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IS THE UNIVERSE INFINITE OR IS IT JUST REALLY BIG?

Some of the great mathematicians killed themselves. The lore is that their theories drove them mad, though I suspect they were just lonely, isolated by what they knew. Sometimes I feel the isolation. I'd like to describe what I can see from here, so you can look with me and ease the solitude, but I never feel like giving rousing speeches about billions of stars and the glory of the cosmos. When I can, I like to forget about maths and grants and science and journals and research and heroes.

Boltzmann is the one I remember most and his student Ehrenfest. Over a century ago the Viennese-born mathematician Ludwig Boltzmann (1844–1906) invented statistical mechanics, a powerful description of atomic behaviour based on probabilities. Opposition to his ideas was harsh and his moods were volatile. Despondent, fearing disintegration of his theories, he hanged himself in 1906. It wasn't his first suicide attempt, but it was his most successful. Paul Eherenfest (1880–1933) killed himself nearly thirty years later. I looked at their photos today and searched their eyes for depression and desperation. I didn't see them written there.

My curiosity about the madness of some mathematicians is morbid but harmless. I wonder if alienation and brushes with insanity are occupational hazards. The first mathematician we remember encouraged seclusion. The mysterious Greek visionary Pythagoras (about 569 BC– about 475 BC) led a secretive, devout society fixated on numbers and triangles. His social order prospered in Italy millennia before labour would divide philosophy from science, mathematics from music. The Pythagoreans believed in the mystical meaning of numbers and developed a religion tending towards occult numerology. Their faith in the sanctity of numbers was shaken by their own perplexing mathematical discoveries. A Pythagorean who broke his vow of secrecy and exposed the enigma of numbers that the group had uncovered was drowned for his sins. Pythagoras killed himself too. Persecution may have incited his suicide, from what little we know of a mostly lost history.

When I tell the stories of their suicide and mental illness, people always wonder if their fragility came from the nature of the knowledge – the knowledge of nature. I think rather that they went mad from rejection. Their mathematical obsessions were all-encompassing and yet ethereal. They needed their colleagues beyond needing their approval. To be spurned by their peers meant death of their ideas. They needed to encrypt the meaning in others' thoughts and be assured their ideas would be perpetuated.

I can only write about those we've recorded and celebrated, if posthumously. Some great geniuses will be forgotten because their work will be forgotten. A bunch of trees falling in a forest fearing they make no sound. Most of us feel the need to implant our ideas at the very least in others' memories so they don't expire when our own memories become inadequate. No one wants to be the tree falling in the forest. But we all risk the obscurity ushered by forgetfulness and indifference.

I admit I'm afraid sometimes that no one is listening. Many of our scientific publications, sometimes too formal or too obscure, are read by only a handful of people. I'm also guilty of a self-imposed separation. I know I've locked you out of my scientific life and it's where I spend most of my time. I know you don't want to be lectured with disciplined lessons on science. But I think you would want a sketch of the cosmos and our place in it. Do you want to know what I know? You're my last hope. I'm writing to you because I know you're curious but afraid to ask. Consider this a kind of diary from my social exile as a roaming scientist. An offering of little pieces of the little piece I have to offer.

I will make amends, start small, and answer a question you once asked me but I never answered. You asked me once: what's a universe? Or did you ask me: is a galaxy a universe? The great German philosopher and alleged obsessive Immanuel Kant (1724–1804) called them universes. All he could see of them were these smudges in the sky. I don't really know what he meant by calling them universes exactly, but it does conjure up an image of something vast and grand, and in spirit he was right. They are vast and grand, bright and brilliant, viciously crowded cities of stars. But universes they are not. They live in a universe, the same one as us. They go on galaxy after galaxy endlessly. Or do they? Is it endless? And here my troubles begin. This is my question. Is the universe infinite? And if the universe is finite, how can we make sense of a finite universe? When you asked me the question I thought I knew the answer: the universe is the whole thing. I'm only now beginning to realize the significance of the answer.

3 SEPTEMBER 1998

Warren keeps telling everyone we're going back to England, though, as you know, I never came from England. The decision is made. We're leaving California for England. Do I recount the move itself, the motivation, the decision? It doesn't matter why we moved, because the memory of why is paling with the wear. I do remember the yard sales on the steps of our place in San Francisco. All of my coveted stuff. My funny vinyl chairs and chrome tables, my wooden benches and chests of drawers. It's all gone. We sit out all day as the shade of the buildings is slowly invaded by the sun and we lean against the dirty steps with some reservation. Giant coffees come and go and we drink smoothies with bee pollen or super blue-green algae in homage to California as the neighbourhood parades past and my pile of stuff shifts and shrinks and slowly disappears. We roll up the cash with excitement, though it is never very much.

When it gets too cold or too dark we pack up and go back inside. I'm trying to finish a technical paper and sort through my ideas on infinity. For a long time I believed the universe was infinite. Which is to say, I just never questioned this assumption that the universe was infinite. But if I had given the question more attention, maybe I would have realized sooner. The universe is the three-dimensional space we live in and the time we watch pass on our clocks. It is our north and south, our east and west, our up and down. Our past and future. As far as the eye can see there appears to be no bound to our three spatial dimensions and we have no expectation for an end to time. The universe is inhabited by giant clusters of galaxies, each galaxy a conglomerate of a billion or a trillion stars. The Milky Way, our galaxy, has an unfathomably dense core of millions of stars with beautiful arms, a skeleton of stars, spiralling out from this core. The earth lives out in the sparsely populated arms orbiting the sun, an ordinary star, with our planetary companions. Our humble solar system. Here we are. A small planet, an ordinary star, a huge cosmos. But we're alive and we're sentient. Pooling our efforts and passing our secrets from generation to generation, we've lifted ourselves off this blue and green watersoaked rock to throw our vision far beyond the limitations of our eyes.

The universe is full of galaxies and their stars. Probably, hopefully, there is other life out there and background light and maybe some ripples in space. There are bright objects and dark objects. Things we can see and things we can't. Things we know about and things we don't. All of it. This glut of ingredients could carry on in every direction forever. Never ending. Just when you think you've seen the last of them, there's another galaxy and beyond that one another infinite number of galaxies. No infinity has ever been observed in nature. Nor is infinity tolerated in a scientific theory – except we keep assuming the universe itself is infinite.

It wouldn't be so bad if Einstein hadn't taught us better. And here the ideas collide so I'll just pour them out unfiltered. Space is not just an abstract notion but a mutable, evolving field. It can begin and end, be born and die. Space is curved, it is a geometry, and our experience of gravity, the pull of the earth and our orbit around the sun, is just a free fall along the curves in space. From this huge insight people realized the universe must be expanding. The space between the galaxies is actually stretching even if the galaxies themselves were otherwise to stay put. The universe is growing, ageing. And if it's expanding today, it must have been smaller once, in the sense that everything was once closer together, so close that everything was on top of each other, essentially in the same place, and before that it must not have been at all. The universe had a beginning. There was once nothing and now there is something. What sways me even more, if an ultimate theory of everything is found, a theory beyond Einstein's, then gravity and matter and energy are all ultimately different expressions of the same thing. We're all intrinsically of the same substance. The fabric of the universe is just a coherent weave from the same threads that make our bodies. How much more absurd it becomes to believe that the universe, space and time could possibly be infinite when all of us are finite.

So this is what I'll tell you about from beginning to end. I've squeezed down all the facts into dense paragraphs, like the preliminary squeeze of an accordion. The subsequent filled notes will be sustained in later letters. You could say this is the story of the universe's topology, the branch of mathematics that governs finite spaces and an aspect of spacetime that Einstein overlooked. I don't know how this story will play itself out, but I'm curious to see how it goes. I'll try to tell you my reasons for believing the universe is finite, unpopular as they are in some scientific crowds, and why a few of us find ourselves at odds with the rest of our colleagues.

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