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Chapter One

Introduction

This chapter presents the motives, as well as a limited literature summary, for both the applied and the theoretical control design problems that we pursue in this book.

1.1 STRING-ACTUATED MECHANISMS

In this section, we introduce four string-actuated mechanisms: mining cable elevators, deep-sea construction vessels (DCVs), unmanned aerial vehicles (UAVs), and oil-drilling systems.

Mining Cable Elevators

In mining exploitation, a cable elevator, which is used to transport the cargo and miners between the ground and the working platform underground, is an indispensable piece of equipment [99]. The mining cable elevator is a cable-actuated moving load system. A common arrangement is a single-drum system [178]: a single-cable mining elevator comprising a driving winding drum, a steel wire cable, a head sheave, and a cage. The cable plays a vital role in mining elevators because its advantages of low bending and torsional stiffness, resisting relatively large axial loads, are helpful to heavy-load and large-depth transportation. However, the compliance property, or stretching and contracting abilities of cables, tends to cause mechanical vibrations, especially when the elevator is running at high speed, which leads to tension oscillations and premature fatigue fracture [87, 88, 91]. Therefore, the importance of suppressing the vibrations and tension oscillations cannot be overestimated, for the safety of both personnel and profitability. The economical and convenient way to suppress the vibrations is by designing an appropriate control input without modifying the original structure of the mining cable elevator. In chapter 2, the control design for axial vibration suppression of a high-speed, single-cable mining cable elevator is presented.

For operation at a greater depth, such as over 2000 m, and carrying a heavier load, the single-cable elevator is not suitable. Because a very thick cable is required to bear the heavy load, such a thick cable, at high bending, suffers from problems in the winding on the winder drum. A dual-cable mining elevator [37], shown in figure 3.1 in chapter 3, is proposed to solve this problem, removing the requirement of a very thick cable because two cables tow the cage. However, an imbalance problem, such as cage roll, frequently appears in the dual-cable elevator [184], as shown in figure 3.1, for which, taut cables are used as flexible guide rails because traditional steel rails come with a high cost of manufacture and installation in deep mines. Cage roll will increase the differences in oscillation tension between two cables and

enlarge the oscillation amplitude of the tension, which accelerates premature fatigue and requires inspections and costly repairs. One feasible arrangement to balance the cage roll and suppress the vibrations and tension oscillations in cables of the dual-cable mining elevator is to design and apply additional control forces through actuators at floating head sheaves, as shown in figure 3.1. The control design for suppressing of the axial vibrations and tension oscillations and balancing the cage roll in a dual-cable mining elevator is presented in chapter 3.

In an actual operating environment, the cage is usually subject to uncertain airflow disturbances. Additionally, the flexible guides, with their uncertain properties, may affect the smooth and steady running of the mining cable elevator. These factors inspired the control designs in chapters 4 and 5.

Deep-Sea Construction Vessels

In deep-sea oil exploration, some equipment, such as a subsea manifold, a subsea pump station, and a subsea distribution unit along with associated foundations, flow lines and umbilicals, is installed at designated locations [167, 168] around the drill center on the seafloor. The installation of the equipment is completed by a DCV [168, 182] because the installation sites are located outside a radius of 45 m from the floating drilling platform (see figure 2 in [168]) and cannot be accessed by the huge floating drilling platform, which has limited access and mobility [168], and because some of the equipment, such as flow lines and umbilicals, is installed in advance to prepare to hook up the floating drilling platform when it arrives. A DCV is shown in figure 6.1 in chapter 6, where the top of the cable is attached to a crane on a vessel at the ocean surface, and the cable's bottom is attached to equipment to be installed at the sea floor. The traditional method in underwater installation by DCVs is to regulate the vessel dynamics position and manipulate the crane to obtain the desired heading for the payload [94]. Such a method is not suitable for the deeper water construction in offshore oil drilling (more than 1000 m) because the cable is very long when the payload is near the seabed, which would introduce large cable oscillations [94, 182], causing a large offset between the payload and the desired heading position of the crane—namely, the designated installation location. In chapter 6, the control forces at the onboard crane are designed to reduce the cable oscillations and then place the equipment in the target area on the sea floor. In chapter 7, we employ a piecewise-constant control input that is more suitable for the massive ship-mounted crane and compensate for delays in the transmission of sensing signals from the seabed to the vessel on the ocean surface through acoustic devices.

Unmanned Aerial Vehicles

In addition to DCVs, the control design in chapter 6 can also be applied to unmanned aerial vehicles (UAVs) used to aid delivery to dangerous and inaccessible areas, such as to flood, earthquake, fire, and industrial disaster victims [70, 140]. Food and first-aid kits are tied to the bottom of a cable, whose other end is attached to the UAV. The swing/oscillation of the cable-payload may appear during the transportation motion due to the properties of the cable and external disturbances, such as wind, which may cause damage to the suspended object, the environment, and the people nearby [70]. At the end of the transport motion, when the UAV arrives at the location directly over the rescue site and is ready to land the aid supplies,

the suspended object naturally continues to swing [140], which makes precisely placing the aid supplies at the target position difficult. Therefore, rapid suppression of the oscillations of the cable and suspended object through a control force provided by the rotor wings of the UAV is required. In addition to aid delivery in disaster relief, UAV delivery is also used in some commercial cases to reduce labor cost. For example, some companies use UAVs to transport cargo in storehouses or lift and position building elements in architectural construction [191]. Some logistics companies have also begun to use UAVs to deliver packages in small areas [70].

Oil Drilling

Oil-drilling systems used to drill deep boreholes for hydrocarbon exploration and production often suffer severe vibrations, which can cause the premature failure of drilling string components, damage to the borehole wall, and problems with precise control [98]. The vibrations also cause significant wastage of drilling energy [53]. The suppression of vibrations in the oil-drilling system is thus required for the economic interest and improvement of drilling performance [156].

There are three main types of vibration in oil-drilling systems [154]: vertical vibration, also called the bit-bouncing phenomenon, lateral vibration due to an out-of-balance drill string, which is called whirl motion, and torsional vibration, which appears due to friction between the bit and the rock. This nonlinear torsional interaction between the drill bit and the rock will cause the bit to slow down and even stall while the rotary table is still in motion. Once enough energy is accumulated, the bit will suddenly be released and start rotating at very high speed before slowing down again [24], settling into a limit cycling motion. This is called the stick-slip phenomenon. Several physical laws of bit-rock friction [156] are used to roughly describe stick-slip behavior in the oil-drilling system, such as the velocity-weakening law [31], the stiction plus Coulomb friction model [160], a class of Karnopp model [38, 107, 136], and so on. The stick-slip oscillations lead to instability from the lower end to travel up the drill string to the rotary table, which results in distributed instabilities and is the primary cause of fatigue to the drill collar connections as well as damage to the drill bit [154]. Therefore, suppressing torsional vibrations (stick-slip oscillations) in the oil-drilling system is important. In addition waves at the sea surface causing a heaving motion of the drilling rig [1] in an offshore rotary oil-drilling system [186] introduce an external disturbance at the bit, which is another instability source.

As will be seen in chapter 8, the designed control input at the rotary table goes down from the rotating table, through the drill string, to the drill bit, to eliminate the stick-slip instability and, as a result, reduce the oscillations of the angular displacement and velocity of the drill bit.

1.2 HYPERBOLIC PDE-ODE SYSTEMS

The dynamics of the aforementioned string-actuated mechanisms are governed by hyperbolic partial differential equation-ordinary differential equation (PDE-ODE) systems. The design of controllers for such hyperbolic PDE-ODE systems requires boundary control approaches because the control input can only be applied at one end of the string in such mechanisms. In this section, we review boundary control

of elementary wave PDE-ODE systems, as well as of a class of coupled first-order hyperbolic PDE models, with the possible inclusion of an ODE in cascade with the hyperbolic PDEs.

Wave PDE-ODE Systems

A wave PDE-ODE system serves as a basic model of a string-actuated mechanism in which the wave PDE describes a vibrating string (without in-domain damping), and the attached payload is modeled as an ODE. Classical results on backstepping boundary control for wave PDEs with anti-damping terms in domain or on the uncontrolled boundary are found in [121, 162, 165]. In the past decade, many results on boundary control of wave PDE-ODE systems have been reported. The very first result on boundary control of a wave PDE-ODE plant was presented in [114], where the interconnection is of the Dirichlet type. Boundary control design for a wave PDE-ODE cascade system with a Neumann-type interconnection was also addressed in [170]. The boundary control problem was also tackled in [18, 28, 29] for a wave PDE-nonlinear ODE system.

Coupled First-Order Hyperbolic PDEs

For the sake of greater clarity in control design and analysis, wave PDEs can be converted to a class of heterodirectional coupled first-order hyperbolic PDE systems via the Riemann transformations [26]. Especially when considering the in-domain viscous damping terms describing the string material damping, there would exist in-domain coupling in the resulting coupled first-order hyperbolic PDE systems [147], which makes the control design more challenging. Some theoretical results on boundary control of coupled first-order hyperbolic PDEs have emerged over the last decade. The basic boundary stabilization problem of 2×2 coupled linear transport PDEs was addressed in [32, 177] by backstepping, based on which the extended results on boundary control of these 2×2 systems were presented in [4, 39]. The sliding mode approach and the proportional integral (PI) controller design applied to the control of such a 2×2 system was also considered in [127] and [173], respectively. Boundary control of the 2×2 transport PDE system was further extended to that of an $n + 1$ system in [50]. For a more general coupled transport PDE system where the number of PDEs in either direction is arbitrary, a boundary stabilization law was first designed by backstepping in [96], which is a systematic framework for control design for this class of systems, and other extended results were proposed in [6, 14, 40, 41, 97]. In addition to the applications to string/cable models, the boundary control design for coupled first-order hyperbolic PDEs has also been applied to water-level dynamics [45, 47, 51, 143, 142] and traffic flow [101, 194, 195, 197].

In the past five years, some results on the control of linear coupled hyperbolic PDEs cascaded with ODEs have also been reported. The first is [48], which addressed the state-feedback stabilization of a general linear hyperbolic PDE-ODE coupled system. The state-feedback boundary control design of a 2×2 linear hyperbolic PDE-ODE coupled system with nonlocal terms was also dealt with in [169]. Based on the observer design, in [44] an output-feedback controller with anti-collocated measurements was proposed to stabilize general linear heterodirectional hyperbolic PDE-ODE systems with spatially varying coefficients.

1.3 “SANDWICH” PDEs

In the results discussed in section 1.2, the control input enters the PDE boundary directly, neglecting the inertia and dynamics of the actuator. However, in some applications, actuator dynamics cannot be neglected, especially when the actuator has its own considerable inertia. Incorporating the actuator dynamics into the control design of the string-actuated mechanisms modeled by hyperbolic PDE-ODE systems gives rise to a more challenging problem: control of what we call *sandwich* ODE-PDE-ODE systems.

The first backstepping state-feedback control design for *sandwich* hyperbolic systems was proposed in [113], which considered a transport PDE-ODE system with an integration (first-order ODE) at the input of the transport PDE. Also, the control problem of an ODE with input delay and unmodeled bandwidth-limiting actuator dynamics, which is represented by an ODE-transport PDE-ODE system where the input ODE is first order, was addressed in [118]. The boundary control design of a transport PDE sandwiched between two ODEs describing actuator and sensor dynamics was also proposed in [8]. Regarding coupled transport PDEs, state-feedback control of heterodirectional coupled transport PDEs sandwiched between two ODEs was proposed in [151, 152, 183]. Adding observer designs, output-feedback control of this type of sandwich systems was developed in [43, 49, 180, 181]. Parameter identification of a drill string, modeled as a wave PDE sandwich system from experimental boundary data, was studied in [150]. Regarding other types of PDEs, boundary control of viscous Burgers PDE, heat PDE, and n coupled parabolic PDE sandwich systems was also addressed in [130, 179] and [42], respectively. A fairly complete theory for boundary control of sandwich hyperbolic PDEs is derived in chapters 9–12, including basic design, delay compensation, event-triggered design, and design with nonlinearities.

1.4 ADVANCED BOUNDARY CONTROL OF HYPERBOLIC PDEs

Apart from the basic boundary control designs mentioned in sections 1.3 and 1.2, in this section we review some extended results on disturbance attenuation, adaptive control, delay compensation, and event-triggered control for hyperbolic PDEs.

Disturbance Rejection/Cancellation

Most research on disturbance rejection and adaptive cancellation for PDE systems focuses on disturbances collocated with control. Sliding mode control (SMC) schemes designed for heat, Euler-Bernoulli beam, and Schrödinger equations with boundary input disturbances were presented in [73, 76, 189]. The internal model principle [63] on the basis of the estimation/cancellation strategy was utilized in the beam equation [145]. For wave PDEs, adaptive disturbance cancellation was used in the output-feedback asymptotic stabilization of one-dimensional wave equations subject to harmonic disturbances at the controlled end and at the measured output in [82, 83] and [81], respectively. By applying the active disturbance rejection control method introduced by Han [86] for ODEs, state-feedback

or output-feedback control designs for wave PDEs with matched disturbances were presented in [60, 74, 75, 80, 172, 198].

The task of rejection or adaptive cancellation of unmatched disturbances—that is, the disturbances anti-collocated with the control input—is more difficult. While several results for this task have been developed for ODE systems, such as those found in [78, 129, 192] and so on, the literature is less ample in PDE systems, where the disturbance is on the far (distal) end from the control input. A state-feedback controller that practically stabilizes a Schrödinger equation-ODE cascade system in the presence of an unmatched disturbance assumed to be small and measurable was presented in [100]. An output-feedback controller was designed for output reference tracking in a wave equation with an anti-collocated harmonic disturbance at a stable damping boundary in [84]. The output regulation problem for a wave equation with a harmonic anti-collocated disturbance at a free boundary was further dealt with in [85]. In chapters 4, 5, and 8, the asymptotic rejection and adaptive cancellation of unmatched disturbances in hyperbolic PDEs are proposed, respectively, along with applications in the control of disturbed mining cable elevators and oil-drilling systems.

Adaptive Control

Three traditional adaptive schemes for PDEs with uncertain parameters are the Lyapunov design, the passivity-based design, and the swapping design [112, 124]. Using the three design methods initially developed for ODEs in [122], the same three adaptive control approaches were proposed for parabolic PDEs in [123, 163, 164]. For adaptive control of hyperbolic PDEs, many results based on the three conventional methods have also been achieved, as follows. In [24, 25, 26, 117], adaptive control laws were presented for a one-dimensional wave PDE that had an actuator on one boundary and an anti-damping instability with an unknown coefficient on the other boundary. The first result on adaptive control of general first-order hyperbolic partial integro-differential equations was proposed in [20]. An adaptive boundary control design of coupled first-order hyperbolic PDEs with uncertain boundary and spatially varying in-domain coefficients was developed in [5]. In [7], two adaptive boundary controllers of coupled hyperbolic PDEs with unknown in-domain and boundary parameters were proposed using identifier and swapping designs, respectively. More adaptive control results of coupled first-order hyperbolic PDEs have been collected in [9]. Adaptive control design methods for hyperbolic PDEs are employed in chapters 5 and 8 to deal with parameter uncertainties in mining cable elevator and oil-drilling systems. Also, an event-triggered adaptive controller is proposed in chapter 13.

Recently, a new adaptive scheme using a regulation-triggered batch least-squares identifier was introduced in [102, 104], which has some advantages over the traditional adaptive approaches, such as guaranteeing exponential regulation of the states to zero as well as finite-time convergence of the estimates to the true values. This method has been successfully applied in the adaptive control of a parabolic PDE where the unknown parameters are the reaction coefficient and the high-frequency gain [106]. Regarding hyperbolic PDEs, using a scalar least-squares identifier updated at a sequence of times with fixed intervals, backstepping adaptive boundary control of a first-order hyperbolic PDE with an unknown transport speed was proposed in [11]. In chapters 14 and 15, adaptive controllers based on batch least-squares identifiers are designed for 2×2 hyperbolic PDE-ODE systems where

the transport speeds of both transport PDEs and the coefficients of the in-domain couplings are unknown, respectively.

Delay Compensation

Time delays often exist in practical control systems and may destroy system stability [79]. For example, in a subsea installation by a DCV, sensor delays [94] exist due to the fact that the sensor signal is transmitted over a large distance from the seafloor to the vessel on the ocean surface, through a set of acoustic devices. Such sensor signal transmission may result in information distortion or even make the control system lose stability. Therefore, the time delay is a vital issue that should be considered in the control design.

Recently, boundary control designs for hyperbolic PDEs have been proposed that take time delays into consideration. For example, delay-robust stabilizing feedback control designs for coupled first-order hyperbolic PDEs were introduced in [12, 13], achieving robustness to small delays in actuation. In order to compensate arbitrarily long delays, a delay compensation technique was developed in [116, 125], where the delay was captured as a transport PDE, and the original ODE plant with a sensor delay was treated as an ODE-transport PDE cascaded system in the controller and observer designs. The observer was built as a “full-order” type, which estimates both the plant states and the sensor states, compared with some classical results on delay-compensated observer designs [3, 27, 67], which only estimate plant states—namely, observers of the “reduced-order” type. While compensation for arbitrarily long delays by this technique are commonly available for finite-dimensional systems, very few examples exist where such compensation has been achieved for PDEs, including parabolic (reaction-diffusion) PDEs [108, 115]. Delay compensation for the wave PDEs with arbitrarily long delays is more complex than that for reaction-diffusion PDEs. The primary reason is the second-order-in-time character of the wave equation. In [119], by treating the delay as a transport PDE and applying a backstepping design, a boundary controller was developed for a wave PDE with compensation for an arbitrarily long input delay and with a guarantee of exponential stability for the closed-loop system. In chapter 10, we design a delay-compensated control scheme for a sandwich hyperbolic PDE in the presence of a sensor delay of arbitrary length by treating the delay as a transport PDE.

Event-Triggered Control

When implementing the designed PDE control laws in an actual mining cable elevator, two challenges caused by high-frequency elements in the control law appear: 1) the massive actuator, comprising a hydraulic cylinder and head sheave, shown in figure 11.1 in chapter 11, is incapable of supporting the high-frequency control signal, and 2) the high-frequency components in the control input may in turn become vibration sources for the cable. It is thus necessary to reduce the actuation frequency and ensure the effective suppression of the vibrations in the mining cable elevator. Designing sampling schemes to apply to the control input is a potential solution. Designs of sampled-data control laws of parabolic and hyperbolic PDEs were presented in [64, 105] and [35, 103], respectively. Compared with the periodic sampled-data control, event-triggered control, where the input to the massive actuator is changed only at the necessary times determined by an event-triggering mechanism that acts by evaluating the operation of the elevator, is more feasible for

the mining cable elevator from the point of view of energy saving. This motivates us to design event-triggered PDE backstepping control laws.

Most of the current designs on event-triggering mechanisms are for ODE systems, such as those in [69, 93, 133, 159, 171]. Designs of event-based controls for PDE systems are still rare in the existing literature. There exist results on the distributed (in-domain) control of PDEs, such as [158, 193]. For boundary control, an event-triggered feedback law was proposed for a reaction-diffusion PDE in [58]. For first-order linear hyperbolic PDEs with dissipativity boundary conditions, an event-triggered boundary control law was originally proposed in [55, 56]. Furthermore, a state-feedback event-based boundary controller for a class of 2×2 coupled linear hyperbolic PDEs was designed in [57]. Based on the observer design, the output-feedback event-triggered boundary control of 2×2 coupled linear hyperbolic PDEs was developed in [54]. In chapter 11, an event-triggered backstepping boundary control law is derived for a sandwich hyperbolic PDE, and an adaptive event-triggered boundary controller is further developed in chapter 13.

1.5 NOTES

Frequently used notations in this book are given next, with more specialized notational conventions introduced in the coming chapters.

The partial derivatives and total derivatives are denoted as

$$f_x(x, t) = \frac{\partial f}{\partial x}(x, t), \quad f_t(x, t) = \frac{\partial f}{\partial t}(x, t),$$

$$f'(x) = \frac{df(x)}{dx}, \quad \dot{f}(t) = \frac{df(t)}{dt}.$$

By $C^k(A)$, where $k \geq 1$, we denote the class of continuous functions on A . The single bars $|\cdot|$ denote the Euclidean norm for a finite-dimensional vector $X(t)$. In contrast, norms of functions (of x) are denoted by double bars. For $u(x, t)$, $x \in [0, D]$, by default, $\|\cdot\|$ denotes the L^2 -norm of a function of x , namely,

$$\|u(\cdot, t)\| = \sqrt{\int_0^D u(x, t)^2 dx},$$

and Sobolev norms such as $H^1[0, D]$ or even $H^2[0, D]$ are defined by

$$\|u(\cdot, t)\|_{H^1} = \sqrt{\int_0^D u(x, t)^2 dx + \int_0^D u_x(x, t)^2 dx},$$

$$\|u(\cdot, t)\|_{H^2} = \sqrt{\int_0^D u(x, t)^2 dx + \int_0^D u_x(x, t)^2 dx + \int_0^D u_{xx}(x, t)^2 dx},$$

and the ∞ -norm is denoted by

$$\|u(\cdot, t)\|_{\infty} = \sup_{x \in [0, D]} \{|u(x, t)|\}.$$

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