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Introduction

Mircea Pitici

This anthology is the 11th in the annual series of *The Best Writing on Mathematics*. It contains pieces originally published in late 2018 and throughout 2019 in various venues, including specialized print and online magazines, research journals, newspapers, books, and collections of conference proceedings. The volume should be considered by the readers in conjunction with the other ten previously published in the series.

Overview of the Volume

In a piece eerily reminding us of the current coronavirus health crisis, Steven Strogatz recounts the little-known contribution of differential equations to virology during the HIV crisis and makes the case for considering calculus among the heroes of modern life.

Peter Denning and Ted Lewis examine the genealogy, the progress, and the limitations of complexity theory—a set of principles developed by mathematicians and physicists who attempt to tame the uncertainty of social and natural processes.

In yet another example of fusion between ideas from mathematics and physics, Bruce Boghosian describes how a series of simulations carried out to model the long-term outcome of economic interactions based on free-market exchanges inexorably leads to extreme inequality and to the oligarchical concentration of wealth.

Stan Wagon points out the harmonic-average intricacies, the practical paradoxes, and the policy implications that result from using the miles-per-gallon measure for the fuel economy of hybrid cars.

Jørgen Veisdal details some of the comparative reasoning supposed to take place in majoritarian democracies—resulting in electoral strategies that lead candidates toward the center of the political spectrum.

In an autobiographical piece, John Baez narrates the convoluted professional path that took him, over many years, closer and closer to algebraic geometry—a branch of mathematics that offers insights into the relationship between the classical mechanics and quantum physics.

Erica Klarreich explains how Hao Huang used the combinatorics of cube nodes to give a succinct proof to a long-standing computer science conjecture that remained open for several decades, despite many repeated attempts to settle it.

A graph-based explanation, combined with a stereographic projection, also helped Richard Montgomery solve one of the questions posed by the dynamical system formed by three masses moving under the reciprocal influences of their gravitational pulls, also known as the three-body problem.

Chris King, who created valuable online resources freely available to everyone, describes the algebraic iterations that lead to families of fractal-like, visually stunning geometric configurations and stand at the confluence of multiple research areas in mathematics.

In the next contribution to our volume, Jim Henle presents several paper-and-pencil games selected from the vast collection invented by Sid Sackson.

Dave Linkletter breaks the classic Rubik's cube apart and, using the mechanics of the cube's skeleton, counts for us the total number of possible configurations; then he reviews a collection of mathematical questions posed by the toy—some answered and some still open.

Colin Adams introduces with examples, defines, and discusses several important properties of the hyperbolic 3-manifold, a geometric notion both common to our physical environment and difficult to understand in its full generality.

In a similar geometric vein, with yet more examples, physical models, and definitions, followed by applications, Boris Odehnal presents an overview of higher dimensional geometries.

With linguistic flourishes recalling Fermat's cryptic style, James Propp traces the history of two apparently disconnected results in the theory of numbers—which, surprisingly, turned out to be strongly related—and tells us how an amateur mathematician used the parallelism to prove one of them.

Patrick Honner works out in several different ways a simple multiplication example to compare the computational efforts required by the

algorithms used in each case and to illustrate the significant benefits that result when the most efficient method is scaled up to multiply big numbers.

Ben Orlin combines his drawing and teaching talents to prove that ignorance of widely known mathematics can be both hilariously ridiculous and academically rewarding!

Donald Teets's piece is entirely concerned with the young Karl Friedrich Gauss's contribution to the history of the Christian calendar.

Paul Thagard proposes five conjectures (and many more puzzling questions) on the working of mathematics in mind and society and formulates an eclectic metaphysics that affirms both realistic and fictional qualities for mathematics.

Mark Colyvan asserts that explanation in mathematics—unlike explanation in sciences and in general—is neither causal nor deductive; instead, depending on the context, mathematical explanation provides either local insights that connect similar mathematical situations or global answers that arise from non-mathematical phenomena.

Gerry Hahn, Necip Doganaksoy, and Bill Meeker call (as they have done over a long period of time) for improving statistical inquiry and analysis by using new tools—such as tolerance and prediction intervals, as well as a refined analysis of the role of sample size in experiments.

More Writings on Mathematics

Readers of this series of anthologies know that in each volume I offer many other reading suggestions from the recent literature on mathematics: book titles in the Introduction and articles in the section on Notable Writings, toward the end of the volume. As a matter of principle, I never included in these lists materials I have not seen; thus, my ability to keep up with the literature has been considerable affected by the health crisis that closed university campuses and libraries during the spring of 2020. I thank the authors and the publishers who sent me books over the last year; complete references are at the end of this introduction.

To start my book recommendation list, special mention deserves—for exceptional illustrations and insightful contributions—the collective volume published by the Bodleian Library with the title *Thinking 3D*, edited by Daryl Green and Laura Moretti. Also—for visual aspect,

inspired humor, and teaching insights—Ben Orlin's books *Math with Bad Drawings* and *Change Is the Only Constant*.

Excellent expository introductions to specific topics are Julian Havil's *Curves for the Mathematically Curious*, Steven Strogatz's *Infinite Powers*, and (slightly more technical) David Feldman's *Chaos and Dynamical Systems*.

In applied mathematics and connections to other domains, we have *The Mathematics of Politics* by Arthur Robinson and Daniel Ullman, *Modelling Nature* by Edward and Michael Gillman, *Data Analysis for the Social Sciences* by Douglas Bors, *Islands of Order* by Stephen Lansing and Murray P. Cox, *Producers, Consumers, and Partial Equilibrium* by David Mandy, and *Ranking: The Unwritten Rules of the Social Game We All Play* by Péter Érdi. Featuring mathematics in astronomy are *Finding Our Place in the Solar System* by Todd Timberlake and Paul Wallace and *Our Universe* by Jo Dunkley; on mathematics in military affairs, *The (Real) Revolution in Military Affairs* by Andrei Martyanov. Two expository statistics books are *The Art of Statistics* by David Spiegelhalter and *Statistics in Social Work* by Amy Batchelor; and an excursion into computer science is *Computational Thinking* by Peter Denning and Matti Tedre.

Interdisciplinary with historical elements but also with ramifications in contemporary affairs are *Proof! How the World Became Geometrical* by Amir Alexander and *How Charts Lie* by Alberto Cairo. Several books last year were dedicated to the increasing role of algorithms in daily social affairs, including *Algorithmic Regulation* edited by Karen Yeung and Martin Lodge, *The Ethical Algorithm* by Michael Kearns and Aaron Roth, and *The Information Manifold* by Antonio Badia.

A few recent books on the history of mathematics are *Power in Numbers* by Talithia Williams, *Bernard Bolzano* by Paul Rusnock and Jan Šebestík, and *David Hume on Miracles, Evidence, and Probability* by William Vanderburgh. Also historical, with strong reciprocal influences between mathematics and the cultural, social, and linguistics contexts are *Disharmony of the Spheres* by Jennifer Nelson, *Republic of Numbers* by David Lindsay Roberts, and *Roads to Reference* by Mario Gómez-Torrente. In logic and philosophy of mathematics is *Reflections on the Foundations of Mathematics* edited by Stefania Centrone, Deborah Kant, and Deniz Sarikaya. Mathematical notions in practical philosophy appear in *Measurement and Meaning* by Ferenc Csatári and in *Conscious Action Theory* by Wolfgang Baer.

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I hope that you, the reader, will enjoy reading this anthology at least as much as I enjoyed working on it. I encourage you to send comments, suggestions, and materials I might consider for (or mention in) future volumes to Mircea Pitici, P.O. Box 4671, Ithaca, NY 14852; or electronic correspondence to mip7@cornell.edu.

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