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**Theorem.** Let  $x$  be real. If  $x^3 - 6x^2 + 11x - 6 = 2x - 2$ , then  $x = 1$  or  $x = 4$ .

*Proof.* By subtraction,  $x^3 - 6x^2 + 9x - 4 = 0$ , which factors as  $(x - 1)^2(x - 4) = 0$ .  $\square$

**One-Line**

## One-Line

Like poets, mathematicians often strive for economy, and the one-line proof is a sort of monostich. Even the abbreviation QED for *quod erat demonstrandum* (“which was to be shown”), which traditionally marks the end of a proof, is deemed too prolix by modern standards. Instead we find the tombstone  $\square$ , sometimes called the halmos, after the Hungarian American mathematician Paul Halmos who first incorporated it into mathematical writing. Economy is an ideal that extends from proofs to larger scale works. Once, in *The Bulletin of the American Mathematical Society*, there appeared a research article written by two number theorists that, in its entirety, consisted of only two sentences.<sup>13</sup> I guess the authors couldn’t agree on one.

In its cryptic way, the one line here does at least give the reader something to do. Go ahead, combine like terms with like—and look, divide out these common factors.

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