the wings,[8] and a wing with 8-deg leading-edge sweep angle. Wing mounting of the engines provides several advantages in this aircraft class, including good ground clearance, reduced cabin noise, and low contri-

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[8] A *nacelle* is a streamlined fairing over an engine.

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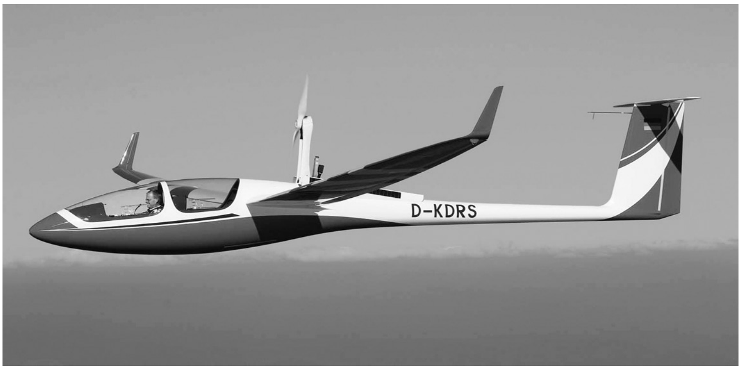

FIGURE 1.2-4. Schleicher *ASG 32* Motor Glider. (courtesy of Alexander Schleicher Segelflugzeugbau)

FIGURE 1.2-5. Honda *HA-420 HondaJet*. (courtesy of Honda)

bution to drag. The *HA-420* carries one or two crew members and up to six passengers in its 11.5-m-long fuselage. The wingspan, area, and aspect ratio are 12.1 m, 15.6 m^2, and 9.4, while the minimum and maximum masses are 3,267 and 4,854 kg. Economical cruise speed at 9,144-m (30,000-ft) altitude is 782 km/hr (422 kt), and the maximum Mach number is 0.72. With a full fuel load, maximum range of the *HondaJet* is 2,660 km (1,435 mi).

Turboprop Commuter Aircraft

The *ATR 72* (Fig. 1.2-6) exemplifies the many twin-turboprop designs used by regional airlines to move dozens of passengers on short trips. The *ATR 72* has a "T" tail and a high, straight wing with span, area, and aspect ratio of 27.05 m, 61 m^2, and 12, respectively. The craft is 27.2-m long, and it carries 70 passengers plus a crew of two; its empty and full masses are 13,311 and 23,000 kg. Typical cruise speed is 510 km/hr (275 kt), corresponding to $\mathcal{M} = 0.36$. The maximum operating altitude is 7,620 m (25,000 ft), and the maximum range is 1,715 km (1,065 mi). Eighty percent of the *ATR 72* structure is aluminum, and the remainder is composite. Each engine produces 1,846 kW (2,475 hp). The aircraft has an active noise and vibration suppression system for quieting the passenger cabin.

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FIGURE 1.2-6. *ATR 72* Turboprop Regional Airliner. (courtesy of ATR)

FIGURE 1.2-7. Embraer *E-190* Jet Transport Aircraft. (courtesy of Embraer)

Small Jet Transport Aircraft

With four-across seating, the 100-passenger Embraer *E-190* (Fig. 1.2-7) is representative of small narrow-body commercial jet transports. First flown in 2004, it has an uncomplicated swept-wing design with two engines mounted in nacelles beneath the wings. The configuration can be characterized as a wing, tubular fuselage, and empennage. Most *E-190*s are fitted with winglets for reduced drag and improved fuel efficiency. The wingspan and fuselage length are 28.7 m and 36.2 m, while the wing area and aspect ratio are 92.5 m^2 and 8.9. Each of the *E-190*'s turbofan engines produces 89 kN (20,000 lb) of maximum thrust. The aircraft has a digital flight deck, fly-by-wire pitch, yaw, and engine controls, and mechanical roll control. The best cruising Mach number is 0.78, and the maximum altitude is 12,000 m. Maximum range with 107 passengers is over 2,450 km.

Medium Jet Transport Aircraft

The Airbus *A320neo* (Fig. 1.2-8) is an outgrowth of the original *A320*, first flown in 1987. The *A320* was the first commercial jet to use a DFBW control system and sidearm hand controllers in the cockpit, and it was the first to use composite materi-

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FIGURE 1.2-8. Airbus *A320neo* Jet Transport Aircraft. (courtesy of Airbus)

als for primary structures. Like its predecessors, the *A320neo* has an advanced technology wing, winglets at its wing tips, and a high degree of redundancy in flight control. Hydraulically powered flight control surfaces include spoilers for roll control and speed braking, leading-edge slats for increased lift at higher angles of attack, and conventional surfaces for roll, pitch, and yaw control. There are mechanical backups for rudder control and pitch trim. The single-aisle *A320neo* carries up to 195 passengers, depending on configuration. The operating empty mass is 85,900 kg, and normal maximum takeoff mass is 79,000 kg. The wingspan and fuselage length are 35.8 m and 37.6 m, while the wing area, sweep angle, and aspect ratio are 122.4 m^2, 25 deg, and 10.5. Each of the *A320neo*'s engines produces 120.6 kN (27,120 lb) of maximum thrust. The optimal cruising Mach number is 0.78, and the maximum range is 6,300 km.

Large Jet Transport Aircraft

At 76.7 m, the Boeing *777-9* is the longest twin-engine aircraft built to date. It has a maximum takeoff mass of over 351,500 kg, and an empty mass of 181,400 kg (Fig. 1.2-9). It is powered by two 470-kN (105,000-lb)-thrust turbofan engines mounted in nacelles under the wing. With 414 passengers, the aircraft has a range of over 13,940 km (7,525 nm), assuming flight at the economic cruising Mach number of 0.84. The service ceiling is 13,135 m (43,100 ft). The *777-9* has wingtips that fold upwards on the ground to reduce the wingspan at airport gates. Fully extended, the span is 71.8 m (64.8 m when folded), giving an aspect ratio of 9.1 and an area of 516.7 m^2. The span of the horizontal tail is more than double the wingspan of the Cirrus *SR20*.

First flown in 1994, the "*Triple 7*" is certified to be flown by a two-person crew. The wing, mounted low on the fuselage, has a leading-edge sweep angle of 31.5 deg. Like other large aircraft with long, flexible wings, the *777* has two sets of roll controls: ailerons near the wing tips for low-speed roll control and mid-wing flaperons

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FIGURE 1.2-9. Boeing *777-9* Jet Transport. (courtesy of Boeing)

plus spoilers for high-speed roll control with reduced wing-bending moments. The *777* was the first U.S. commercial transport to use DFBW controls, with a triply redundant system; each of these strings uses a different microprocessor chip set and is internally triply redundant to minimize the impact of hardware or software failures. While control signaling is electronic, control surface actuators are powered hydraulically. The *777-9* uses a greater percentage of carbon fiber–composite material than in previous models.

Fighter/Attack Aircraft

The Lockheed-Martin *F-35 Lightning II* is a single-engine, single-seat, high-performance aircraft intended for ground attack and air-to-air combat (Fig. 1.2-10). It is designed for low observability ("stealth") as well as high performance. There are three variants with 80–90-percent commonality: a multi-role conventional aircraft (F-35A, Fig. 1.2-10), a short-takeoff- and- vertical- landing (STOVL) version (F-35B), and a carrier-based version (F-35C). The main turbofan engine for all variants produces an unaugmented maximum thrust of about 120 kN (28,000 lb) and 190 kN (43,000 lb) with afterburner. The *F-35A* is 15.4-m long, and it has an 11-m wingspan. The empty mass is 10,000 kg, while the maximum takeoff mass is 31,750 kg. Given its wing area of 43 m^2, the aspect ratio is 2.7. The *F-35*'s maximum Mach number is 1.6, although its typical maneuvering speed is in the transonic range below $\mathcal{M} = 1$. Dimensions of the *F-35B* are similar to those of the *F-35A*, though lift-fan engine and attendant structure increase the empty weight. The *F-35C* has a larger wing that folds for storage on an aircraft carrier, as well as larger stabilizing and control surfaces, strengthened landing gear, increased fuel capacity, and a tail hook for arrested landing. Distinguishing features include twin vertical tails, side-mounted air inlets, and trapezoidal wing and horizontal tail.

Jet Trainer Aircraft

The Boeing/SAAB *T-7A Red Hawk* jet trainer, the production version of the USAF *T-X* prototype (Fig. 1.2-11), is currently under development. The general configuration of the *T-X* is similar to that of the Boeing *F/A-18 E/F*, with a length of 14 m, wing area of 28 m^2, and wing span of 10 m. It is powered by a single General Electric *F404* afterburning turbofan engine with dry thrust of 49 kN (11,000 lbf).

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FIGURE 1.2-10. Lockheed Martin *F-35A Lightning II* Aircraft. (courtesy of Lockheed Martin)

FIGURE 1.2-11. USAF *T-X* Prototype Jet Trainer. (courtesy of Boeing/SAAB)

Hybrid Wing Body Aircraft

Aircraft with bodies that smoothly blend into the wing have less *wetted area* (i.e., total surface area) than "wing-barrel fuselage" configurations with the same total volume, reducing skin friction drag. The fuselage and engine nacelles of the Mach 3.3 Lockheed Martin *SR-71* (Fig. 1.2-12) were blended into the wing not only to reduce skin friction but to reduce shock wave interactions in supersonic flight. Last flown by the National Aeronautics and Space Administration (NASA) in 1999, the aircraft had length, wingspan, and wing area of 32.7 m, 16.9 m, and 170 m^2. Maximum takeoff and empty masses were 78,020 and 30,620 kg. Each afterburning turbojet engine produced a maximum thrust of 151 kN. The service ceiling was 26,000 m.

The Northrop Grumman *B-2 Spirit* (Fig. 1.2-13) is a high-subsonic hybrid wing-body bomber aircraft with an unrefueled range of 11,500 km and a payload of 18,000 kg. The angular planform, with straight edges and sharp corners, and the

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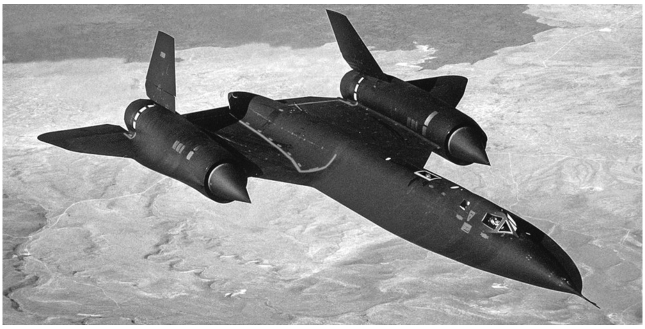

FIGURE 1.2-12. Lockheed Martin *SR-71 Blackbird* reconnaissance aircraft. (courtesy of Lockheed Martin)

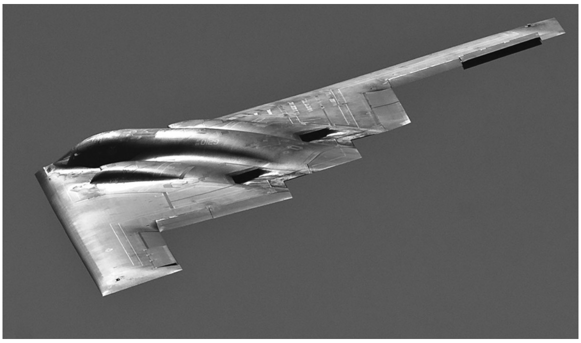

FIGURE 1.2-13. Northrop Grumman *B-2 Spirit* Bomber Aircraft. (courtesy of Northrop Grumman)

lack of a vertical tail are beneficial characteristics for low observability, although the latter provides a unique flight control challenge: providing adequate yaw stability and control. An active system using deflection of outboard "drag rudders" achieves this goal. There are four control surfaces on each wing, with elevator, aileron, and rudder functions achieved by blending commands to these surfaces. The aircraft's wingspan, wing area, and length are 52.4 m, 465 m^2, and 21 m. The *B-2* is powered by four 77-kN (17,300-lb)-thrust engines.

NASA and several aircraft manufacturers have studied a variety of hybrid transport aircraft concepts, including an electrically driven model that blends the fans into the aft body area (Fig. 1.2-14). Passengers would have a wide theater-style cabin, and the body itself would contribute to overall lift in cruising flight.

Supersonic and Hypersonic Transport Aircraft

The Aérospatiale/British Aerospace *Concorde* ceased operation in 2003 (Fig. 1.2-15). Maximum Mach number was 2.2, and typical cruising speed was 2,180 km/hr (1,350 mph). The cruising altitude was over 15,000 m (50,000 ft). Supersonic cruising

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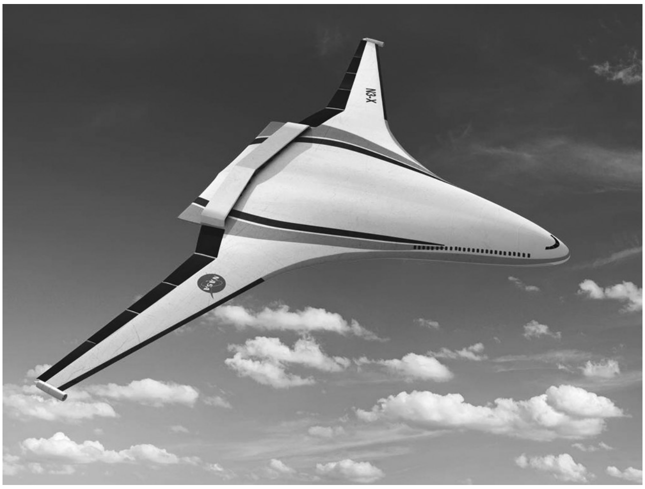

FIGURE 1.2-14. Hybrid Wing Body Transport Concept. (courtesy of NASA)

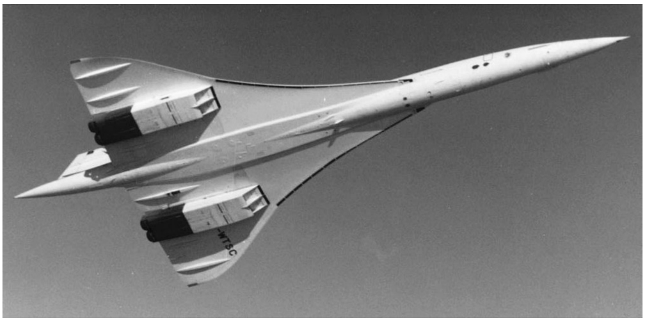

FIGURE 1.2-15. Aérospatiale/British Aerospace *Concorde* Commercial Airliner. (courtesy of Aérospatiale/British Aerospace)

range with a 10,100-kg payload and reserve fuel was 6,300 km (3,900 mi). *Concorde* was powered by four afterburning turbojet engines, each generating 169 kN (38,050 lb) of thrust with 17-percent afterburning for cruise. Afterburning was available for takeoff, acceleration through the transonic region, and supersonic cruise. The aircraft was 61.7-m long, had a wingspan of 25.6 m, and a wing area of 358.3 m^2. Empty and maximum takeoff masses were 79,265 kg and 181,435 kg. *Concorde* had a significant sonic boom, which disallowed it from flying supersonically over land.

There is renewed interest in developing supersonic transports. The proposed Boom Supersonic *Overture* (cover illustration) would be smaller than *Concorde* and fly at Mach 1.7. Other designs reduce shock-wave overpressure at ground level by extending and shaping the slender nose (Fig. 1.2–16).

Hypersonic transport aircraft represent an increased technological challenge. In one concept (Fig. 1.2-17), the cruise Mach number would be 5 and cruising altitude would be 29 km (95,000 ft). Such an aircraft would use turboramjet engines,

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FIGURE 1.2-16. Low-Boom Supersonic Transport Concept. (courtesy of NASA)

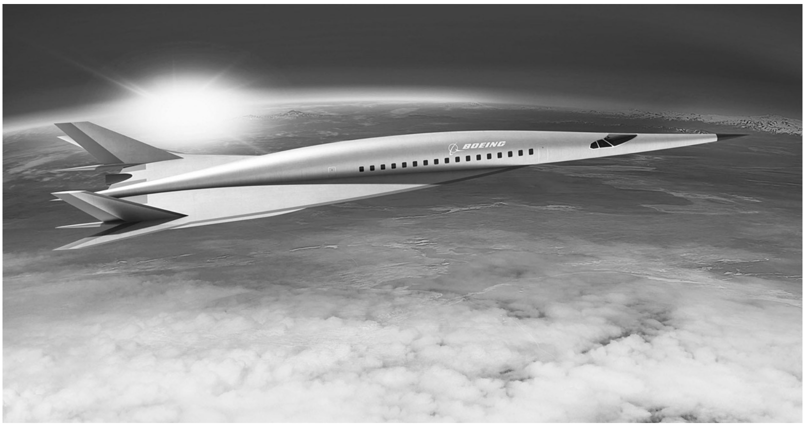

FIGURE 1.2-17. Hypersonic Transport Concept. (courtesy of Boeing)

compound designs that perform as turbofans at low speed and ramjets at high speed. Somewhat ironically, a hypersonic transport might produce a lower sonic boom overpressure than an SST at the earth's surface due to its higher cruising altitude; however, the boom still could be a problem during supersonic climb and descent.

Lifting Reentry Spacecraft

The Boeing *X-37B* is an uncrewed *orbital test vehicle* (OTV) that transitions from spacecraft to aircraft on returning from orbit to earth. It reenters the atmosphere at $\mathcal{M}=25$ and high angle of attack to dissipate heat efficiently without damaging the structure and then glides to a horizontal landing (Fig. 1.2-18). The OTV's planform and aerodynamic control surfaces are functionally similar to those of the Space Shuttle, with the addition of a "V" tail that assists pitching and yawing stability at low angles of attack. Like the Shuttle, it has high drag and low lift-to-drag ratio (see Fig. 1.3-4), thus gliding at a steep flight path angle. It is 8.9-m long, and it has a wingspan of 4.55 m^2. Its launch mass is about 4,990 kg.

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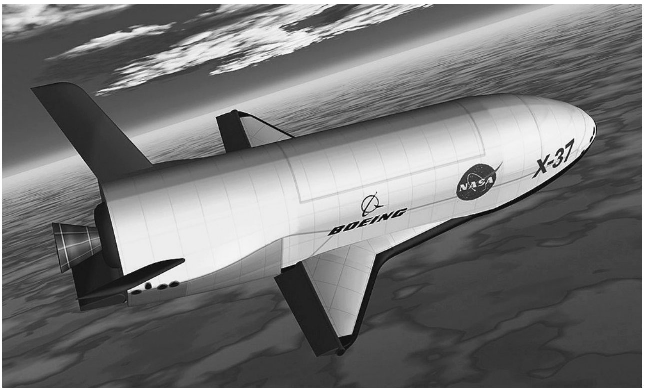

FIGURE 1.2-18. NASA/Boeing *X-37B* Orbital Test Vehicle. (courtesy of NASA)

1.3 The Mechanics of Flight

Aircraft flight is described by the principles of *classical mechanics* [G-1]. We introduce qualitative notions of mechanics in this section and provide important details in ensuing chapters. *Mechanics* deals with the motions of *objects* that possess a substantive scalar inertial property called *mass m* that expresses resistance to acceleration produced by an external *force*. Objects may be modeled as individual, infinitesimal *particles* (also called *point masses*) or assemblages of particles called *bodies*. The translational motions of point masses and bodies are of interest, as both occupy positions in three-dimensional space (r_1, r_2, r_3) and may possess three linear velocity components relative to some *frame of reference*. Orthogonal scalar position and velocity components (v_1, v_2, v_3) can be combined in three-dimensional column vectors[9] as

$$\mathbf{r} \triangleq \begin{bmatrix} r_1 \\ r_2 \\ r_3 \end{bmatrix}; \quad \mathbf{v} \triangleq \begin{bmatrix} v_1 \\ v_2 \\ v_3 \end{bmatrix} \qquad (1.3\text{-}1, 1.3\text{-}2)$$

(See Scalars, Vectors, and Matrices for additional details.) The combination of position and velocity is referred to as the six-dimensional *dynamic state* of the particle $\mathbf{x}_{particle}$:

$$\mathbf{x}_{particle} = [\mathbf{r}^T \quad \mathbf{v}^T]^T \qquad (1.3\text{-}3)$$

The product of an object's mass and velocity $m\mathbf{v}$ is called its *translational momentum*, also a three-dimensional vector. Three-dimensional *forces* **f** may act on an object to change its translational momentum, which otherwise remains constant relative to an *inertial* (*absolute* or *un-accelerated*) frame of reference.

---

[9] A *vector* such as **v** is an *ordered set* of *n* scalar quantities, that is, components that appear in a defined order. By convention, the default array is a *column* of these quantities. The *transpose* of a vector $\mathbf{v}^T$ is a *row array* of the same quantities; hence, with $n = 3$, $\mathbf{v}^T = [v_1 \quad v_2 \quad v_3]$. The symbol "$\triangleq$" means "defined as."

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Unlike point masses, *rigid bodies* have three-dimensional shape and volume; the *position* of a body is defined by the coordinates of a particular reference point on or in the body, such as its *center of mass* (*c.m.*, the "balance point"). Translational velocity of the body refers to the velocity of the *c.m.* with respect to the inertial frame. The angular orientation and rotational motion of a body also are important descriptors of its physical state. Instantaneous angular orientation $\boldsymbol{\Theta}$ can be portrayed by angles between the body's coordinate frame and the inertial frame of reference. Subject to complexities that are explained in Chapters 2 and 3, it suffices here to describe orientation as *yawing*, *pitching*, and *rolling angles*, angles that one might experience on a ship in stormy seas. The three components of *angular velocity* $\boldsymbol{\omega}$ convey the body's rotational rate about the *c.m.* in the inertial frame.

Angular momentum, the rotational equivalent of translational momentum, remains unchanged unless a *moment* (a force that is displaced from the *c.m.* by a distance called the *moment arm* and that is perpendicular to the moment arm) acts on the body. A body can be characterized by six rotational inertial properties called *moments* and *products of inertia*. The former three portray the direct relationship between angular rate and momentum about a rotational axis, while the latter three establish coupling effects between axes.

Together with the translational state, the rotational state defines the *dynamic state of the rigid body* $\mathbf{x}_{body}$, a 12-dimensional vector

$$\mathbf{x}_{body} = [\mathbf{r}^T \quad \mathbf{v}^T \quad \boldsymbol{\Theta}^T \quad \boldsymbol{\omega}^T]^T \tag{1.3-4}$$

Subsequently, we shall define lower- and higher-dimension state vectors with redefined translational and rotational components that address reduced-order problems, computational efficiency, and flexible-body motions.

Scalars, Vectors, and Matrices

Scalar: Usually lowercase italic font. Common arithmetic applies for real-valued scalars.

$$a=12; \quad b=7; \quad c=a+b=19; \quad x=a+b^2=12+49=61$$

Vector: Usually lowercase bold font. The default form of a vector is an $(n\times 1)$ column of scalars.

$$\mathbf{a}=\begin{bmatrix}2\\-7\\16\end{bmatrix}; \quad \mathbf{x}=\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}; \quad \mathbf{y}=\begin{bmatrix}a\\b\\c\\d\end{bmatrix}$$

Vector Transpose: Elements of the column vector are expressed as a $(1\times n)$ row of scalars.

$$\mathbf{x}^T=[x_1 \quad x_2 \quad x_3]$$
$$\mathbf{x}=[x_1 \quad x_2 \quad x_3]^T$$

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Vector Addition: Vectors must be of the same dimensions and are added term-by-term.

$$\mathbf{x} = \begin{bmatrix} a \\ b \end{bmatrix}; \quad \mathbf{z} = \begin{bmatrix} c \\ d \end{bmatrix}$$

$$\mathbf{x} + \mathbf{z} = \begin{bmatrix} a + c \\ b + d \end{bmatrix}$$

Multiplication by a scalar: The product is associative, commutative, and distributive.

$$a\mathbf{x} = \mathbf{x}a = \begin{bmatrix} ax_1 \\ ax_2 \\ ax_3 \end{bmatrix}$$

$$a(\mathbf{x} + \mathbf{y}) = (\mathbf{x} + \mathbf{y})\,a = (a\mathbf{x} + a\mathbf{y})$$

$$a\mathbf{x}^T = [ax_1 \quad ax_2 \quad ax_3]$$

Scalar (or Dot, or Inner) Product: The transpose of one vector multiplied term-by-term by a *conformable* column vector (i.e., of same dimension).

$$\mathbf{a}^T\mathbf{b} = \mathbf{a} \cdot \mathbf{b} = [a_1 \quad a_2 \quad a_3] \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = a_1 b_1 + a_2 b_2 + a_3 b_3$$

$$\mathbf{x}^T\mathbf{x} = \mathbf{x} \cdot \mathbf{x} = [x_1 \quad x_2 \quad x_3] \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = x_1^2 + x_2^2 + x_3^2$$

Vector Length (or Magnitude): A vector's length is the *absolute value* of the square root of vector's scalar product.

$$\text{abs}(\mathbf{x}) = |\mathbf{x}| = |\mathbf{x}^T\mathbf{x}| = (x_1^2 + x_2^2 + x_3^2)^{1/2}$$

Outer Product: The *outer product* is a column vector multiplied term-by-term by a conformable row vector, producing a matrix.

$$\mathbf{ab}^T = \mathbf{a} \otimes \mathbf{b} = \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} [b_1 \quad b_2 \quad b_3] = \begin{bmatrix} a_1 b_1 & a_1 b_2 & a_1 b_3 \\ a_2 b_1 & a_2 b_2 & a_2 b_3 \\ a_3 b_1 & a_3 b_2 & a_3 b_3 \end{bmatrix}$$

The sum of the diagonal elements is the inner product.

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Matrix: Usually uppercase bold font. A matrix is an $(m \times n)$ array of scalars.

$$\mathbf{A} = \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & k \\ l & m & n \end{bmatrix}$$

Matrix Transpose: The rows and columns of the original $(m \times n)$ matrix are interchanged to produce an $(n \times m)$ array of scalars.

$$\mathbf{A}^T = \begin{bmatrix} a & d & g & l \\ b & e & h & m \\ c & f & k & n \end{bmatrix}$$

Matrix-Vector Product: The rows of an $(m \times n)$ matrix multiply elements of a conformable $(n \times 1)$ column vector, and scalar products are summed. The product is an $(m \times 1)$ vector.

$$\mathbf{y} = \mathbf{A}\mathbf{x} = \begin{bmatrix} 2 & 4 & 6 \\ 3 & -5 & 7 \\ 4 & 1 & 8 \\ -9 & -6 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} (2x_1 + 4x_2 + 6x_3) \\ (3x_1 - 5x_2 + 7x_3) \\ (4x_1 + x_2 + 8x_3) \\ (-9x_1 - 6x_2 - 3x_3) \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix}$$

Determinant of a Square Matrix: The *determinant* of a square $(n \times n)$ matrix is a scalar metric that is formed by products of diagonal elements; it is most readily understood by example:

$$\mathbf{A} \triangleq \begin{bmatrix} a & b \\ c & d \end{bmatrix}; \quad \det(\mathbf{A}) = |\mathbf{A}| = ad - bc$$

$$\mathbf{A} \triangleq \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & i \end{bmatrix}; \quad \det(\mathbf{A}) = (aei + bfg + cdh) - (afh + bdi + ceg)$$

Note that

$$(aei + bfg + cdh) - (afh + bdi + ceg) = a \begin{vmatrix} e & f \\ h & i \end{vmatrix} - b \begin{vmatrix} d & f \\ g & i \end{vmatrix} + c \begin{vmatrix} d & e \\ g & h \end{vmatrix}$$

where the (2×2) determinants are *minors* of the (3×3) determinant; hence, determinants of higher-dimensioned matrices can be formed by recursive expansion in their minors.

Vector Transformation and Inverse: Two $(n \times 1)$ vectors may be related by a square $(n \times n)$ *transformation matrix* such that

$$\mathbf{x}_2 = \mathbf{A}\mathbf{x}_1$$

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If **A** is a nontrivial matrix, and $\det(\mathbf{A}) \neq 0$, $\mathbf{A}^{-1}$ exists and is the inverse of **A**:

$$\mathbf{x}_1 = \mathbf{A}^{-1}\mathbf{x}_2$$

Methods for computing the inverse are discussed in following sections.

Identity Matrix: The *identity matrix* is a square $(n \times n)$ matrix. Its main diagonal elements are one, and its off-diagonal elements are zero.

$$\mathbf{I} = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & \cdots & 1 \end{bmatrix}$$

Given the square matrix **B**, if $\mathbf{B}^{-1}$ exists, then

$$\mathbf{B}^{-1}\mathbf{B} = \mathbf{B}\mathbf{B}^{-1} = \mathbf{I}$$

Mechanics is further broken down into kinematics, statics, dynamics, stability, and control. *Kinematics* is the general description of an object's motions without regard to the forces or torques that may induce change; thus, the geometry and coupling of position and velocity (both translational and rotational) are considered, while the means of effecting change are not. *Statics* addresses the balance of forces and torques with inertial effects to produce equilibrium. An aircraft can achieve static equilibrium as long as neither translational nor angular momentum are changing (e.g., cruising flight); for constant mass and rotational inertial characteristics, this implies unaccelerated flight. *Dynamics* deals with accelerated flight, when momentum is changing with time. While it is possible to achieve a steady, dynamic equilibrium (as in constant-speed turning flight), the more usual dynamics problem concerns continually varying motions in response to a variety of conditions, such as nonequilibrium initial conditions, disturbance inputs, or commanded forces and torques produced by *control effectors*.

Kinematics is well described by algebraic and trigonometric equations. Because many scalar variables often must be addressed simultaneously, matrix-vector notation is helpful for kinematic and other problems. Suppose **x** is a vector with three components $[x_1\ x_2\ x_3]^T$ and **y** is a vector with two components $[y_1\ y_2]^T$ such that

$$y_1 = x_1 + \sin x_2 + \cos x_3 \tag{1.3-5}$$

$$y_2 = \cos x_1 + \sin x_3 \tag{1.3-6}$$

The two equations can be replaced symbolically by the single vector equation

$$\mathbf{y} = \mathbf{f}(\mathbf{x}) = \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} x_1 + \sin x_2 + \cos x_3 \\ \cos x_1 + \sin x_3 \end{bmatrix} \tag{1.3-7}$$

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where $\mathbf{f}(\mathbf{x})$ is a two-component vector $[f_1(\mathbf{x})\ f_2(\mathbf{x})]^T$ comprising the right sides of eq. 1.3-5 and 1.3-6. As a second example, let

$$y_1 = ax_1 + bx_2 + cx_3 \tag{1.3-8}$$

$$y_2 = dx_1 + ex_2 + fx_3 \tag{1.3-9}$$

The two scalar equations are represented by the matrix-vector equation

$$\mathbf{y} \triangleq \begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \mathbf{H}\mathbf{x} = \begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} ax_1 + bx_2 + cx_3 \\ dx_1 + ex_2 + fx_3 \end{bmatrix} \tag{1.3-10}$$

Dynamic equations (or *equations of motion*) for particles and bodies are expressed as *ordinary differential equations*, which have a single independent variable (in this case, time t). In *state-space notation*, the scalar equations of motion are assembled in a single vector equation for the state $\mathbf{x}(t)$,

$$\frac{d\mathbf{x}(t)}{dt} \triangleq \dot{\mathbf{x}}(t) = \mathbf{f}[\mathbf{x}(t), \mathbf{u}(t), \mathbf{w}(t)] \tag{1.3-11}$$

where $\mathbf{f}[\cdot]$ is the vector of dynamic functions, $\mathbf{u}(t)$ is a vector of control inputs, and $\mathbf{w}(t)$ is a vector of disturbance inputs. A first-order differential equation of this form is called a *state-space model* of system dynamics. If the control vector depends upon a pilot command and the state itself (e.g., $\mathbf{u}(t) = \mathbf{u}_{command} + \mathbf{c}[\mathbf{x}(t)]$), the state-space model is

$$\begin{aligned} \frac{d\mathbf{x}(t)}{dt} &\triangleq \dot{\mathbf{x}}(t) = \mathbf{f}\left\langle \mathbf{x}(t), \{\mathbf{u}_{command} + \mathbf{c}[\mathbf{x}(t)]\}, \mathbf{w}(t) \right\rangle \\ &= \mathbf{f}'[\mathbf{x}(t), \mathbf{u}_{command}, \mathbf{w}(t)] \end{aligned} \tag{1.3-12}$$

In other words, $\mathbf{f}'[\cdot]$ subsumes the control effect but retains the general form of eq. 1.3-11. In the more general case, eq. 1.3-12 is integrated to compute a time history of the state:

$$\begin{aligned} \mathbf{x}(t) &= \mathbf{x}(0) + \int_0^t \mathbf{f}(\mathbf{x}(\tau), \mathbf{u}(\tau), \mathbf{w}(\tau))\, d\tau \\ &= \mathbf{x}(0) + \int_0^t \mathbf{f}'[\mathbf{x}(\tau), \mathbf{u}_{command}(\tau), \mathbf{w}(\tau)]\, d\tau \end{aligned} \tag{1.3-13}$$

If there is a static solution to the equations of motion, $d\mathbf{x}(t)/dt = 0$, and eq. 1.3-11 becomes

$$0 = \mathbf{f}[\mathbf{x}_s, \mathbf{u}_c, \mathbf{w}_c] \tag{1.3-14}$$

where $\mathbf{x}_s$ is the static solution for the state, and $\mathbf{u}_c$ and $\mathbf{w}_c$ are constant values. A static solution for the full state is not possible if the aircraft is moving because $\mathbf{r}$ is continually changing (eq. 1.3-4); however, equilibrium of a ninth-order model involving rates and orientation only can be achieved.

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Derivatives and Integrals of Time-Dependent Variables

Derivatives and Integrals of Scalars: A scalar velocity $v(t)$ is the time-rate-of-change (or derivative with respect to time) of a scalar position $r(t)$,

$$v(t) = dr(t)/dt = \lim_{(t_2 - t_1) \to 0} \frac{r(t_2) - r_1(t_1)}{t_2 - t_1}$$

while position is the definite-integral of velocity plus the appropriate constant of integration (i.e., the value of r at the beginning of the integration):

$$r(t) = r(0) + \int_0^t v(\tau)\, d\tau$$

If the velocity is subject to a constant acceleration, $a = f/m$, where f is an applied force and m is the mass of a particle, the scalar equations for velocity and position are

$$\begin{aligned} &v(t) = v(0) + at = v(0) + (f/m)t \\ &r(t) = r(0) + v(0)t + at^2/2 = r(0) + v(0)t + (f/m)t^2/2 \end{aligned}$$

For a falling object, a simplified force model might be based on the state, control, disturbance, and gravitational acceleration g, such as

$$f = (k_r r + k_v v) + (k_u u + k_w w + mg)$$

where the k_i are constants. How would this affect the evolution of $v(t)$ and $r(t)$?

Derivatives and Integrals of Vectors: The derivative (or integral) of a vector is a vector of scalar derivatives (or integrals):

$$\frac{d\mathbf{x}(t)}{dt} \triangleq \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} dx_1(t)/dt \\ dx_2(t)/dt \\ dx_3(t)/dt \end{bmatrix}; \quad \int \mathbf{x}(t)\, dt = \begin{bmatrix} \int x_1(t)\, dt \\ \int x_2(t)\, dt \\ \int x_3(t)\, dt \end{bmatrix}$$

For the previous examples, let $v(t)$ and $r(t)$ be two elements of the state $\mathbf{x}(t)$. For constant acceleration,

$$\frac{d\mathbf{x}(t)}{dt} \triangleq \begin{bmatrix} \dot{v}(t) \\ \dot{r}(t) \end{bmatrix} = \begin{bmatrix} a \\ v(t) \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} v(t) \\ r(t) \end{bmatrix} + \begin{bmatrix} a \\ 0 \end{bmatrix}$$

$$\mathbf{x}(t) = \begin{bmatrix} v(t) \\ r(t) \end{bmatrix} = \begin{bmatrix} v(0) + at \\ r(0) + v(0)t + at^2/2 \end{bmatrix}$$

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For the falling object,

$$\begin{aligned}
\mathbf{x}(t) &= \mathbf{x}(0) + \int_0^t \mathbf{f}[\mathbf{x}(\tau), \mathbf{u}(\tau), \mathbf{w}(\tau)]d\tau \\
&\triangleq \begin{bmatrix} v(t) \\ r(t) \end{bmatrix} = \begin{bmatrix} v(0) \\ r(0) \end{bmatrix} + \int_0^t \begin{bmatrix} [k_r r(\tau) + k_v v(\tau) + k_u u(\tau) + k_w w(\tau) + mg]/m \\ v(\tau) \end{bmatrix} dt \\
&= \begin{bmatrix} v(0) \\ r(0) \end{bmatrix} + \int_0^t \left\{ \begin{bmatrix} k_r/m & k_v/m \\ 0 & 1 \end{bmatrix} \begin{bmatrix} r(\tau) \\ v(\tau) \end{bmatrix} + \begin{bmatrix} [k_u u(\tau) + k_w w(\tau) + mg]/m \\ 0 \end{bmatrix} \right\} dt
\end{aligned}$$

Motions occurring in the vertical plane are called *longitudinal motions*, while those occurring out of the plane are *lateral-directional motions*. Each of these subsets has six dimensions, and in steady, level flight, they are largely uncoupled from each other. The variables of longitudinal motion, related to body axes, are

- Axial velocity, u
- Normal velocity, w
- Pitching rate, q

and their inertial-axis integrals are

- Range, x_I
- Altitude, $-z_I$
- Pitch angle, θ

Translational velocities often are replaced by velocity magnitude V and angle of attack α or flight path angle γ (Fig. 1.3-1).

The lateral-directional variables are

- side velocity, v
- roll rate, p
- yaw rate, r

and their inertial-axis integrals are

- cross range, y_I
- roll angle, ϕ
- yaw angle, ψ

Side velocity is often replaced by sideslip angle β (Fig. 1.3-2). The horizontal angle of the velocity vector with respect to the x_I axis is ξ.

Stability is an important dynamic characteristic that describes the tendency for the aircraft's state to return to an equilibrium condition or to diverge in response to control inputs, disturbances, or initial conditions. *Control* is the critical area of mechanics that develops strategies and systems for achieving goals and assuring stability, given

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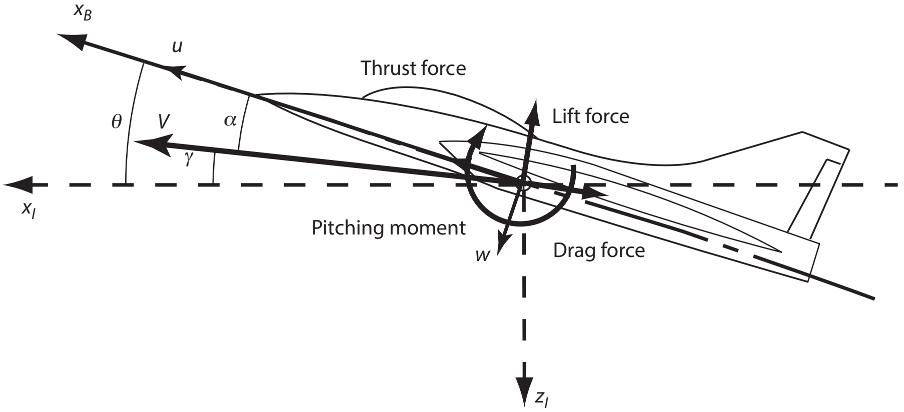

FIGURE 1.3-1. Longitudinal variables, forces, and moment.

an aircraft's mission, disturbances, parametric uncertainties, and piloting tasks. The aircraft's natural stability can be augmented by *feedback control*, in which measured flight motions drive control effectors on a continuing basis.

Example 1.3-1. Longitudinal Motions of a Paper Airplane

The classic dart-shaped paper airplane (Fig. 1.3-3) provides an excellent introduction to flight mechanics, both for numerical analysis and experiment. Starting with a sheet of plain paper, which weighs[10] about 3 grams, the experimental vehicle has a wingspan of 12 cm and a length of 28 cm, yielding aspect ratio AR and wing area S of 0.86 and $0.017\,\mathrm{m}^2$, respectively. Refer to Sections 2.3 and 2.4 for more details about these quantities.

As shown in Chapter 3, the point-mass longitudinal motions of the paper airplane can be predicted by integrating four differential equations; written in vector form, they are

$$\begin{bmatrix} \dot{V}(t) \\ \dot{\gamma}(t) \\ \dot{h}(t) \\ \dot{r}(t) \end{bmatrix} = \begin{bmatrix} \left[C_D \frac{1}{2}\rho V^2(t) S\right] \Big/ m - g\sin\gamma \\ \frac{1}{V}\left\{\left[C_L \frac{1}{2}\rho V^2(t) S\right] \Big/ m - g\cos\gamma\right\} \\ V(t)\sin\gamma(t) \\ V(t)\cos\gamma(t) \end{bmatrix} \tag{1.3-15}$$

where V, γ, h, and r are the airspeed, flight path angle, height, and range, respectively. The parameters of this model are contained in the drag coefficient C_D, lift coefficient C_L, air density ρ ($1.225\,\mathrm{kg/m^3}$ at sea level), reference area S, mass m, and

---

[10] Strictly speaking, its *mass* (not weight) is 3 grams; "weighs" is used here and throughout in a colloquial sense.

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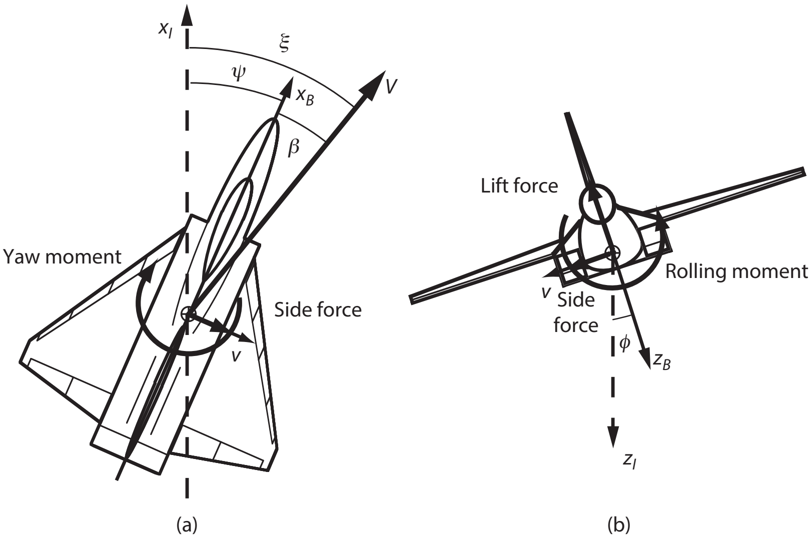

FIGURE 1.3-2. Lateral-directional variables, force, and moments.

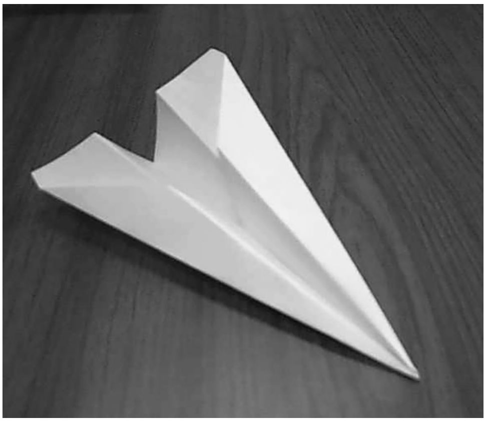

FIGURE 1.3-3. Classic paper airplane.

the acceleration due to gravity $g = 9.807\,\text{m/s}^2$. As explained in Section 2.4, the lift coefficient is modeled as a linear function of the angle of attack α,

$$C_L = \frac{\partial C_L}{\partial \alpha}\alpha \tag{1.3-16}$$

The lift-slope derivative is estimated as

$$\frac{\partial C_L}{\partial \alpha} \triangleq C_{L_\alpha} = \frac{\pi AR}{1+\sqrt{1+(AR/2)^2}}, \text{ per radian} \tag{1.3-17}$$

The drag coefficient is modeled as

$$C_D = C_{D_o} + \varepsilon C_L^2 \tag{1.3-18}$$

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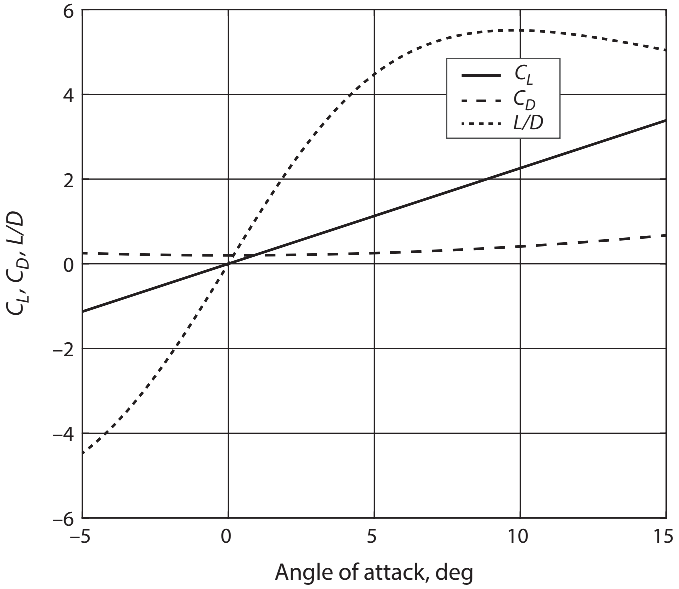

FIGURE 1.3-4. Lift and drag coefficients, lift-to-drag ratio for the paper airplane model.

with $C_{D_o} = 0.02$ and the *induced-drag factor* $\varepsilon = 1/\pi eAR$; e is known as the *Oswald efficiency factor*, estimated to be 0.9 here. In this example, attitude dynamics are neglected, so the angle of attack is taken as the control variable; its trimmed value is a function of the center-of-mass location and the amount of upward deflection of the wing's trailing edge. The lift and drag coefficients, as well as the lift-to-drag ratio, are plotted against angle of attack in Fig. 1.3-4.

Equation 1.3-15 is nonlinear function for several reasons: C_D is a quadratic function of u, and several nonlinear operations involve elements of the state, including the square of x_1, division by x_1, sines and cosines of x_2, and the product of x_1 with functions of x_2.

Three longitudinal paths are typical of paper airplane flight: *constant-angle descent*, *vertical oscillation*, and *loop*. Numerically integrating the equations of our mathematical model produces these paths (eq. 1.3-13), as shown in the accompanying figures. Four cases are simulated:

a) Equilibrium glide at maximum lift-drag ratio, $L/D = C_L/C_D$,

b) Oscillating glide due to zero initial flight path angle,

c) Increased oscillation amplitude due to increased initial speed, and

d) Loop due to a further increase in launch speed.

In all cases $\alpha = 9.3$ deg, yielding the best L/D (=5.2) for the configuration. This angle of attack produces the shallowest equilibrium glide slope (−11 deg) and a constant speed of 3.7 m/s. The equilibrium trajectory is represented by the straight descending path in Fig. 1.3-5a and the constant speed and flight path angle in Fig. 1.3-5b (Case a).

Obtaining this smooth, linear path is dependent on starting out with exactly the right speed and angle; Case b illustrates that having the right speed but the wrong initial angle (0 deg) perturbs the dynamic system, introducing a lightly damped oscillation

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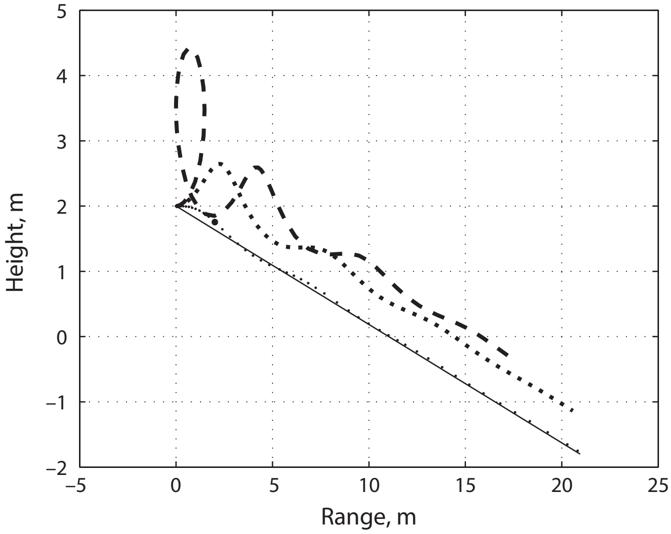
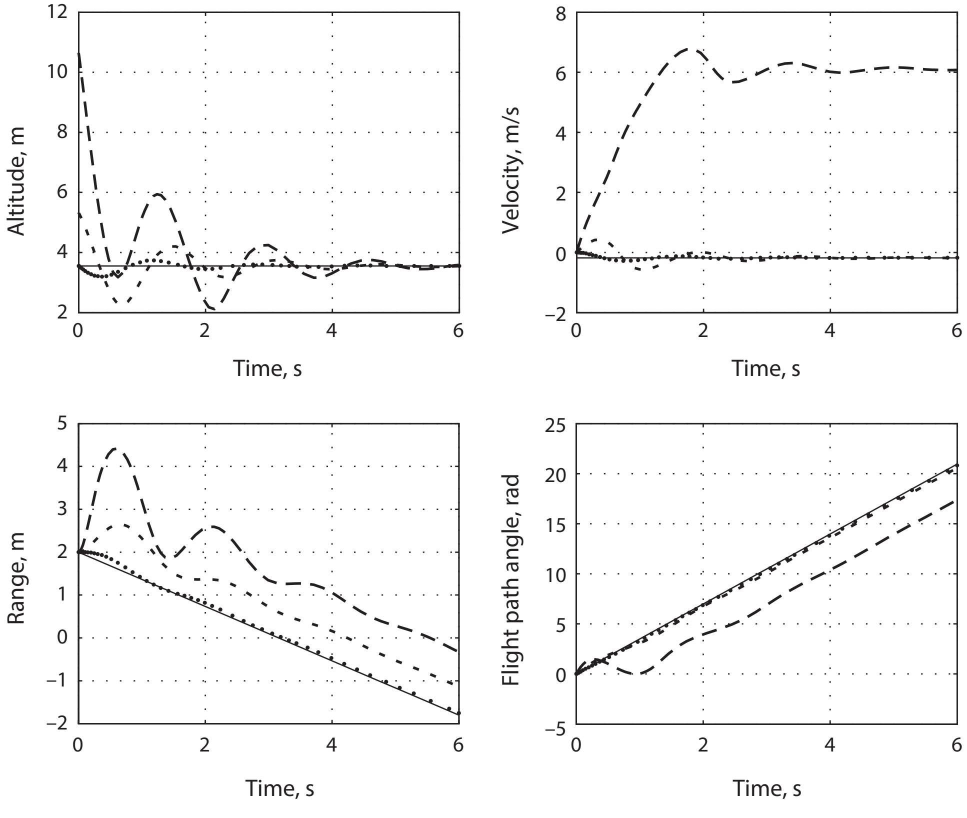


a) Height vs. range of paper airplane flight with different initial conditions.



b) Velocity, flight path angle, altitude, and range vs. time.

FIGURE 1.3-5. Flight of a paper airplane.

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about the equilibrium path. The period of the oscillation is 1.7 s, and the wave shape is very nearly sinusoidal.

The initial-condition perturbation is increased in Case c, where the zero initial angle is retained, and the speed is increased. The vertical oscillation has greater height, speed, and angle amplitude, and the wave shape is perceptibly scalloped in the first two variables. Nevertheless, the rate of amplitude decay and the period are about the same as in the previous case. Further increasing the launch speed results in a loop, followed by sharply scalloped, decaying undulations with a 1.7-s period (Case d). The loop provides an upper limit on the vertical oscillations that can be experienced by an airplane with fixed control settings.

The MATLAB program used to generate these results is described in Appendix D. You can replicate these flight paths with your own paper airplane, although you are not likely to get identical flight test results. The aerodynamic model used here is approximate, and the weight and dimensions of your airplane will be slightly different. It is difficult to trim the paper plane's angle of attack precisely, but it is relatively easy to sweep a range of values by bending the wing's trailing edge up or down. Similarly, launching by hand produces variable starting conditions, and it probably is not possible to launch a paper airplane at speeds much above 5 m/s without inducing significant aeroelastic deformations to the structure, changing its aerodynamics. Perhaps most important, attitude dynamics (not modeled above) play a role, particularly in large-amplitude maneuvers such as the loop. It is tricky to loop the classic dart-shaped paper airplane in practice, but other designs can be looped readily.

1.4 Courses in Flight Dynamics

Flight Dynamics provides a comprehensive reference for understanding aircraft performance, stability, and control; hence, it can be of value in aeronautical or aerospace engineering educational programs. The book presents a core for college-level instruction as well as a resource for practical application in aircraft research and development. Technical concepts range from introductory to advanced, and instructors should not expect to cover all of the material in a single course. Instructors may weave in material not only from this book but also sources that provide specific examples and related perspectives.

A complete course uses multiple sources to address the instructor's objectives. *Flight Dynamics* supports upper-level undergraduate or graduate instruction; thus, students will have familiarity with first-year physics, multivariable calculus, and linear algebra. The author has found Abzug and Larrabee's book [A-1], subtitled *A History of the Technologies That Made Aviation Possible*, to be an invaluable resource in teaching; it provides in-depth case studies that reveal the interplay between mission objectives, aircraft design, flying qualities, and dynamic requirements. The technical reports, papers, notes, memoranda, and special publications (including historical monographs) of NASA and its predecessor, the National Advisory Committee for Aeronautics (NACA), can be accessed via the *NASA Technical Reports Server*, https://ntrs.nasa.gov. Students are encouraged to go to these sources as well as journal papers for added depth and breadth.

At the end of a first (or only) flight dynamics course, students should be able to:

- Understand aircraft configuration aerodynamics, performance, stability, and control
- Analyze and compute mathematical descriptions of the rigid-body motions of aircraft

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- Estimate an aircraft's aerodynamic model from geometric and inertial properties
- Recognize airplane modes of motion and their significance
- Understand experimental methods for measuring aircraft aerodynamics, including flight and wind-tunnel testing
- Appreciate the historical context within which airplanes have evolved to present-day configurations

In addition, students should demonstrate the ability to work in multidisciplinary teams and to present ideas in written and oral assignments.

Lectures for the author's undergraduate course are organized conceptually and do not follow the book outline rigidly. The sections principally referenced in each lecture are identified in Example 1.4-1. A lecturer should not expect to cover all of the material in every noted section, as numerous concepts, graphs, and formulas are included for completeness. The connections between physics and mathematics are made in the adjacent text and should be emphasized in class and assignments.

Example 1.4-1. Syllabus for a First Course in Flight Dynamics

1. Introduction, Mathematical Preliminaries (Sections 1.1, 1.3)
2. Point-Mass Dynamics and Forces (Sections 2.1, 2.2, 2.3, 2.5)
3. Two-Dimensional Low-Speed Aerodynamics (Section 2.4)
4. Three-Dimensional Low-Speed Aerodynamics: 3-D (Section 2.4)
5. Induced Drag and High-Speed Aerodynamics (Section 2.4)
6. Aerodynamic Moments (Sections 2.3, 2.4)
7. Power and Thrust for Cruising Flight (Sections 2.5, 2.6)
8. Cruising Flight Envelope (Section 2.6)
9. Gliding, Climbing, and Turning Flight Performance (Section 2.6)
10. Aircraft Equations of Motion: Translation and Rotation (Sections 3.1, 3.2)
11. Aircraft Equations of Motion: Flight Path Computation (Sections 3.1, 3.2)
12. Aircraft Control Devices and Systems (Sections 3.5, 5.5, 6.5)
13. Linearized Equations of Motion (Sections 3.6, 4.1, 4.3)
14. Linearized Longitudinal Equations of Motion (Sections 5.1, 5.3)
15. Linearized Lateral-Directional Equations of Motion (Sections 6.1, 6.2, 6.3)
16. Analysis of Time Response (Sections 4.2, 4.3)
17. Transfer Functions and Frequency Response (Section 4.4)
18. Root-Locus Analysis of Parameter Variations and Feedback Control (Sections 4.4, 5.2)
19. Advanced Problems of Longitudinal Dynamics (Sections 3.4, 5.4)
20. Advanced Problems of Lateral-Directional Dynamics (Section 6.4)

(continued...)

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