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CHAPTER 1

Introduction

1.1 Motivation

If not for *magmatism*, the Earth might still be an undifferentiated ball of early solar-system dust—a humble cosmic dust-bunny—rather than the majestic planet on which (and about which) the author composes these words. Magmatism comprises melting of rocks and minerals and segregation of that melt from the residual solid. Over the age of the Earth, magmatism has produced the compositionally layered, radial structure of the core, mantle, and crust (and the ocean and atmosphere, too).

Magmatism shapes the planet today. This is especially true at plate-tectonic boundaries where crust is produced by partial melting of the mantle. These boundaries are associated with mantle convection: convergent boundaries (subduction zones) where negatively buoyant lithosphere founders and sinks into the mantle; divergent boundaries (mid-ocean ridges) where the lithosphere is rifted apart by far-field tectonic stresses. In each of these settings, mantle rock that underlies the boundary partially melts; buoyancy of that melt causes it to be transported toward the surface; some of the transported melt fuels volcanism and the production of new crust.

Indeed, magmatism is responsible for the long-term stability of the geological substrates of human life: atmosphere, oceans, and continental crust. While many of the details remain obscure, we understand the basic outline of this multistep process. It begins at mid-ocean ridges, where divergent plates drive mantle upwelling and partial melting. The magma rises to the ridge axis and, through a combination of eruption and intrusion, forms the oceanic crust. The oceanic crust moves along the sea floor as part of a tectonic plate. While doing so, it is altered by interaction with sea water, becoming hydrated and carbonated. This altered crust eventually subducts back into the mantle. As it sinks to higher pressures and temperatures at depth, metamorphic reactions release the volatile elements from the minerals. They flow into the mantle and cause melting. The magma rises to yield volcanism, this time in a subduction setting, and returns some of the volatile elements to the atmosphere. Through a complicated and poorly understood process, this magma evolves to be continental crust.

But some of the water and carbon that were transported to depth by the oceanic crust do not return immediately to the surface. Instead they are trapped in the mantle. Over the age of the Earth, this could bury all of the surface water and dry out the oceans. But magmatism is again the means of escape: mantle rock that upwells and melts beneath mid-ocean ridges releases its water and carbon into the magma, which transports them to the surface and exsolves them into the ocean or atmosphere.

Thus the plate tectonic/mantle convection system brings rocks up from depth to the surface and back down to depth, while magmatism couples this physical rock cycle to the surface environment. Magma is a crucial link in the chemical cycles that enable habitability of Earth. Magma transports heat and chemical elements, affecting the composition of the atmosphere, ocean, and soils, thus shaping the surface environment. It leads to volcanic eruption hazards and volcanic resources such as hydrothermal energy and ore formation. At mid-ocean ridges, it "re-paves" more than half of the solid Earth (i.e., the ocean floor) with a thin veneer of mantle-derived melt every $\sim$100 million years.

But since the discovery of plate tectonics, geodynamicists have mostly focused on the plate motions that arise from mantle convection. Despite the importance of magmatism, much less effort has been expended to systematically understand and quantitatively model the genesis, segregation, and emplacement of magma and the chemistry it transports. An aim of this book is to promote and facilitate a correction of that imbalance.

Magmatism is obviously not unique to Earth. An extremely active magmatic system shapes Io, one of Jupiter's moons. And since ice is nothing more than a low-density rock, it is fair to say that Europa (another Jovian moon where some 20 km of ice floats atop a $\sim$100-km-thick ocean), is or was shaped by magmatism. The same probably applies to countless exoplanets. Indeed, the physical and chemical interactions of liquid magma and its solid residue must be common to condensed planets throughout the universe. It is a primary aim of this book to strengthen the foundation on which our understanding of these interactions are built.

The primary means by which we shall pursue this aim is to collect relevant theory, developed in different scientific fields, published in different journals, with diverse notations, styles, and applications. We shall bring this together into a single, coherent, structured framework of knowledge: on the written page and also, we hope, in your mind. Exercises are provided to help you erect this mental framework; codes are provided to help you explore and extend the concepts addressed here. Hopefully, with this framework established, you will more easily read the related scientific literature on magma/mantle dynamics, more rapidly come to understand the cutting edge of research, and more ably contribute to the advance of the cutting edge through your own research. Perhaps you will discover that parts of the framework established here should be modified or entirely rebuilt to better describe the physical reality of magmatism; please email me if so!

1.2 Basic Physical Considerations

Magmatism on Earth (let alone all other planets) is broader than the physical system considered in this book. Magmatism on Earth includes the early magma ocean, the production and migration of the lava erupted in continental flood basalts, the deep source region of komatiites, the emplacement of batholiths into the continental crust, and the melting of the continental crust itself to form granites. Magma may be present between 410- and 660-km depth in the mantle mantle transition zone and, deeper, at $\sim$2800-km depth above the core–mantle boundary. Indeed, separation of metal from mantle silicates to form the core is a sort of magmatism, as is the solidification of the inner core. And while magmatism, as a category, may exclude the formation of sea ice or meltwater generation in temperate glaciers, they are governed by essentially the same physics.

Although this book develops theory that is relevant to a wide range of phenomena, our primary concern is magmatism in the shallow ($\lesssim$300 km) mantle, often known as the *asthenosphere*,[1] especially in environments such as mid-ocean ridges, subduction zones and mantle plumes. In these contexts, it is thought that the volume fraction of melt remains small, except where magma stalls before being erupted or crystallizing. In particular, volume fractions in the melting regions are small enough that the solid residue forms a contiguous skeleton that transmits stress and may thus have a different pressure than the interstitial liquid magma.

The mantle is solid; it transmits elastic shear waves in response to rapid changes in stress associated with earthquakes. Observed over longer time scales, however, it is fluid. The most obvious consequence of this fluidity is postglacial rebound, which arises from flow of mantle rock toward surface depressions left by retreating ice sheets. This fluid behavior of a polycrystalline aggregate occurs through solid-state deformation known as *creep*. There are various microscopic mechanisms of creep that are active at the grain scale (e.g., diffusion of crystal mass, motion of crystal dislocations); we shall not be concerned with their microscopic details. At the continuum scale, the creeping aggregate can be described with a viscosity that relates the deviatoric stress and strain rate of the continuum. The theory for flow of such a viscous, slowly deforming material is based on the Stokes equation.

Magma is liquid; it has a range of viscosity that, at its lower end where it is associated with low-silica basalts, is the same as that of glucose syrup at room temperature. It forms by melting of more fusible minerals that are distributed throughout the polycrystalline mantle. The magma fills the pore space at the junctions between solid grains. Because of the wetting properties of magma with mantle minerals, the pores form an interconnected network that is permeable even at vanishingly small volume-fractions of melt. Within the pores at the microscopic scale, a Stokes balance of forces applies to the magma, controlling the microscopic pressure and flow velocity. At the continuum scale, however, these microscopic variables cannot be resolved; instead, upscaled variables representing volume-averaged melt velocity and pressure satisfy a modified Darcy's law.

To summarize the physical context: the mantle is a high-viscosity, polycrystalline, creeping solid that forms a contiguous but porous skeleton (also called the solid *matrix*); the magma is a low-viscosity liquid that is transported through the interconnected network of pores between solid grains. Both liquid magma and solid mantle are modeled as fluids. As we shall see in subsequent chapters, the *two-phase flow* of partially molten mantle is governed by a Stokes/Darcy system of coupled partial differential equations (PDEs). Associated with this system are the usual material properties of shear viscosity and permeability, which appear as constitutive laws (or closure conditions) in the model formulation.

Both solid mantle and liquid magma are compressible. Indeed, over the full depth of the mantle, density differences due to isentropic compression are almost 50% of the mean density. However, over the pressure range of the asthenosphere, this compression is small. Here, the relevant comparison is to the density difference between solid and liquid phases that drives melt segregation. Hence, for mechanical models of the asthenosphere, we shall make a Boussinesq approximation, neglecting compressibility except in body-force terms (see chapter 4). For some thermal models, we retain the effect of isentropic compressibility on temperature (see chapter 8).

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[1] Derived from the Greek word *asthenes* meaning weak. The asthenosphere has a lower viscosity that the deeper mantle below it or the lithosphere above it.

It is of fundamental importance that assumptions about (in)compressibility do not preclude *compaction*. Compaction is the consolidation of mantle grains with expulsion of magma; it is represented as a convergent solid flux that is balanced by a divergent liquid flux. Decompaction or dilatation of the mantle is the opposite: solid grains spread apart and magma is locally imbibed, giving a divergent solid flux and convergent liquid flux. To clarify this fundamental concept, consider a mixture of an incompressible solid grains and incompressible liquid. If this mixture is fully enclosed by an impermeable membrane, then it cannot be compacted. However, the same mixture enclosed by a permeable membrane can compact as liquid is expelled through the membrane.

In the context of this book, the resistance to compaction arises from two processes. The first is the expulsion of the liquid phase from the pores between solid grains. Because magma is viscous, its flow is retarded by viscous drag on the surrounding solid; expelling it requires a force that is proportional to its viscosity. However, even if the liquid in the pore network were inviscid (or replaced by a void), a second process would still resist compaction. This process is the grain-scale solid deformation that occurs during closure of pores. The viscous resistance to this deformation is known as the *compaction viscosity*. It is distinct from but related to the better-known *shear viscosity*. Neither the liquid nor the compaction viscosity appear in the theory of single-phase mantle dynamics. In fact, the mechanics of viscous compaction is a special feature of partially molten rock that gives rise to much of its interesting behavior.

Of course melting and solidification, which transfer mass between liquid and solid phases, are also crucial to the physics of the partially molten mantle and they feature prominently in the mathematical description of the system. This brings into play a range of thermal physics and chemical thermodynamics related to phase change. The latter is usually known as petrology; it is a field that is older and more developed than geodynamics. Petrological studies of mantle-derived crystalline rocks and lavas give us detailed knowledge of the chemical and mineralogical composition of the mantle and its melts. The solid mantle is, in fact, a grain aggregate of a variety of mineral phases, each with different properties.

The dominant mineral in the asthenospheric mantle is olivine. Olivine forms a solid solution between a magnesian end member (forsterite, Mg_2SiO_4, about 90% of olivine by mass) and a ferrous end-member (fayalite, Fe_2SiO_4). Forsterite has a very high melting temperature (about 1900 °C at atmospheric pressure) and hence melting of olivine contributes little to magma production. Pyroxenes, although less abundant by volume, are the dominant contributor to mantle partial melting. They are more silicious than olivine and more chemically diverse because they allow more chemical substitutions. Importantly, they can incorporate more water and other impurities than olivine. The magnesium–iron solid-solution series of pyroxene has end members of enstatite ($MgSiO_3$) and ferrosilite ($FeSiO_3$), but pyroxenes typically also include some calcium (e.g., $CaMgSi_2O_6$–$CaFeSi_2O_6$) and up to 10% aluminum. Clinopyroxenes (cpx) and orthopyroxenes (opx) are distinguished by their crystal structure, but also by their fusibility; cpx accommodates calcium and melts more readily than opx. Finally, the asthenospheric mantle contains aluminous minerals including garnet (higher pressure), spinel (intermediate pressure), and plagioclase (lower pressure). Although volumetrically unimportant, these minerals support a huge range of chemical substitutions and hence can exert a disproportionate control on the chemistry of melts.

The relative proportions of olivine, clino- and orthopyroxene, and garnet/spinel/plagioclase vary widely within mantle rocks. Melting tends to strip out the aluminous phase and the clinopyroxene first, then the orthopyroxene, eventually leaving only olivine. Mantle rocks with more cpx are thus considered *fertile* (capable of producing more melt) while those containing mostly olivine are considered *refractory*. A typical fertile peridotite, a spinel lherzolite, might consist of about 66% olivine, 24% opx, 8% cpx, and 2% spinel;[2] in contrast, a refractory peridotite such as dunite would be at least 90% olivine with the remainder composed mostly of opx.

The solid mantle is thus a rather complicated physical entity, comprising multiple mineral phases that vary significantly in their composition and proportion. We shall sidestep much of this complexity by treating the solid mantle as a single, mechanically uniform phase. The mechanical transport properties of that phase are most consistent with olivine, the dominant mineral of the shallow mantle and the mineral commonly used in laboratory deformation experiments. When we consider the thermochemistry of melt production, however, some of the petrological terminology and concepts will reappear in the discussion.

The liquid phase is, in a limited way, simpler than the solid.[3] For most cases of relevance, it is a solution of any chemical components that are rejected by the solid. This includes portions of the oxides of silica, magnesium, iron, calcium, and aluminum, as well as solid-incompatible components such as sodium, potassium, carbon, and water. The magma that is produced by melting of mantle rocks is said to be *basaltic* in composition, meaning that it is the liquid parent of the rock basalt. Basaltic lavas erupt at mid-ocean ridges and ocean islands (e.g., Hawaii), volcanos that directly tap magma produced in the mantle. These lavas carry encoded information about their source and path of transport; they deposit that information where it can be sampled and measured.

The natural system outlined above operates according to a vast and complex set of physical and chemical processes and properties: continuum mechanics, microstructural mechanics, thermal and chemical transport, mineralogy, chemical thermodynamics/petrology, and more. These are embedded in the evolving context of planetary geology and tectonics. Some of the observable consequences of the operation of this system are well understood and explained; many are not. Quantitative theory and models can facilitate quantitative tests of hypothetical explanations of observations. Perhaps more importantly, models can sharpen the questions raised by observations, making hypotheses testable by discovering corollary predictions. However, to be tractable and comprehensible (and hence useful), our theoretical treatment of the magma/mantle system will necessarily simplify aspects, even though more sophisticated theory (i.e., theory that describes the some aspect of the physics in more detail[4]) may be available. It is possible (and, indeed, beneficial) to develop and analyze models more complex than those developed here. But in doing so, it is also possible to create models that cannot be understood in simple terms and hence cannot be validated, even though they may be more "realistic." There is little point in such an exercise. This book aims to provide the theoretical basis for incrementally adding complexity (and realism) to models. An aim of equal importance, however, is to provide a basis of physical understanding of the mathematical models. It is on this basis that we will interpret the model behavior when new physics or chemistry is incorporated.

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[2] See Wilson [1989] for a introduction to igneous petrology of the mantle.

[3] A caveat: there is no broadly accepted thermochemical model for silicate liquids, whereas for most silicate crystals, a variety of competing thermochemical models exist. Understanding the molecular structure of silicate melts is an area of current research that may help to improve thermochemical theory.

[4] As is the case, for example, of the chemical thermodynamics of mantle melting.

One final, basic physical consideration is important to bear in mind. The theory formulated and analyzed here is a *continuum theory* that is based on volume-averaging of the grain-scale, microscopic physics on a scale much larger than individual mantle grains. It represents fundamental conservation principles (mass, momentum, energy) that are true to an extremely good approximation. But that does not imply that this theory is entirely correct. Representing the microscopic physics at the continuum scale is fraught with the potential for error. For example, approximating the volume average of the product of microscopic variables with the product of their volume averages neglects microscopic correlations between variables. This is a classical problem for mean-field theories, including closures such as the viscosity of an aggregate. Furthermore, in some cases there may be no clear separation of scales between microscopic and continuum. For example, coherent alignment of microscopic pores could lead to anisotropic or discontinuous transport properties at the continuum scale. Moreover, it is plausible that large-scale discontinuities (e.g., magma-filled dikes) could initiate from such features. Such emergent features would have important consequences at the continuum scale but cannot be readily deduced from the continuum model. It is thus important to understand and regularly re-evaluate the assumptions involved in upscaling from the microscopic to the continuum scale.

1.3 Research Questions and Applications

Open questions about terrestrial magmatism motivate this book and the theory that it describes. However, with one exception, it is not our aim to present research that addresses those questions; for that, the reader is directed to the scientific literature and encouraged to take matters into her own hands. But in thinking about the physics and mathematics that is introduced below, it is helpful to have in mind some of the motivating questions. This section provides a brief and incomplete overview.

The exception, alluded to above, is the fundamental question *how does magma move through the asthenosphere and adjust to varying physical and chemical conditions en route?* This book provides a detailed account of a particular hypothesis developed in response to that question. The hypothesis states that magma in the asthenosphere moves by porous flow through a solid matrix that can compact and deform according to viscous rheological laws. This hypothesis is broadly considered to be correct, even if direct evidence to support it is lacking. Of course, direct, in situ evidence of physical processes in the asthenosphere is exceedingly rare. Hence, to test this hypothesis, it is necessary to evaluate its quantitative predictions against the observations that are available.

The porous-flow hypothesis is not the only concept for melt transport in the asthenosphere. Transport through an emergent network of veins and dikes was proposed by Sleep [1984] and developed by Sleep [1988] and, more rigorously, by Rubin [1998]. This hypothesis states that veins form and grow in a partially molten rock under external deviatoric stress. If a vein forms in a plane normal to the least compressive stress (most tensile deviatoric stress), magma flows into the vein due to the difference between the ambient pore pressure and that stress. At the same time, the melt pressure in the vein exceeds the least compressive stress, so the vein dilates and grows. Once the vein has reached a critical vertical extent, the buoyancy of the enclosed melt drives crack opening at the top and crack closing at the bottom; the vein moves upward as a dike (a *magma-fracture*, analogous to a hydrofracture) while it draws additional melt from the surrounding porous medium. These mechanics are certainly relevant for shallow environments with larger deviatoric stress [Rivalta et al., 2015] or capped zones of large magma overpressure [Havlin et al., 2013]. They may also be relevant for discrete heterogeneities [Sleep, 1984] and for mantle depths to about 30 km, depending on the deviatoric stress and background permeability [Nicolas and Jackson, 1982; Nicolas, 1986; Ito and Martel, 2002]. Indeed, seismological evidence in the form of earthquake locations that traverse the subduction-zone mantle wedge support the idea of a brittle mechanism [White et al., 2019]. However, the hypothesis of melt extraction from the asthenosphere by diking has received much less attention than the porous flow theory considered in this book. If diking in the asthenosphere is consistent with observations (more on this below), then its distinct physics must be embedded in the physics considered here.

A second research question, related to the first, is *what is the rate of buoyancy-driven melt transport through the asthenosphere and what mechanics control this rate?* Two lines of observational evidence suggest that this rate is faster than that predicted by diffuse porous flow. The first is disequilibrium in the uranium-series isotopes measured in young lavas.[5] Elemental fractionation of parent and daughter nuclides leads to secular disequilibrium in decay rates, but equilibrium is restored over a time-scale proportional to the half-lives of the elements. Some of these half-lives are of the order of kiloyears to tens of kiloyears, and so observed disequilibria constrain fractionation events to have taken place within that time frame. The hypothesis that fractionation occurs at small melt fractions at great depth in the mantle then suggests that melt transport must be rapid enough to preserve disequilibria. Elliott and Spiegelman [2003] provide an overview and references. U-series disequilibrium in Icelandic lavas indicates melt ascent rates of tens to hundreds of meters per year [Stracke et al., 2006]. Studies for ocean-island basalts and island arcs [Claude-Ivanaj et al., 1998; Turner et al., 2004] come to similar conclusions. There is uncertainty, however, in these interpretations that is associated with the various model assumptions required. For example, in a lithologically heterogeneous mantle, partition coefficients and patterns of melt transport may be substantially more complex than those envisioned by simple models [Weatherley and Katz, 2016].

The second line of observational evidence for fast melt transport is from the reconstruction of eruption rates during and after Icelandic deglaciation. Jull and M^{c}Kenzie [1996] summarized early observations of enhanced volcanic output and showed that they can be explained by accounting for the mantle decompression melting associated with removal of a 2-km-thick ice sheet. If this ice disappears over $\sim$1000 yr, the melting rate beneath it should increase by a factor of $\sim$30. The lag of the volcanic output time series with respect to deglaciation was used by Maclennan et al. [2002] to constrain the rate of buoyancy-driven melt transport to a minimum of $\sim$50 m/yr; to better fit the observations, their model indicated speeds $>$100 m/yr. Eksinchol et al. [2019] developed a more sophisticated model accounting for spatiotemporal evolution of ice removal and mantle isostatic rebound to predict trace-element concentrations. They obtained a best fit to lanthanum concentration data with an average melt speed of 100 m/yr; larger speeds gave a worse fit. However, both of these models assume quasisteady melt transport—that the time scale for adjustment to steady-state melt transport is negligible relative to the time scale of deglaciation. Using a time-dependent model of melt transport, Rees Jones and Rudge [2020] showed that unsteady effects could be

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[5] Models of U-series are developed in section 11.5

mapped onto faster melt speeds in a quasi-steady model. They concluded that the maximum steady-state melt speed beneath Iceland is $\sim$30 m/yr, and that speeds greater than about 10 m/yr are expected beneath most of the global mid-ocean ridge system. These estimates are consistent with constraints from observed U-series disequilibria.

A null hypothesis for melt transport in the mantle is *diffuse porous flow*. This assumes spatially uniform melting and purely vertical melt transport. Assuming magma is produced by upwelling mantle in which the various minerals are uniformly distributed and in contact with each other, we expect melting to be broadly distributed and to create a pervasive network of interconnected pores. The buoyancy of the melt would drive it to segregate vertically. Simple melting column models (as in Chapter 11) tell us that the ratio of the liquid to solid upwelling speeds scales like the maximum degree of melting (say about 20%) divided by the maximum porosity (say about 1%). Hence for mantle upwelling at 10 cm/yr, diffuse porous flow predicts a melt speed of $\sim$2 m/yr (see section 11.1). This is an order of magnitude lower than the observational estimates noted above. One possible resolution to this discrepancy is that porous flow is not diffuse, but is instead *channelized*. Such channelized flow has been inferred from geological observations [Kelemen et al., 1995a] and shown to be consistent with uranium-series disequilibria [Jull et al., 2002; Elliott and Spiegelman, 2003]. It was predicted with reactive-flow theory assuming a homogeneous mantle source [Aharonov et al., 1995, and see Chapter 12] and with numerical models of a heterogeneous source [Weatherley and Katz, 2012]. Volatile components such as water and CO_2 in the mantle source may promote deep channelization [Keller and Katz, 2016]. These models all utilize highly simplified mantle thermochemistry, however, leaving open questions of how magmatic channelization works and whether it is quantitatively consistent with observations including that of rapid melt extraction.

A third research question is *by what forces and processes does lateral melt transport occur?* The buoyancy of liquid magma in the asthenosphere (see section 5.4.1) explains its gravity-driven, vertical ascent through the permeable solid. But volumetric and geochemical evidence indicates that the magma erupted from volcanoes is not simply sourced from a vertical column beneath them—it comes from a volume of mantle much broader than the volcano itself [e.g., Behn and Grove, 2015]. The lateral transport associated with this pooling of magma is referred to as *melt focusing* and is often invoked in tectonic-scale models of magmatism and volcanism. However, the mechanics of melt focusing remain a subject of debate, which has played out mostly in the context of mid-ocean ridge models. Early work by Spiegelman and M^cKenzie [1987] and Morgan [1987] linked magma focusing to the dynamic pressure gradient generated by plate-driven corner flow of the solid phase (see sections 3.3 and 13.2), but this mechanism relies on a large and roughly uniform asthenospheric viscosity that is inconsistent with many estimates. Lateral transport through a high-porosity, high-permeability channel along the sloping base of the lithosphere was proposed by Sparks and Parmentier [1991] and has been a dominant paradigm in recent years; it is described in sections 11.4 and 13.3. In a magnetotelluric study along the Mid-Atlantic Ridge, Wang et al. [2020] obtain inversion results with a striking indication of a sublithospheric channel. In contrast, magnetotelluric tomography on the East Pacific Rise by Key et al. [2013] find no evidence for sublithospheric focusing. The magnetotelluric imaging results from Key et al. [2013] depict a distribution of melt within the asthenosphere that is close to what might be predicted under a third mechanism for focusing, proposed more recently by Turner et al. [2017] and Sim et al. [2020]. In this case, the compaction associated with melt extraction creates pressure gradients that drive lateral flow. Other ideas have been proposed and there is no reason to exclude the possibility that multiple mechanisms contribute.

The research question of *how does heterogeneity of lithology (and hence fusibility) of the mantle affect melting and melt transport?* is something of an "elephant in the room" for all of the questions above. Over hundreds of millions of years (probably a few billions), plate tectonics and magmatism have fractionated mantle components to produce oceanic crust, coated it with sediments, altered it with sea water, and subducted it back into the mantle. This rock, of distinct chemistry and lithology, is stirred by mantle convection but not homogenized. Geochemical evidence shows that it is recycled and remelted beneath plate boundaries and at hot spots [e.g., Stracke, 2012]. The characteristics of this heterogeneity, in terms of its compositions, shapes, and length scales, and volume fraction, are poorly known. Even less clear is what effect it might have on melt transport. Sleep [1984] argued that it could nucleate veins and dikes in the mantle; Richter and Daly [1989] highlighted the potential for reactive porous flow effects. In the petrologically simplified context of a mantle with two chemical components, these effects were explored by Weatherley and Katz [2012] and Jordan and Hesse [2015], who predicted that channelization of flow can occur. The implications of heterogeneity (in two plausible, hypothesized forms drawn from a large possible space) for mid-ocean ridges were investigated by Katz and Weatherley [2012], demonstrating the potential for heterogeneous networks of melt-transport channels to control melting and melt extraction. To what extent these or other results are representative of the natural system of heterogeneous, partially molten asthenosphere remains an open question.

It is broadly clear, however, that melt transport affects how mantle heterogeneity is expressed in basalts that erupt at the surface. Hence petrological and geochemical inferences of mantle heterogeneity rely on assumptions about transport and mixing [e.g., Stracke and Bourdon, 2009]. Thus a fifth research question, which arises in attempting to interpret the chemical signature(s) of erupted basalts, is *what are the distinct contributions of the heterogenous mantle source and the spatially variable extraction process?* This question folds in all of the questions above in that it links the dynamics to the observable chemistry of erupted lava. Channelized melt flow, melt focusing, flow in veins and dikes, and other flow complexities will all have geochemical consequences [e.g., Spiegelman and Kelemen, 2003; Behn and Grove, 2015; Sleep, 1984], and those will play out in the context of the heterogeneity that magma inherits from its mantle source [Weatherley and Katz, 2016]. Addressing this question requires models that couple geochemical transport (see chapter 9) with melting and two-phase flow [Richter, 1986; Navon and Stolper, 1987]. But there is an additional challenge: transport pathways are sensitive to partitioning of elements between the liquid and solid [Spiegelman, 1996]. It is simple to assume that trace-element concentrations in magma are in equilibrium with the solid residue of melting, according to partitioning coefficients, and that these melts are instantaneously extracted with no further interaction with the solid. Although this approach has had success in explaining the systematics of geochemical observations [e.g., White et al., 1992], there are theoretical reasons to doubt its validity. Foremost among these is the extremely slow diffusion of trace elements through the interior of solid grains [van Orman et al., 2001] that explains their observed chemical zonation. Disequilibrium models that account for this diffusion have been formulated [e.g., Kenyon, 1990; Iwamori, 1992; Qin, 1992], but it remains an open question whether their complexity improves the skill of predictions. Moreover, extraction without chemical interaction during transport has obvious conceptual difficulties [Navon and Stolper, 1987]. Hence there remains a great and largely unrealized potential for the systematics residing in abundant geochemical data to constrain the style and pathways of melt transport.

All the melt erupted from volcanoes has traversed the cold thermal boundary layer beneath the surface of the solid Earth. In the lithosphere, temperatures fall below the solidus of rocks of magmatic composition, precluding transport by porous flow. Moreover, lithospheric rocks are cold enough that creeping deformation is negligible on the time scale of melt tranport. Plentiful observations show that under these conditions, magma moves through pressure-driven fractures, dikes, and sills [Rivalta et al., 2015]. What is less clear, however, is *how does transport of magma work at the base of the lithosphere, at the transition between porous flow beneath and brittle fracture above?* This has broad implications that include the geochemistry of lavas, frequency and style of volcanic eruptions, the spatial distribution of volcanism, and the creation of mantle heterogeneity. Two-phase models of viscous deformation predict melt pooling and crystallization at the lithosphere–asthenosphere boundary (LAB) [e.g., Ghods and Arkani-Hamed, 2000; Katz, 2008; Keller et al., 2017]. In contrast, Havlin et al. [2013] hypothesized that accumulated, overpressured melt at the LAB will readily enter the lithosphere through dikes, where it will freeze. One means to addressing this problem is through the use of continuum, two-phase models that capture elastic/brittle deformation in addition to viscous flow. Keller et al. [2013] demonstrated feasibility of such models and showed the emergence of remarkably dike-like features, but further work is needed to validate this approach and explore the behavior of models in tectonically realistic contexts.

Other current research questions are derived from the consideration of particular tectonic environments. Mid-ocean ridges can be modeled with the simplest boundary conditions and material properties; they are the best-studied tectonic environment (see chapters 11and 13). Subduction zones are of great interest because of their role in forming continental crust, element cyling through the deep Earth, and the hazards associated with their volcanoes (and earthquakes). The two-phase dynamics of subduction zones is complicated: a water- and carbon-rich liquid enters the mantle wedge from the relatively cold slab and percolates toward higher temperatures, where it promotes *flux melting* at the volatile-saturated solidus. The aqueous liquid becomes hydrous magma; its viscosity and density increase. It is driven upward by buoyancy and eventually interacts with the lithosphere. Cooler temperatures lead to magma evolution and lateral transport. Some magma is erupted but much is frozen into the lithosphere. This vague but probably accurate overview leaves many open questions. Where in the wedge does flux melting take place? What role does decompression melting play? Is melt transport channelized? How much lateral melt flow occurs and by what mechanisms? What roles do buoyancy forces play in shaping the solid flow? Does the subducting slab itself or its lamination of sediments ever melt? Do the sediments rise into the wedge as diapirs? What is the chemistry of the residue that is dragged back into the mantle? What are the fluxes of volatile elements through subduction zones and, in particular, into the deep mantle? In summary, *what are the two-phase dynamics of subduction zones and how does this relate to chemical cycling through the mantle?* There is a vast literature that considers these and other questions; it will not be reviewed here. Recent numerical models of two-phase flow, however, have developed the framework in which such questions can be studied quantitatively [Wilson et al., 2014; Cerpa Gilvonio et al., 2017]. But these models exclude phase change, considering only transport of the aqueous liquid. Going beyond this requires stable and efficient numerical simulations of the nonlinear interactions between melt flow, chemical reaction, and thermal evolution in the context of large temperature and compositional gradients. Developing these is a major challenge.

Magmatism may also occur at the bottom of the mantle, above the core–mantle boundary [Fiquet et al., 2010]. Stixrude and Karki [2005] showed that melts in the deep mantle are more compressible and probably higher density than the solid residue, so they would be expected to sink toward the core. This has led to the hypothesis that ultralow seismic velocity zones (ULVZ) above the core–mantle boundary are regions of dense, pooled melt [reviewed by McNamara, 2019]. Melt at the core–mantle boundary, if it exists, may be the last remanents of a basal magma ocean that has slowly crystallized over the age of the Earth [Labrosse et al., 2007], or may be the product of mantle dynamics and heat derived from the core. In any case, the physical and chemical conditions just above the core–mantle boundary are highly uncertain, making two-phase models of this region almost entirely unconstrained. In this context where there are many open research questions, anything but the simplest models are potentially misleading.

A colder and more readily observable context where two-phase dynamics may be crucial is in glaciers and ice sheets. Glacial ice is a monomineralic, polycrystalline solid—a rock, broadly construed. When ice melts at grain boundaries, pores form with a small dihedral angle (see chapter 5), creating a permeable network. Indeed, an early derivation of the equations of two-phase flow by Fowler [1984] was motivated by the water/ice problem [see also Schoof and Hewitt, 2016]. Glacial ice at the melting temperature, called *temperate ice*, is now recognized as a significant part of ice sheets. It is found near the bed, where the ice is heated by frictional dissipation and geothermal heating, and insulated from the cold atmosphere [Hewitt and Schoof, 2017]. It is also found at the shear margins of ice streams, where viscous dissipation supplies heat [Jacobson and Raymond, 1998]. Englacial pore water may significantly decrease the viscosity of temperate ice relative to cold ice [Duval, 1977], and this may lead to a positive feedback that sharpens the margins of ice streams [Haseloff et al., 2019]. The behavior of partially molten, temperate ice remains little explored, however, leaving open the broad question *what is the role of temperate ice in the dynamics of glaciers and ice sheets?* This also applies to the multiphase dynamics of firn, the unconsolidated sediment of snow and recrystallized grains that accumulates on the surface of glaciers and ice sheets [Meyer and Hewitt, 2017].

Interaction of liquid and solid phases is ubiquitous in cold, crustal rocks and there is a vast literature on crustal hydrology. In this context, multiphase flow means that there are multiple fluid phases present (the crust is modeled as an elastic solid). Most interest has focused on problems of reactive flow and chemical transport—for example, the motion of contaminants or hydrocarbons through the subsurface. What is less common but more relevant for this book is research that addresses the mechanical interaction between phases. This can occur when reaction causes density changes that promote solid microfracture [Yakobson, 1991], when fluid flow causes solid dissolution and modified permeability [Hoefner and Fogler, 1988; Hinch and Bhatt, 1990], or when liquid overpressure creates macroscopic fractures [Fyfe, 2012]. A research question that relates to the present topic is *under what conditions does crustal flow and reaction enhance permeability and hence promote flow and when does it have the opposite effect?* Evans et al. [2018] developed models extended from the theory of magma/mantle dynamics that address this in the context of carbon sequestration in ultramafic crustal rocks [see also Malthe-Sørenssen et al., 2006; Røyne et al., 2008; Rudge et al., 2010]. In Evans et al. [2020], they followed on to study how the volume change of reaction can cause microfracture that promotes further reaction. The mechanics of the rock in this context is poroelastic [e.g., Biot, 1941; MacMinn et al., 2016], whereas that of the asthenosphere can be considered poroviscous [e.g., M^{c}Kenzie, 1984; Bercovici et al., 2001a].

In the planetary context, rock is often considered to be simultaneously viscous and elastic. The relative weighting of these mechanisms in the response to stress depends on the time scale of the process that is forcing deformation. On the time scale of seismic waves, elasticity is dominant (but seismic attenuation indicates that viscous mechanisms are present too). On the time scale of mantle convection, viscous flow is dominant. But at the intermediate time scale of tidal deformation, both viscous and elastic mechanisms can play a role (though the details of this combination remain a subject of research [Bierson and Nimmo, 2016; Renaud and Henning, 2018]). Dissipation of heat by tidal deformation attests to the importance of viscous mechanisms. With the increasing number of known exoplanets, an emergent research question is *how do tidal dissipation, mantle convection and magmatism interact to determine the structure and dynamics of tidally heated bodies?* In Jupiter's moon Io, as an extreme example, tidal dissipation causes partial melting throughout the silicate mantle [Peale et al., 1979]. Mantle convection transports heat too slowly to achieve a thermal balance under such rapid heating, but magmatic production and segregation can keep pace [Moore, 2003]. Hence heat export relies on transport of magma across the cold lithosphere and out of volcanoes, a process termed heat-piping [O'Reilly and Davies, 1981]. But heat-piping must be inefficient on Io to prevent lithospheric growth to a thickness much greater than observed [Spencer et al., 2020a]. In this context, where magma transport dominates the heat budget and promotes lithological stratification that is potentially unstable [Spencer et al., 2020b], what is the role of mantle convection?

The questions posed in this section are merely a sample of a much larger set that arises from the interaction of liquids and solids in Earth and planets. Theory developed in this book is directly relevant to some of these questions, and indirectly relevant to others. But the concepts and mathematical framework introduced here can provide a base from which to extend into related areas of theory. And, of course, for any particular question arising from natural observations, the theoretical tools that are readily available may guide—but should never constrain—the hypotheses posed to answer it.

1.4 About This Book

1.4.1 OVERVIEW OF THE ORGANIZATION AND CONTENT

The next chapter provides a brief history of the theory of the two-phase dynamics of partially molten asthenosphere in terms of the key investigators, influential or innovative publications, and ongoing themes.

Chapter 3 is a very brief review of single-phase mantle convection. The governing equations are presented without derivation and it is assumed that the reader is already familiar, having previously studied them elsewhere. They are used to illustrate some physical/mathematical concepts that are relevant for two-phase flow. A solution for isoviscous, incompressible Stokes flow is derived to model mantle flow beneath a mid-ocean ridge. This result is used later in the book as a background on which melt transport is computed.

Chapter 4 considers the equations for conservation of mass and momentum of the partially molten aggregate. Mathematical tools and notation are introduced to describe the physics at the grain scale; volume averages then lead to PDEs for continuum variables. Physical arguments are presented that justify assumptions required for computing these averages. In particular, we discuss the dominant theory for the interphase force, exerted by the liquid on the solid and vice versa. General viscous constitutive laws are derived without specifying how viscosities depend on other parameters. These components are then assembled into the full system of equations representing conservation of mass and momentum for the liquid and solid phases. The chapter concludes with a discussion of special cases in which the full system can be simplified into recognizable forms.

In chapter 5 we consider the material properties of partially molten mantle, providing more detail than was presented in chapter 4 but much less books that focus on, for example, rock mechanics. The chapter begins with a discussion of the physics at the microscopic scale that governs the evolution of grain size and the wetting of olivine grains by basaltic melt. An idealized model of pore geometry is developed. This is followed by short studies of the permeability, the shear and compaction viscosity of the aggregate, and the liquid viscosity. Finally we discuss the melting rate, though a more detailed treatment is deferred to later in the book.

Chapters 6 and 7 concern solutions of the governing equations under idealized conditions. These solutions are useful for developing an understanding of the physical behaviour that is encoded in the governing equations and also for recognizing characteristic solutions that may appear in more complex models. Chapter 6 focuses on elucidating compaction and the compaction length—a length scale that emerges from liquid–solid interaction in the framework of two-phase fluid dynamics. We consider canonical problems including magmatic solitary waves. To focus on the key physics, models presented are one-dimensional.

In chapter 7 we consider the role of shear in establishing pressure gradients that drive liquid segregation. The models are motivated by laboratory experiments on partially molten rocks in which deformation leads to an instability and the emergence of high-porosity sheets oriented at a low angle to the shear plane. Models in this chapter are two-dimensional and hence the sheets appear as bands. Although the compaction length also features prominently, this chapter primarily examines how the viscosity of the two-phase aggregate can feed back into the dynamics.

Chapters 8 and 9 develop theory for conservation of energy and conservation of mass for chemical species. In the former, much of the development is in casting the first law of thermodynamics in terms of different variables: internal energy, enthalpy, temperature and entropy. The latter chapter develops the governing equations for conservation of species mass, looking at the different cases of species: thermodynamic components, trace elements and radiogenic trace elements. In both chapters, relatively simple applications are developed as demonstrations of the physics.

Chapter 10 provides a brief introduction to modeling the thermochemistry of mantle petrology. It sets out a simple but useful approach, based on ideal solution theory, for approximating the equilibrium phase fractions and compositions in a two-phase system with an arbitrary number of chemical components. An extension to disequilibrium thermodynamics is also discussed.

Chapter 11 develops one-dimensional models of melting and buoyancy-driven melt segregation. These *column models* impose upwelling of the solid phase to drive the melting. Conservation of energy and species mass are used to couple the thermochemistry of melting to the mechanics of flow. These models demonstrate that Darcy drag is the key force resisting buoyancy over most of the partially molten region. The chapter also considers the decompaction boundary layer beneath the lithosphere. It concludes with a consideration of uranium-series disequilibrium in the context of column models.

Chapter 12 further analyzes the buoyancy-driven vertical segregation of magma, focusing on the reactive corrosivity of upwelling melts and its dynamical consequences. The analysis demonstrates the tendency for reactive localization of magma into high-flux channels with compacted regions between them.

Chapter 13 concerns application of the foregoing theory to models of tectonic scale processes and, in particular, to mid-ocean ridges. It begins with a rescaling and simplification of the equations, assuming small porosity. This leads to a system of equations with a partial decoupling of compaction from large-scale shear flow, which simplifies calculations and enables re-use of existing codes for single-phase Stokes problems. Armed with this approximate formulation, the chapter revisits the mid-ocean ridge mantle-flow problem from chapter 3 and layers on top of it a calculation of magmatic segregation. It then develops a simple model for melt focusing based on a high-porosity channel along the lithosphere–asthenosphere boundary, where freezing leads to an impermeable barrier to vertical flow. The chapter concludes with a discuss of the enthalpy method, an approach to directly coupling the fluid dynamics with the thermochemistry.

Chapter 14 concludes the book with a brief introduction to the numerical methods that have been used to solve the equations of magma/mantle dynamics. It focuses on the finite difference/volume method but also introduces the finite-element method, demonstrating the their effectiveness on benchmark problems.

1.4.2 REFERENCES TO THE LITERATURE

Most of the chapters of this book are written with minimal reference to the literature. This should not be taken as evidence that the theory and ideas presented originate from the author; most of them do not. Rather, references are omitted from the main text to avoid distraction from the flow of concepts and connections. The hope is that by reading this book, one can rapidly obtain the background knowledge that is required for comprehension of the specialist literature.

A section entitled "literature notes" is provided at the end of each chapter. These sections serve two purposes. First, they cite and describe the publications from which the content of the relevant chapter was derived. Second, they provide some advice about further reading: publications that extend the theory, or that approach it from an experimental or observational perspective.

Decisions about what to include and exclude from the literature notes are subjective and based on incomplete information. There is no clear line dividing what is relevant and what is irrelevant. And while the author has a reasonable knowledge of the literature that is immediately related to the book, there are undoubtedly publications that have escaped his attention. Lastly, of course, the relevant literature evolves as new contributions are added and older ones are reevaluated.

1.4.3 MATHEMATICAL NOTATION

This is, predominantly, a book about fluid dynamics. Thus it is appropriate that we exercise care in our use of the word *fluid*. We shall consider a fluid to be a substance that flows in response to a stress applied over a sufficient duration of time. The mantle, which is a solid, behaves elastically when stress fluctuates over short time scales (e.g., seismic waves).[6] In response to the long-term stresses associated with mantle convection and plate tectonics, the mantle flows: the rate of strain is proportional to the stress, though this relationship is not always linear. Since liquid magma is obviously a fluid, we are dealing with a system of two fluids. It is therefore sensible to avoid using "fluid" as a label for either phase.[7] Instead, we opt for the labels

$$\ell \rightarrow \text{liquid phase, magma;}$$
$$s \rightarrow \text{solid phase, mantle.}$$

This has the potential to become confusing in the context of subduction zones, where the water-rich material released by the subducting slab is referred to as "fluid" to distinguish it from silicate-rich magma. However for present purposes, our priority is to avoid misuse of the term "fluid."

The phase labels will be applied to variables as superscripts. For example, the density of liquid is ρ^ℓ while that of the solid is ρ^s. We can apply this to phase fractions too: ϕ^ℓ and ϕ^s are the phase fractions of liquid and solid, respectively. We shall always assume that the liquid and solid together occupy the full space, meaning that there are no voids or other phases present; hence we take $\phi \equiv \phi^\ell$ such that $\phi^s = (1 - \phi)$. The $\equiv$ sign signifies an equality that is true by definition of one of the variables (ϕ in this case). Using the phase fractions, we can define the meaning of the overline notation,

$$\overline{\rho} \equiv \phi\rho^\ell + (1-\phi)\rho^s.$$

The symbol $\overline{\rho}$ thus represents the *phase-averaged* or *bulk* density. An overline on any symbol (or group of symbols) will have the equivalent meaning for that quantity. We also define the phase difference with a Δ,

$$\Delta\rho \equiv \rho^s - \rho^\ell,$$

in terms of the solid-minus-liquid difference. We shall retain this definition in chapter 10 even though it is inconsistent with thermodynamic convention.

Superscripts will also be used to represent exponents. A symbol that would require multiple superscripts will be placed into parentheses, e.g., $\left(q^\ell\right)^2$. Finally, superscripts will be used to index the entries in a series expansion. In this case, the index itself will appear in parentheses, e.g.,

$$q = q^{(0)} + \epsilon q^{(1)} + \epsilon^2 q^{(2)} + \ldots$$

is an expansion of q in powers of a small parameter, $\epsilon \ll 1$. We will refer to terms in such power series according to their order, meaning their approximate size, and represent this with the notation $\mathcal{O}()$. Hence the $q^{(0)}$ term is $\mathcal{O}^{(1)}$, the $q^{(1)}$ term is $\mathcal{O}(\epsilon)$, and so on. The same notation will be used more broadly to represent the approximate size of any quantity.

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[6] Though there is some component of anelastic deformation in seismic waves, leading to dispersion and attenuation.

[7] In the literature on magma/mantle dynamics, f (fluid) has typically been adopted as the label for the magma and m (matrix) for the mantle.

Vectors and tensors will be represented by bold symbols, with no notational distinction between them. Hence it will be up to the reader to make this distinction according to the context in which the symbol is used. Superscripts on vectors and tensors will typically represent the phase: $\boldsymbol{v}^i$ is the velocity vector of phase i, where i can be s or ℓ. Subscripts are used to denote components of a vector or tensor. Hence v_j^ℓ is the jth component of the liquid velocity. Generally we will avoid index notation for vector and tensor operations, except where it contributes to the clarity of a calculation. Furthermore, it shall be convenient to treat sets of chemical concentrations as pseudo-vectors. For example, c_j^i is the concentration of the j^{th} chemical species in phase i. Concentration pseudo-vectors cannot be considered true vectors because the algebraic operations of vector calculus don't apply (e.g., change of coordinates).

We shall reserve the non-italic symbol d as a prefix to denote infinitesimal quantities. Thus $\mathrm{d}Q$ is an arbitrarily small but nonzero change in Q. Other details of notation will be addressed where they arise in the book.

Occasionally it will be useful to rescale variables, usually with the aim of non-dimensionalizing them. If a variable q has a value that is typically of the same order as $[q]$, then we can define a rescaled, non-dimensional variable $q' \equiv q/[q]$. Hence the notation $[q]$ means a dimensional constant that represents the scale of the variable q. Typically we will drop the $'$ on rescaled variables to avoid clutter.

Table 1 lists important symbols used throughout the book.

1.5 The Way Forward?

Research on the two-phase dynamics of the magma/mantle system is still in its early stages, despite the basic theory having been around for more than 30 years. The field is now advancing rapidly, with progress in the theoretical development, constitutive hypotheses, numerical methods and software, and applications to the Earth and other planets. This book makes no attempt to explain developments at the cutting edge, some of which may soon be superseded by others. Instead it provides a foundation of knowledge from which the reader can access, critically evaluate, and contribute to research at the cutting edge. Thus this book is a starting point and/or reference point for the reader.

There are legitimate scientific reasons to question some of the theory that is developed here. Of course the validity of the basic conservation principles, for present purposes, is entirely assured. However, the physical assumptions that are used to adapt those principles to the magma/mantle system are difficult to robustly validate. This validation must ultimately come from a comparison of model predictions with measurements of the natural system being modelled. Those measurements can be obtained in laboratory experiments or from the natural world. But the solid Earth that we seek to model is a complicated and inaccessible place; our observations of it are made from the surface and are generally only proxies for the dynamics, convolving those dynamics in time or space or both. Interpretation of proxy or laboratory data in terms of the two-phase dynamics in the mantle is far from straightforward, but in fact this should be the goal of geoscientists who work with the theory.

Theory that is inconsistent with the natural system is incorrect or incomplete. Deficiencies of theory, where they can be identified, provide the impetus and direction for developments that bring it closer into line with the truth. However, one should also be cognisant that inconsistency with nature is not always straightforward to prove, especially when the constraints derived from observations are interpretations (i.e., models) themselves. We thus face the challenge of testing theory against observations of complex proxies that are, at best, integral constraints on the dynamics. To overcome this, one must take a multidisciplinary approach; it is insufficient to apply any single observational constraint independently. Instead one should test models of two-phase magma/mantle against a diverse dataset including geophysical, geochemical and laboratory (rock mechanics and petrology) data. But to test the theory, one must generate testable predictions from the theory. Hence it is insufficient for the community of geoscientists working with this theory to solve only the PDEs associated with the fluid dynamics. We must go further to quantitatively predict the implications for, e.g., radiogenic disequilibrium of the uranium-series isotopes, seismic anisotropy of partially molten regions, crustal thickness and trace element composition, porosity distribution in deformation experiments on partially molten rock, and more.

To achieve such predictions we require associated theory, some of which is discussed in the chapters of this book. But we also need multi-pronged analysis of the theory. Numerical solutions are necessary; obtaining them efficiently in the multi-physical, multi-dimensional context of tectonic-scale applications is currently a key challenge. But numerical solutions are not enough. We also need analytical solutions of representative problems of reduced complexity to enable mathematical insights into the problem, and we need basic scaling analysis to explain the robust features of the numerical and analytical models. It is not necessary for each geoscience practitioner to do all of these things simultaneously, but progress will require that all are aware of and knit together the results from each of these perspectives and, crucially, from the observations too—a tall order indeed.

That said, I think the chapters below will also demonstrate the inherent value of and interest in the mathematical analysis and extension of the PDEs. Hopefully this book will make the theory more accessible to analysts who are not particularly interested in partial melting of the Earth (or any other planet). And hopefully their results will provide tools and insights that can be adopted by geoscientists.

For the author (and perhaps for the reader), a fundamental point is as follows. Igneous products (lavas, magmas, residues) that are exposed on the surface of the solid Earth and other planets carry chemical, textural, and thermal information from the planet's interior. Measurements of these lavas are valuable in their own right, but without other constraints, only non-unique inferences about the interior of the planet and its evolution can be drawn from such measurements. The theory of magma/mantle dynamics introduces constraints in the form of conservation of mass, momentum, and energy and it incorporates explicit, quantitative statements of material and transport properties. The inclusion of such information doesn't guarantee a unique fit to the data, but it should reduce the range of possible explanations and rule out those that are physically inconsistent. Moreover, such model calculations can provide corollary predictions that are testable with new observations. This complementarity can sharpen the scientific question and eventually hone in on an answer.

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