Contents

Color insert follows page 240

Lockdown Mathematics: A Historical Perspective

Viktor Blåsjö

Isolation and Productivity

"A mathematician is comparatively well suited to be in prison." That was the opinion of Sophus Lie, who was incarcerated for a month in 1870. He was 27 at the time. Being locked up did not hamper his research on what was to become Lie groups. "While I was sitting for a month in prison . . ., I had there the best serenity of thought for developing my discoveries," he later recalled [11, pp. 147, 258].

Seventy years later, André Weil was to have a very similar experience. The circumstances of their imprisonments—or perhaps the literary tropes of their retellings—are closely aligned. Having traveled to visit mathematical colleagues, both found themselves engrossed in thought abroad when a war broke out: Lie in France at the outbreak of the Franco-Prussian War, and Weil in Finland at the onset of World War II. They were both swiftly suspected of being spies, due to their strange habits as eccentric mathematicians who incessantly scribbled some sort of incomprehensible notes and wandered in nature without any credible purpose discernible to outsiders. Both were eventually cleared of suspicion upon the intervention of mathematical colleagues who could testify that their behavior was in character for a mathematician and that their mysterious notebooks were not secret ciphers [11, pp. 13–14, 146–147; 13, pp. 130–134].

Weil was deported back to France, where he was imprisoned for another few months for skirting his military duties. Like Lie, he had a productive time in prison. "My mathematics work is proceeding beyond my wildest hopes, and I am even a bit worried—if it's only in prison that I work so well, will I have to arrange to spend two or three months locked up every year?" "I'm hoping to have some more time here to finish in peace and quiet what I've started. I'm beginning to think that nothing is more conducive to the abstract sciences than prison." "My sister says that when I leave here I should become a monk, since this regime is so conducive to my work."

Weil tells of how colleagues even expressed envy of his prison research retreat. "Almost everyone whom I considered to be my friend wrote me at this time. If certain people failed me then, I was not displeased to discover the true value of their friendship. At the beginning of my time in [prison], the letters were mostly variations on the following theme: 'I know you well enough to have faith that you will endure this ordeal with dignity.' . . . But before long the tone changed. Two months later, Cartan was writing: 'We're not all lucky enough to sit and work undisturbed like you.'" And Cartan was not the only one: "My Hindu friend Vij[ayaraghavan] often used to say that if he spent six months or a year in prison he would most certainly be able to prove the Riemann hypothesis. This may have been true, but he never got the chance."

But Weil grew weary of isolation. He tried to find joy in the little things: "[In the prison yard,] if I crane my neck, I can make out the upper branches of some trees." "When their leaves started to come out in spring, I often recited to myself the lines of the *Gita*: '*Patram puspam phalam toyam* . . .' ('A leaf, a flower, a fruit, water, for a pure heart everything can be an offering')." Soon he was reporting in his letters that "My mathematical fevers have abated; my conscience tells me that, before I can go any further, it is incumbent upon me to work out the details of my proofs, something I find so deadly dull that, even though I spend several hours on it every day, I am hardly getting anywhere" [13, pp. 142–150].

Judging by these examples, then, it would seem that solitary confinement and a suspension of the distractions and obligations of daily life could be very conducive to mathematical productivity for a month or two, but could very well see diminishing returns if prolonged. Of course, it is debatable whether coronavirus lockdown is at all analogous to these gentleman prisons of yesteryear. When Bertrand Russell was imprisoned for a few months for pacifistic political actions in 1918, he too "found prison in many ways quite agreeable. . . . I read enormously; I wrote a book, *Introduction to Mathematical Philosophy*." But his diagnosis of the cause of this productivity is less relatable, or at least I have yet to hear any colleagues today exclaiming about present circumstances that "the holiday from responsibility is really delightful" [9, pp. 29–30, 32].

Mathematics Shaped by Confinement

"During World War II, Hans Freudenthal, as a Jew, was not allowed to work at the university; it was in those days that his interest in mathematics education at primary school level was sparked by 'playing school' with his children—an interest that was further fueled by conversations with his wife." This observation was made in a recent editorial in *Educational Studies in Mathematics* [1]—a leading journal founded by Hans Freudenthal. Coronavirus lockdown has put many mathematicians in a similar position today. Perhaps we should expect another surge in interest in school mathematics among professional mathematicians.

Freudenthal's contemporary Jakow Trachtenberg, a Jewish engineer, suffered far worse persecution, but likewise adapted his mathematical interests to his circumstances. Imprisoned in a Nazi concentration camp without access to even pen and paper, he developed a system of mental arithmetic. Trachtenberg survived the concentration camp and published his calculation methods in a successful book that has gone through many printings and has its adherents to this day [12].

Another Nazi camp was the birthplace of "spectral sequences and the theory of sheaves . . . by an artillery lieutenant named Jean Leray, during an internment lasting from July 1940 to May 1945." The circumstances of the confinement very much influenced the direction of this research: Leray "succeeded in hiding from the Germans the fact that he was a leading expert in fluid dynamics and mechanics. . . . He turned, instead, to algebraic topology, a field which he deemed unlikely to spawn war-like applications" [10, pp. 41–42].

An earlier case of imprisonment shaping the course of mathematics is Jean-Victor Poncelet's year and a half as a prisoner of war in Russia. Poncelet was part of Napoleon's failed military campaign of 1812 and was only able to return to France in 1814. During his time as a prisoner, he worked on geometry. Poncelet had received a first-rate education in mathematics at the École Polytechnique, and his role in the military was as a lieutenant in the engineering corps. In his Russian prison, he did not have access to any books, so he had to work out all the mathematics he knew from memory. Perhaps it is only because mathematics lends itself so well to being reconstructed in this way that Poncelet ended up becoming a mathematician; other scientific or engineering interests would have been harder to pursue in isolation without books. The absence of books for reference would also naturally lead to a desire to unify geometrical theory and derive many results from a few key principles in Poncelet's circumstances. This is a prominent theme in early nineteenth-century geometry overall; it was not only the imprisoned who had this idea. But it is another sense in which Poncelet could make a virtue out of necessity with the style of mathematics he was confined to during his imprisonment.

The same can be said for another characteristic of early nineteenth-century geometry, namely, the prominent role of visual and spatial intuition. This too was a movement that did not start with Poncelet, but was fortuitously suited to his circumstances. Consider, for instance, the following example from the *Géométrie descriptive* of Monge, who had been one of Poncelet's teachers at the École Polytechnique. Monge was led to consider the problem of representing three-dimensional objects on a plane for purposes of engineering, but he quickly realized that such ideas can yield great insights in pure geometry as well, for instance, in the theory of poles and polars, which is a way of realizing the projective duality of points and lines. The foundation of this theory is to establish a bijection between the set of all points and the set of all lines in a plane. Polar reciprocation with respect to a circle associates a line with every point and a point with every line as follows. Consider a line that cuts through the circle (Figure 1). It meets the circle at two points. Draw the tangents to the circle through these points. The two tangents meet in a point. This point is the pole of the line. Conversely, the line is the polar of the point.

But what about a line outside the circle (or, equivalently, a point inside the circle, Figure 2)? Let L be such a line. For every point on L

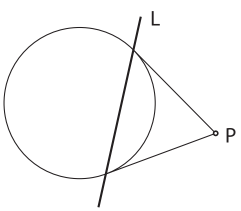

FIGURE 1. Polar reciprocation with respect to a circle: simplest case. Points P outside the circle are put in one-to-one correspondence with lines L intersecting the circle.

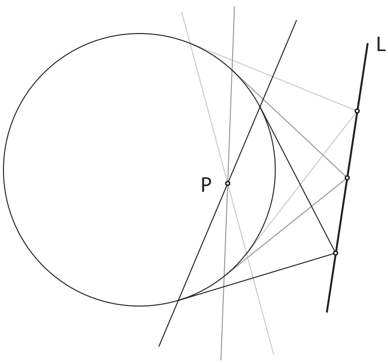

FIGURE 2. Polar reciprocation with respect to a circle: trickier case. Points *P* inside the circle are put in one-to-one correspondence with lines *L* that don't intersect the circle. The mapping works because of the collinearity of the meeting points of the tangents: a nontrivial result that becomes intuitively evident by introducing the third dimension and viewing the figure as the cross section of a configuration of cones tangent to a sphere.

there is a polar line through the circle, as above. We claim that all these polar lines have one point in common, so that this point is the natural pole of *L*. Monge proves this by cleverly bringing in the third dimension. Imagine a sphere that has the circle as its equator. Every point on *L* is the vertex of a tangent cone to this sphere. The two tangents to the equator are part of this cone, and the polar line is the perpendicular projection of the circle of intersection of the sphere and the cone. Now consider a plane through *L* tangent to the sphere. It touches the sphere at one point *P*. Every cone contains this point (because the line from any point on *L* to *P* is a tangent to the sphere and so is part of the tangent cone). Thus, for every cone, the perpendicular projection of the intersection with the sphere goes through the point perpendicularly below *P*, and this is the pole of *L*, and *L* is the polar of this point. *QED*

One is tempted to imagine that Poncelet was forced to turn to this intuitive style of geometry due to being deprived of pen and paper, just as Trachtenberg had to resort to mental arithmetic. But this is a half-truth at best, for Poncelet evidently did have crude writing implements at his disposal: the prisoners were allocated a minimal allowance, for which he was able to obtain some sheets of paper, and he also managed to make his own ink for writing [5, p. 20].

Ibn al-Haytham is another example of a mathematician starting out as an engineer and then turning increasingly to mathematics while in confinement. Early in his career, he devised an irrigation scheme that would harness the Nile to water nearby fields. When his plans proved unworkable, "he feigned madness in order to escape the wrath of the Caliph and was confined to a private house for long years until the death of the tyrannical and cruel ruler. He earned his livelihood by copying in secret translations of Euclid's and Ptolemy's works" [7, p. 156]. Euclidean geometry and Ptolemaic astronomical calculations are certainly better suited to house arrest scholarship than engineering projects. One may further wonder whether it is a coincidence that Ibn al-Haytham, who was forced to spend so many sunny days indoors, also discovered the camera obscura and gave it a central role in his optics.

From these examples, we can conclude that if coronavirus measures are set to have an indirect impact on the direction of mathematical research, it would not be the first time lockdown conditions have made one area or style of mathematics more viable than another.

Newton and the Plague

Isaac Newton went into home isolation in 1665, when Cambridge University advised "all Fellows & Scholars" to "go into the Country upon occasion of the Pestilence," since it had "pleased Almighty God in his just severity to visit this towne of Cambridge with the plague" [14, p. 141]. Newton was then 22 and had just obtained his bachelor's degree. His productivity during plague isolation is legendary: this was his *annus mirabilis*, marvelous year, during which he made a number of seminal discoveries. Many have recently pointed to this as a parable for our time, including, for instance, the *Washington Post* [3]. The timeline is none too encouraging for us to contemplate: the university effectively remained closed for nearly two years, with an aborted attempt at reopening halfway through, which only caused "the pestilence" to resurge.

It is true that Newton achieved great things during the plague years, but it is highly doubtful whether the isolation had much to do with it, or whether those years were really all that much more *mirabili* than others. Newton was already making dramatic progress before the plague broke out and was on a trajectory to great discoveries regardless of public health regulations. Indeed, Newton's own account of how much he accomplished "in the two plague years of 1665 & 1666" attributes his breakthroughs not to external circumstances but to his inherent intellectual development: "For in those days I was in the prime of my age for invention & minded Mathematicks & Philosophy more then at any time since" [15, p. 32].

"Philosophy" here means physics. And indeed, in this subject Newton did much groundwork for his later success during the plague years, but the fundamental vision and synthesis that we associate with Newtonian mechanics today was still distinctly lacking. His eventual breakthrough in physics depended on interactions with colleagues rather than isolation. In 1679, Hooke wrote to Newton for help with the mathematical aspects of his hypothesis "of compounding the celestiall motions of the planetts of a direct motion by the tangent & an attractive motion towards the centrall body." At this time, "Newton was still mired in very confusing older notions." To get Newton going, Hooke had to explicitly suggest the inverse square law and plead that "I doubt not but that by your excellent method you will easily find out what that Curve [the orbit] must be." Only then, "Newton quickly broke through to dynamical enlightenment . . . following [Hooke's] signposted track" [2, pp. 35–37, 117].

Newton later made every effort to minimize the significance of Hooke's role. Indeed, Hooke was just one of many colleagues who ended up on Newton's enemies list. This is another reason why Newton's plague experience is a dubious model to follow. Newton could be a misanthropic recluse even in normal times. When Cambridge was back in full swing, Newton still "seldom left his chamber," contemporaries recalled, except when obligated to lecture—and even that he might as well have done in his chamber for "ofttimes he did in a manner, for want of hearers, read to the walls" [4, n. 11]. He published reluctantly, and when he did, Newton "was unprepared for anything except immediate acceptance of his theory": "a modicum of criticism sufficed, first to incite him to rage, and then to drive him into isolation" [14, pp. 239, 252]. With Hooke, as with so many others, it may well be that Newton only ever begrudgingly interacted with him in the first place for the purpose of proving his own superiority. But that's a social influence all the same. Even if Hooke's role was merely to provoke a sleeping giant, the fact remains that Newton's *Principia* was born then and not in quarantine seclusion.

In mathematics, it is accurate enough to say that Newton "invented calculus" during the plague years. But he was off to a good start already before then, including the discovery of the binomial series. In optics, Newton himself said that the plague caused a two-year interruption in his experiments on color that he had started while still at Cambridge [6, p. 31]. Perhaps this is another example of pure mathematics being favored in isolation at the expense of other subjects that are more dependent on books and tools.

Home isolation also affords time for extensive hand calculations: a self-reliant mode of mathematics that can be pursued without library and laboratory. Newton did not miss this opportunity during his isolation. As he later recalled, "[before leaving Cambridge] I found the method of Infinite series. And in summer 1665 being forced from Cambridge by the Plague I computed y^{e} area of y^{e} Hyperbola . . . to two & fifty figures by the same method" [14, p. 98]. Newton's notebook containing this tedious calculation of the area under a hyperbola to 52 decimals can be viewed at the Cambridge University Library website [8].

References

1 Arthur Bakker and David Wagner, Pandemic: Lessons for today and tomorrow? *Educational Studies in Mathematics*, 104 (2020), 1–4, https://doi.org/10.1007/s10649-020-09946-3.
2 Zev Bechler, Ed., *Contemporary Newtonian Research*, Reidel, Dordrecht, Netherlands, 1982.
3 Gillian Brockell, During a pandemic, Isaac Newton had to work from home, too. He used the time wisely, *Washington Post*, March 12, 2020.
4 I. Bernard Cohen, *Newton, Isaac,* Dictionary of Scientific Biography, Vol. 10, Charles Scribner's Sons, New York, 1974.
5 Isidore Didion, *Notice sur la vie et les ouvrages du Général J.-V. Poncelet*, Gauthier-Villars, Paris, 1869.
6 A. Rupert Hall, *Isaac Newton: Adventurer in Thought*, Cambridge University Press, Cambridge, U.K., 1992.
7 Max Meyerhof, Ali al-Bayhaqi's Tatimmat Siwan al-Hikma: A biographical work on learned men of the Islam, *Osiris* 8 (1948), 122–217.
8 Isaac Newton, MS Add.3958, 79r ff., https://cudl.lib.cam.ac.uk/view/MS-ADD-03958/151.
9 Bertrand Russell, *The Autobiography of Bertrand Russell: 1914–1944*, Little Brown and Company, Boston, 1968.
10 Anna Maria Sigmund, Peter Michor, and Karl Sigmund, Leray in Edelbach. *Mathematical Intelligencer* 27 (2005), 41–50.
11 Arild Stubhaug, *The Mathematician Sophus Lie: It Was the Audacity of My Thinking*, Springer, Berlin, 2002.
12 Jakow Trachtenberg, *The Trachtenberg Speed System of Basic Mathematics*, Doubleday and Company, New York, 1960.
13 André Weil, *The Apprenticeship of a Mathematician*, Birkhäuser, Basel, Switzerland, 1992.
14 Richard S. Westfall, *Never at Rest: A Biography of Isaac Newton*, Cambridge University Press, Cambridge, U.K., 1983.
15 D. T. Whiteside, Newton's Marvellous Year: 1666 and All That, *Notes and Records of the Royal Society of London* 21(1) (1966), 32–41.