

Contents

Preface	xiii
1 Introduction	1
1.1 The world of galaxies	1
1.2 A brief tour of galaxy observations	6
PART I Basics of galactic dynamics	
2 Gravitation	17
2.1 Matter and the gravitational field	18
2.2 Spherical systems: Newton's shell theorems	22
2.3 Circular velocity and dynamical time	26
2.4 Examples of spherical potentials	28
2.5 Exercises	38
3 Elements of classical mechanics	42
3.1 Mechanics of a particle	42
3.2 Escape velocity	45
3.3 Lagrangian formulation of classical mechanics	47
3.4 Hamiltonian mechanics	49
3.5 Exercises	59
4 Orbits in spherical mass distributions	60
4.1 General properties of orbits in spherical potentials	60
4.2 Orbits in specific spherical potentials	64
4.3 Integrals of motion	74
4.4 Actions and angles in spherical potentials	78
4.5 Numerical orbit integration	82
4.6 Exercises	88
5 Equilibria of collisionless stellar systems	92
5.1 Collisionless vs. collisional dynamics	93
5.2 The virial theorem	97
5.3 The collisionless Boltzmann equation	100
5.4 The Jeans equations	103
5.5 The Jeans theorem	109

5.6	Spherical distribution functions	111
5.7	Exercises	128
6	Masses of spherical stellar systems	131
6.1	Dark matter in the Coma cluster	131
6.2	The escape velocity near the Sun and the total mass of the Milky Way	132
6.3	The Local Group timing argument	134
6.4	The mass of the Milky Way's dark matter halo	136
6.5	The masses of dwarf spheroidal galaxies using the Jeans equation	138
6.6	The Wolf mass estimator	141
6.7	Ultra-diffuse galaxies and Dragonfly 44	144
6.8	Estimating masses with multiple populations: cusps vs. cores in dwarf spheroidals	147
6.9	Exercises	151
 PART II Disk galaxies		
7	Gravitation in galactic disks	155
7.1	The observed structure of disk galaxies	156
7.2	Simple gravitational potentials for flattened mass distributions	159
7.3	Gravitational potentials from disk density distributions	170
7.4	Exercises	181
8	The kinematics and dynamics of galactic rotation	185
8.1	Velocity fields in external galaxies	185
8.2	Rotation curves of external galaxies	193
8.3	The kinematics of the Milky Way's interstellar medium	203
8.4	Observing differential rotation locally: The Oort constants	209
8.5	Exercises	214
9	Orbits in disks	217
9.1	Motion in the meridional plane	218
9.2	The separability of disk orbits	222
9.3	Close-to-circular orbits: The epicycle approximation	227
9.4	Action-angle coordinates in and around disks	231
9.5	Exercises	242
10	Equilibria of galactic disks	247
10.1	The axisymmetric Jeans equations and the asymmetric drift	249
10.2	Distribution functions for razor-thin disks	251

10.3	The velocity distribution in the solar neighborhood	259
10.4	The vertical equilibrium state of disks	269
10.5	Exercises	279
11	Chemical evolution in galaxies	283
11.1	Chemical enrichment processes in galaxies	283
11.2	Classical models of galactic chemical evolution	289
11.3	The evolution of abundance ratios	305
11.4	Exercises	321
PART III Elliptical galaxies		
12	Gravitation in elliptical galaxies and dark-matter halos	329
12.1	The three-dimensional structure of elliptical galaxies and dark-matter halos	330
12.2	Gravitational potentials for mildly-flattened and triaxial mass distributions	338
12.3	Multipole and basis-function expansions	351
12.4	N -body modeling	363
12.5	Exercises	378
13	Orbits in triaxial mass distributions and surfaces of section	380
13.1	Surfaces of section	380
13.2	Orbits in two-dimensional, non-axisymmetric systems	384
13.3	Chaos in axisymmetric systems	389
13.4	Orbits in triaxial systems	395
13.5	Exercises	412
14	Equilibria of elliptical galaxies and dark-matter halos	416
14.1	The tensor virial theorem	417
14.2	Are elliptical galaxies flattened by rotation?	424
14.3	Schwarzschild modeling	428
14.4	Exercises	444
15	Gravitational lensing	448
15.1	The lensing equation	451
15.2	The lensing and Fermat potentials	456
15.3	Magnification and shear	469
15.4	The occurrence of images and the critical and caustic curves . . .	481
15.5	Exercises	492
16	Mass modeling in elliptical galaxies	500

16.1 Supermassive black holes at the centers of galaxies	500
16.2 Schwarzschild modeling and the structure of elliptical galaxies . .	510
16.3 Jeans Anisotropic Modeling (JAM)	516
16.4 The inner orbital structure of elliptical galaxies	518
16.5 The stellar and dark mass profiles of elliptical galaxies	523
16.6 Exercises	529

PART IV Galaxy formation and evolution

17 Formation of dark-matter halos	539
17.1 Cosmological evolution of small perturbations	540
17.2 Spherical collapse of overdensities in the Universe	562
17.3 Virialization	565
17.4 The halo mass function	578
17.5 Exercises	584
18 Extragalactic astronomy	589
18.1 Stellar-population synthesis modeling	589
18.2 Dark-matter halos and their galaxies	609
18.3 Galaxy classification	623
18.4 Galaxy-scaling relations	633
18.5 Star formation in galaxies	640
18.6 Exercises	647
19 Hierarchical galaxy formation	650
19.1 The role of mergers in galaxy evolution	652
19.2 Dark-matter halo growth by mergers and accretion	659
19.3 Galaxy growth by gas accretion	674
19.4 Dynamical processes	687
19.5 Exercises	713
20 Internal evolution in galaxies	718
20.1 The (in)stability of disks	721
20.2 Bars	746
20.3 Spiral structure	771
20.4 Exercises	797

APPENDICES

A Coordinate systems	801
A.1 Generalities	801

CONTENTS	xi
A.2 Positions in the Milky Way	802
A.3 Velocities in the Milky Way	804
B Mathematical background	806
B.1 Vector calculus	806
B.2 The calculus of variations and the Euler–Lagrange equation . . .	808
B.3 Special functions	809
B.4 Solving integral and differential equations	823
C The general theory of relativity and galaxies	827
C.1 Einstein’s field equations and geodesic motion	828
C.2 The Newtonian limit	839
C.3 Gravitational light bending and the Shapiro delay	846
C.4 Homogeneous and isotropic cosmological models	849
Bibliography	859
Index	917

Chapter 1

Introduction

Galaxies. They are some of the most beautiful and fascinating objects in the Universe. They come in a large variety of sizes and shapes, colors and brightnesses, from majestic spirals to seemingly boring ellipticals, massive cluster members to dinky little satellites only seen nearby. Every new way that astronomers devise to look at the sky reveals a different view of galaxies, from the familiar optical and near-infrared images dominated by starlight, to the dust emission in the far infrared, and the high-energy processes that shape the far reaches of the wavelength scale. Remarkably, much of the structure of galaxies can be understood using simple physics and the goal of this book is to provide an introduction to the quantitative understanding of galactic structure, formation, and evolution that is derived primarily from chasing down the effects of gravity on stellar systems. Of course, while gravity is arguably the most important fundamental force on the scale of galaxies, it does not suffice to understand the full picture of galaxies and their formation and evolution. The physics of radiation, hydrodynamics, nuclear fusion, chemistry, etc. play a fundamental role as well and we will also discuss their important contributions to shaping galaxies.

1.1 THE WORLD OF GALAXIES

Before we start our sojourn in the realm of galaxies, let's take a quick tour of the types of galaxies found in the Universe. Figure 1.1 displays two very beautiful, famous galaxies: M51 and Cen A. M51 is a *grand-design spiral* galaxy. While the origin of spiral structure in galaxies remains an open problem with likely a variety of solutions, in this case the beautiful spiral structure is most probably caused by the response to the gravity from the companion galaxy at the top of the image. Spiral structure, besides looking pretty, is an important driver of star formation, as the gravitational force of the spiral compresses gas, leading to the formation of new stars within the spirals. These show up as the compact dots along the spiral arms in Figure 1.1, which are small clusters of young stars, traced by their ionized HII emission here.

Cen A is one of the brightest galaxies in the sky and an example of an S0 galaxy, a galaxy somewhere between the extremes of elliptical and spiral galaxies. Its exact classification is still a matter of debate and this illustrates the difficulty we face when studying galaxies. The only data we have is a single snapshot in time from a single perspective, with very limited information on its three-dimensional structure.



Figure 1.1: M51 (NASA/JPL-Caltech/Univ. of Arizona/DSS/SST) and Cen A (NASA/DOE/Fermi LAT Collaboration, Capella Observatory).

Because no galaxy is a fully relaxed system, *all* galaxies are evolving and many are going through dramatic transformations. Cen A has probably endured a merger in its recent past, leading to a burst of star formation and the fueling of an active galactic nucleus (AGN)—a supermassive black hole with a bright accretion disk—at its center.

While most of the famous galaxies are as large as those pictured in Figure 1.1, many other galaxies are quite a bit smaller. We find many examples of these in our own backyard, as satellite galaxies of the Milky Way. Figure 1.2 shows two examples: the Large Magellanic Cloud (LMC) and the Fornax dwarf galaxy (not to be confused with the Fornax cluster!). The LMC is one of the closest neighbors of the Milky Way and the largest of its satellite galaxies—galaxies caught in the web of the Milky Way’s gravitational attraction. The LMC is part of a pair of galaxies that fell into the Milky Way’s gravitational field together and its irregular appearance is in large part due to the tidal interactions with its sibling, the Small Magellanic Cloud, and with the Milky Way. The LMC has a prominent bar at its center and was likely a barred spiral galaxy before it started interacting with the Milky Way. The LMC is about 10 times smaller in size and mass than the Milky Way. It has a lot of gas and many areas of active star formation. Some of the most famous star-forming regions are in the LMC.

Fornax is an example of a dwarf spheroidal galaxy, which are the smallest galaxies. Such galaxies are much more diffuse, as can be seen by comparing the Fornax dwarf galaxy to the pictures of M51 or Cen A above, and their masses are dominated by dark matter, making them some of the best places we have to constrain the fundamental

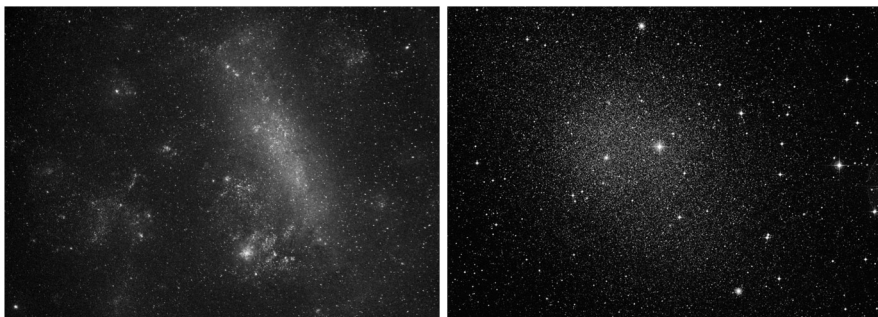


Figure 1.2: The Large Magellanic Cloud (credit: Dylan O'Donnell) and the Fornax dwarf spheroidal (ESO/Digitized Sky Survey 2).

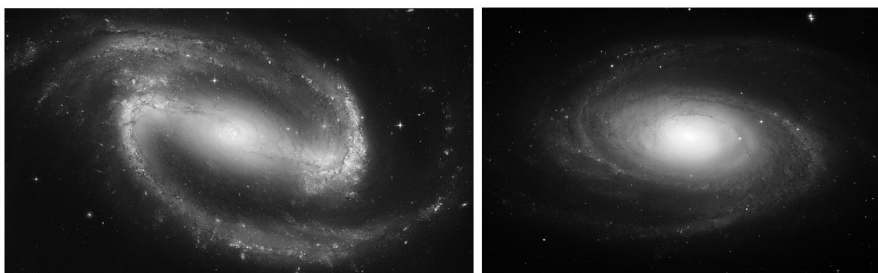


Figure 1.3: NGC 1300 and M81 (both NASA, ESA, and The Hubble Heritage Team STScI/AURA).

nature of dark matter. Fornax in particular is a bit of a dynamical celebrity, because it has a number of globular clusters. If the dark-matter profile in the centers of galaxies has the expected, cuspy shape, then the process of dynamical friction should have caused these clusters to have long since spiraled into the center (Tremaine 1976b). That Fornax still has these globular clusters therefore indicates that the dark-matter density is lower than expected at its center.

Circling back to Milky-Way-sized galaxies, Figure 1.3 gives two more examples of large disk galaxies: NGC 1300 and M81. The most obvious property of NGC 1300 is the rectangular-ish feature at its center, with two prominent spiral arms emanating from its ends. This feature is called a *bar* (for obvious reasons) and it is a common feature of disk galaxies. Overall, about 30% of disk galaxies in the local Universe have strong bars, a fraction that goes down as we look further into the past, showing that bars are a relatively recent (in cosmic time units!) feature of galaxies. Our own Milky Way has a bar at its center, the exact properties of which are still up for debate, but it is far less strong than the bar in NGC 1300. Bars form naturally during the evolution of disk galaxies.

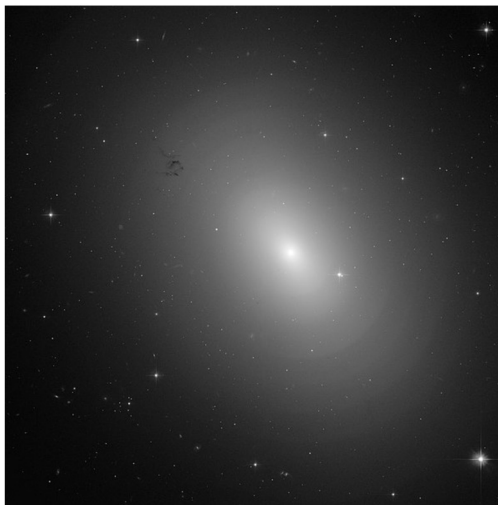


Figure 1.4: NGC 3923 (NASA/ESA Hubble Space telescope).

M81 is a spiral galaxy without an obvious bar at its center. Instead, it has a prominent *bulge*, a spheroidal concentration of mostly old stars, and tightly wound spirals. M81 is part of a group of galaxies much like the Local Group, the set of galaxies that includes the Milky Way and the Andromeda galaxies (which we will typically refer to as M31), their satellites, and some more small galaxies associated with them. M81 interacts with M82 and NGC 3077, a good reminder that almost no galaxy lives in isolation.

We could keep showing pictures of barred and spiral disk galaxies (and of barred-spiral disk galaxies!), but let's look at an example of an elliptical galaxy: NGC 3923 in Figure 1.4. NGC 3923 is a more-interesting-than-usual example of an elliptical galaxy, which normally have smooth, elliptical light distributions without any features. Even though most elliptical galaxies might at first glance therefore seem a little boring, their internal orbital structure can be quite complex. NGC 3923 is especially interesting, because it has multiple shell-like features in its outer parts; such galaxies are known as *shell galaxies*. The presence of shells indicates that the stars are arranged in a structure more akin to layers than in a smooth, continuous distribution like in other elliptical galaxies. Such shell-like features are common in elliptical galaxies, with an estimated 50% of elliptical galaxies displaying faint shells. The shells are an indication that NGC 3923 has experienced a merger with a small satellite and the distribution of orbits of stars in the satellite got re-arranged to give rise to a density profile with sharp edges.

Galaxies are not just made up of single stars, but also contain stellar clusters of various sorts. Because the evolution of these clusters is intimately related to the evolution of their host galaxies, we will also give them some attention in this book. Two

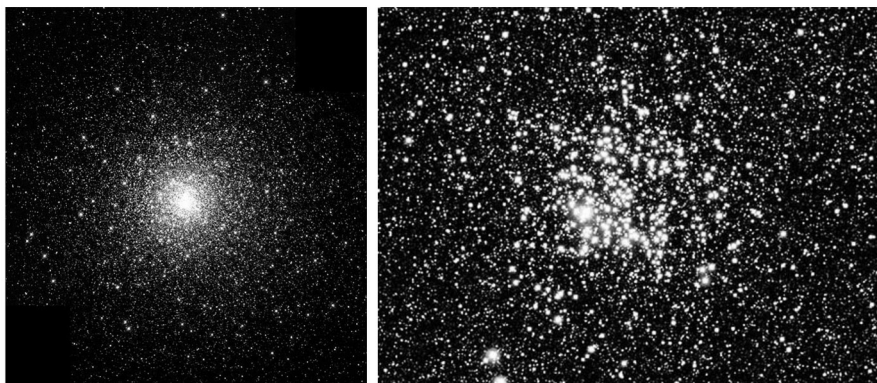


Figure 1.5: M80 (NASA, The Hubble Heritage Team, STScI/AURA) and M11 (NASA).

of the main types of stellar clusters are globular clusters and open clusters. Figure 1.5 gives an example of one of each: the globular cluster M80 and the open cluster M11. It is immediately clear that these are two quite different beasts. Globular clusters are extremely dense, with typical sizes of a few to 10 pc and containing upwards of ten thousand stars (up to a million). The center of M80 is a blob of light, even in this picture taken by the Hubble Space Telescope, because there are too many stars close together to be able to resolve all of them as individual stars in this image. M80 is one of the most massive of the ≈ 150 globular clusters in the Milky Way. Globular clusters are old and are found in almost all galaxies, with the number of globular clusters in a galaxy scaling with the total mass (e.g., Harris et al. 2013). Because they are so dense, globular clusters display a rich dynamical phenomenology that is quite different from that of galaxies. Dynamics and stellar evolution are also intimately linked in globular clusters, making them a fine laboratory for stellar evolution and potentially factories for binary black holes such as those seen by LIGO. Globular clusters can also get disrupted by tidal interactions with their host galaxies, leading to the production of narrow stellar streams.

Open clusters are very different from globular clusters. The main difference is that they are much less dense and many of them are only marginally bound. Open clusters dot the disks of star-forming galaxies. They consist of stars born together in a (typically recent) star-formation event, have typical sizes of about 10 pc, and contain a few hundred to tens of thousands of stars. Open clusters are generally only a few hundred million years old, although some survive for many billions of years. The open cluster pictured in Figure 1.5, M11, is about 250 million years old and is a rich open cluster with about 3,000 stars. Open clusters are an important laboratory for simple stellar evolution. Open clusters get easily disrupted by tidal forces and only the most tightly bound therefore survive for billions of years.



Figure 1.6: NGC 4565, an edge-on disk galaxy (ESO).

1.2 A BRIEF TOUR OF GALAXY OBSERVATIONS

To set the stage for the material covered in this book, we start with a high-level overview of the various components that make up galaxies. The objective here is not to provide a detailed and exhaustive overview of galaxy phenomenology (that will be done in later chapters!), but merely to briefly discuss the size, shape, and contribution to the overall mass budget of different galactic components.

We will focus our overview on large disk galaxies. Large disk galaxies are one of the most important type of galaxies for various reasons: approximately half of all galaxies in the local Universe are disk galaxies, and such galaxies appear to be most efficient at converting gas into stars. Therefore, most stars in the local Universe live in the disks of large disk galaxies. We also live close to the mid-plane of a large disk galaxy, the Milky Way. This provides us with a detailed, close-up view of the structure of such a galaxy. In the Milky Way, we can observe stars from the most luminous giants to the lowest-mass M dwarfs, from the center of the Galaxy to the outskirts of the spherical stellar halo surrounding the disk, and we can study gas in its various atomic and molecular phases. Because galactic disks are “dynamically cold”, meaning that the ratio of their velocity dispersion to the typical velocity of a star is small, they are also prone to instabilities that give rise to bars, spiral structure, etc., which are phenomena that we will discuss in later chapters.

Figure 1.6 shows a typical disk galaxy, NGC 4565, seen edge-on. NGC 4565 displays all of the components of a typical disk galaxy. Its structure is dominated by a disk consisting of stars and gas, with a protrusion of stars near the center that is called the bulge. The gas itself cannot be seen in this picture, but its presence can

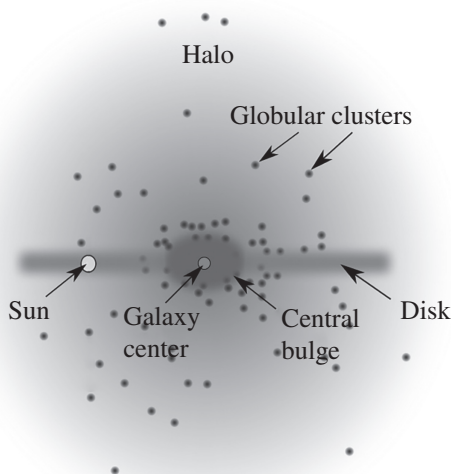


Figure 1.7: A schematic overview of the Milky Way (RJHall at English Wikipedia).

be inferred because gas is traced by dust grains, which redden passing starlight and cause the reddish band close to the disk’s mid-plane. NGC 4565 does have a stellar halo (Harmsen et al. 2017), but it is too faint to be seen in this picture. Also not seen in this picture (and never seen *directly*—whatever that means—in any picture of any galaxy) is the spheroidal distribution of dark matter that surrounds the disk and extends to very large distances from the center.

Figure 1.7 provides a schematic overview of a disk galaxy, the Milky Way in particular (but except for the Sun’s position, this structure applies to all disk galaxies). This illustration shows the disk edge-on and displays the system of globular cluster that surrounds each galaxy, in addition to the components that we already discussed. The Galactic center is also separately emphasized. We have known for about two decades that the centers of galaxies host supermassive black holes (see Chapter 16), which can be active and surrounded by an accretion disk (such accretion disks are a topic that we will not cover in this book) or inactive and often surrounded by a dense *nuclear star cluster*.

1.2.1 The distribution of stars in galaxies

One of the most basic observations about the disks of galaxies is that they are **exponential**. That is, their light profiles $I(R)$ follow an exponential decline with radius $I(R) = I_0 \exp(-R/h_R)$. This had been known in the mid-1900s, but was most famously discussed by Freeman (1970) “On the disks of spiral and S0 galaxies” (oh,

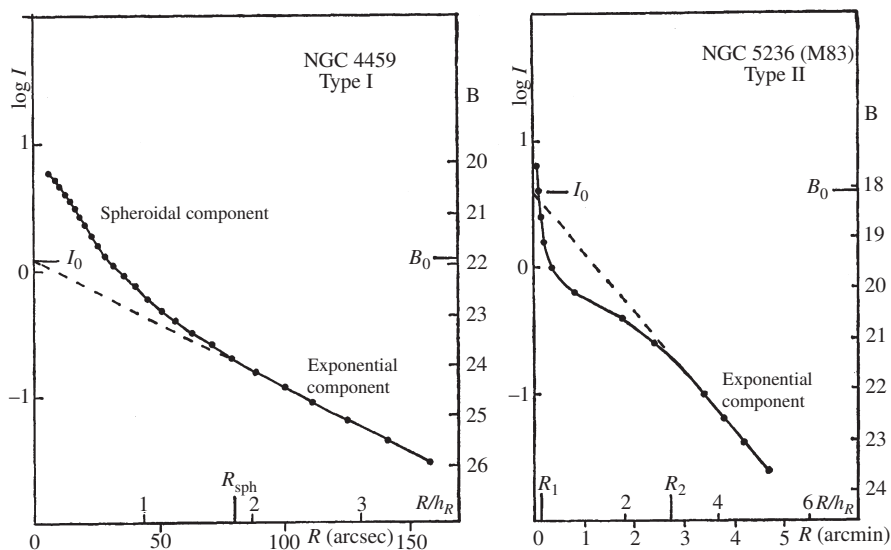


Figure 1.8: Galaxy disks have exponential profiles (Freeman 1970).

to be able to write a paper with a title like that!). Figure 1.8 is the first figure in Freeman’s paper, which shows the surface brightness as a function of radius along the major axis in the blue B band. Because surface brightness is given on the y axis as a logarithmic quantity measured in magnitudes, the linear decline implies an exponential decline of the light profile. Both of the shown galaxies only follow the exponential law over a restricted radial range. Near the center they rise precipitously, due to the presence of the bulge (the “spheroidal component” above); the galaxy on the right also has a flattened part of the light profile near the center, which is not uncommon.

The second most important dimension of a disk is its vertical dimension. Vertical for the moment means perpendicular to the two-dimensional plane of the disk, the direction along which the disk is narrowest. Along the vertical direction, the density also falls off quickly in a roughly symmetric manner; the peak of the vertical density occurs at the **mid-plane**, the zero-point of the vertical coordinate. Like for the radial profile, we look at the vertical profile of the surface brightness to get a sense for what the vertical mass distribution is. Figure 1.9 shows a typical result. This figure shows vertical surface-brightness profiles for NGC 4244 at different distances from its center. Again, we see a linear decline at large distances, which implies an exponential fall-off (because the surface brightness in this figure is again a logarithmic quantity). Near the mid-plane the profile flattens and becomes close to constant. The fit that is shown is a sech^2 profile, a hyperbolic secant squared, which is the equilibrium solution of a self-gravitating isothermal disk, as we will see in Chapter 10.4. As we discussed above and will discuss below in more detail, disks contain gas and stars of various ages with different vertical distributions and the simple isothermal—having

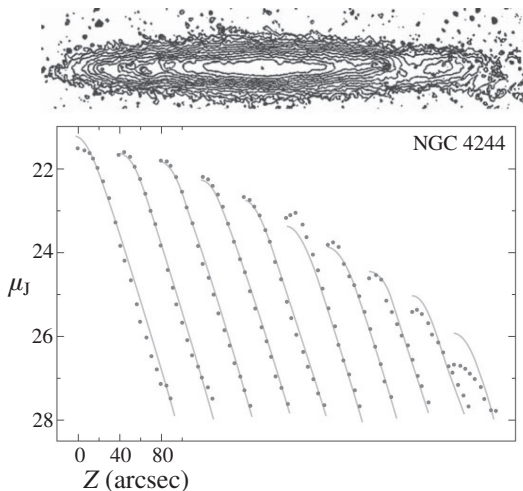


Figure 1.9: NGC 4244: top: a pure-disk galaxy seen edge-on; bottom: vertical surface-brightness profiles at a range of distances from the center; profiles are displaced along the X-axis to avoid overlapping (van der Kruit & Freeman 2011).

the same amount of random motion at each vertical height—can therefore not be entirely correct. The sech^2 profile should therefore not be taken too seriously, but is simply a physically-motivated profile that does a good job of fitting the observations.

The ratio between the scale length h_R of the radial decline and the scale height h_z of the vertical decline is a measure of the thickness of the disk. A typical value is $h_R/h_z \approx 10$, meaning that disks are quite thin.

Whether we use number counts or surface brightness, to translate these observed profiles to mass profiles, we have to assume stellar-population models for how mass is traced by light. This is done by applying a **mass-to-light ratio** M/L that converts the amount of light observed to an amount of (stellar) mass and is usually silently expressed in solar units (the Sun’s mass over the Sun’s luminosity). The stellar mass-to-light ratio is typically assumed to be constant through the galaxy and is $M/L \approx 3$, but it depends on the passband in which the luminosity is measured. The total mass-to-light ratio in galaxies is much larger and depends on position, because in addition to stars, it includes the dark matter, which adds mass, but does not contribute to the luminosity; the total M/L also includes gas, which in broad optical and near-infrared passbands similarly does not contribute to the luminosity, while adding mass (but generally much less mass than either stars or dark matter; see below). The total mass-to-light ratio depends on position because the fraction of total mass that is dark matter and gas strongly depends on position.

While the radial and vertical profiles of the disk are (close to) exponential, the central bulge is not. Bulges typically have profiles similar to elliptical galaxies, which are modeled as **Sérsic profiles**: $\mu(R) = \mu_0 - b_n R^{1/n}$, where μ is the surface brightness,

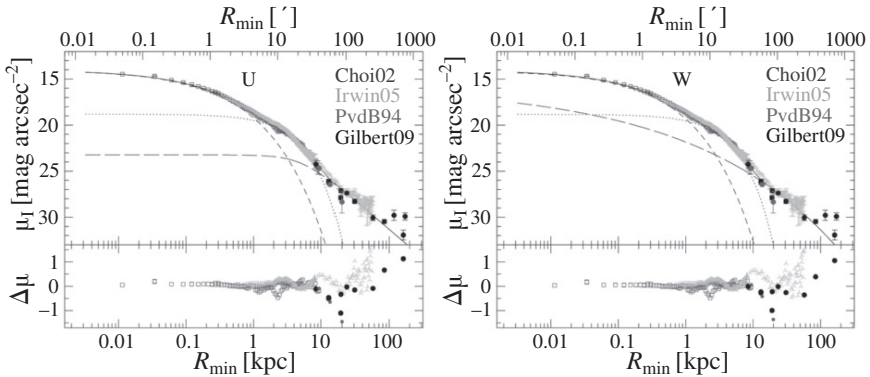


Figure 1.10: Surface-brightness profile of M31 along with a fit with a Sérsic bulge (dashed), exponential disk (dotted) and halo (long dashed) with the combined fit profile as the full curve; halo model is a power-law on the left and Sérsic on the right (Courteau et al. 2011).

μ_0 is the central surface brightness, n is the Sérsic index, and b_n can be computed from the value of n . The case of $n = 4$ is the classic **de Vaucouleurs profile**, which represents the surface brightness profiles of large elliptical galaxies well. Setting $n = 1$, we have an exponential profile.

As an example, Figure 1.10 displays the surface-brightness profile of M31, the closest large disk galaxy outside of the Milky Way. M31’s bulge component is a dashed line in fits to this profile with a Sérsic bulge, exponential disk, and a stellar halo (with two different radial profiles in the two panels). The bulge has a Sérsic index of about 2.2 and dominates within 1 kpc, the very central region of the galaxy. The disk, given by the dotted line, starts to dominate the light outside 1 kpc and does so out to 10 kpc. The stellar halo dominates the light outside the disk region.

Given how prominent the stellar halo is outside the disk region, let’s take a closer look at this component. Figure 1.11 shows deep observations of some external disk galaxies. These observations are deep enough to reveal the low surface-brightness envelope surrounding these disks (the actual disks are painted in from shallower observations). When fitting the radial surface-brightness profile of these galaxies with an exponential-disk plus a bulge, many of these galaxies have excess light at large radii (> 20 kpc) beyond the exponential disk. This light extends in a spheroidal manner, not a disk, and follows an approximate power-law profile. There is great diversity in the profiles of the stellar halos in galaxies. This is because the stellar halo is believed to form primarily from mergers (accreted material from merging satellite galaxies, or material from the main galaxy disturbed by a merger).

While the stellar halo takes up a large volume, its total stellar mass is much smaller than that of the other main galactic components. The stellar halo typically only accounts for a few percent of the stellar mass of a galaxy and much less than that of the

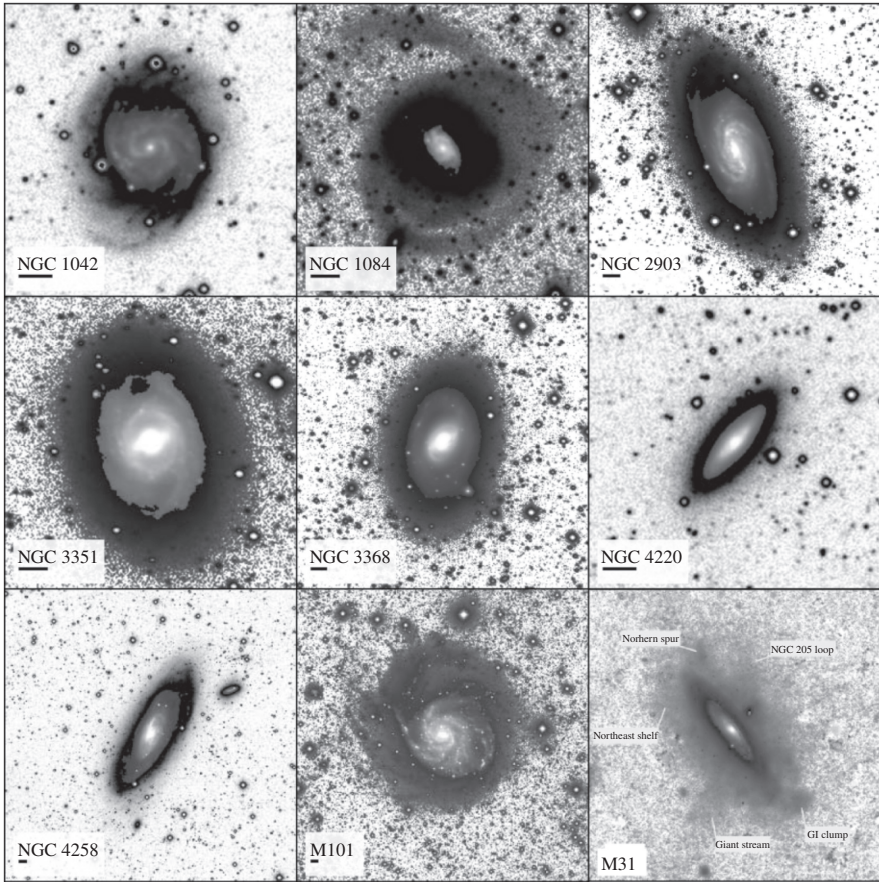


Figure 1.11: Stellar halos (Merritt et al. 2016).

total mass. There is no part of a galaxy where the stellar halo dominates the density, because it is always overwhelmed by either the bulge, disk, dark-matter halo, or the gaseous component.

1.2.2 The distribution of gas in galaxies

All baryonic matter was once in gaseous form, but in large, present-day disk galaxies only about 10% of the baryonic mass is present as cold/warm gas in the interstellar medium. Gas is present in galaxies in different phases and in both atomic and molecular form. Making a precise census of all of the gas in galaxies, in particular the amount of hot halo gas, is difficult.

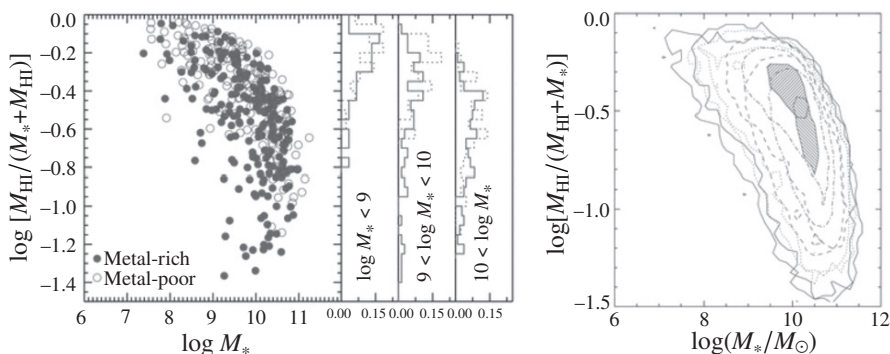


Figure 1.12: The gas content of galaxies (Zhang et al. 2009). Left panel: HI gas mass fraction vs. stellar mass for metal-rich (red) and metal-poor (blue) galaxies, for galaxies with direct HI observations. Right panel: HI mass fraction vs. stellar mass for star-forming galaxies with indirect HI measurements.

Stars form from cold molecular gas. Cold gas is contained in a disk component much like the stellar disk component that we discussed above. Most of the gas is atomic hydrogen, or HI. Figure 1.12 shows the cold-gas content (HI in particular) as a function of stellar mass for a sample of galaxies in the local Universe. The Milky Way’s stellar mass is about $6 \times 10^{10} M_\odot$ or $\log(M_*/M_\odot) \approx 10.8$ (Bovy & Rix 2013); gas makes up about 10% of the baryonic disk mass in the Milky Way. Figure 1.12 shows that the gas fraction rises as one goes to lower mass galaxies and the dynamics of gas and its interaction with stars is thus much more important in lower mass galaxies than it is in the Milky Way.

In the Milky Way, we can make a reasonably complete census of the interstellar medium near the Sun. The main components are: atomic hydrogen (HI), ionized hydrogen (HII), and molecular hydrogen (H_2), contributing $\approx 1.01 \text{ cm}^{-3}$, $\approx 0.015 \text{ cm}^{-3}$, and $\approx 0.15 \text{ cm}^{-3}$, respectively in the mid-plane (McKee et al. 2015). The atomic hydrogen comes in cold and warm phases that constitute $\approx 80\%$ and $\approx 20\%$ of the local HI density. Converting these number densities in the mid-plane to a mass density, we find that the mid-plane gas density is about $0.041 M_\odot \text{ pc}^{-3}$, approximately the same as that in stars ($\approx 0.040 M_\odot \text{ pc}^{-3}$; Bovy 2017b). The molecular gas layer has an exponential scale height that is approximately $h_z = 100 \text{ pc}$, the cold component of HI has about the same thickness, the warm HI has $h_z \approx 300 \text{ pc}$, while the ionized gas is contained in a much thicker layer. The total local gas surface density is $\approx 14 M_\odot \text{ pc}^{-2}$ with HI contributing $\approx 11 M_\odot \text{ pc}^{-2}$, HII $\approx 2 M_\odot \text{ pc}^{-2}$, and H_2 adding $\approx 1 M_\odot \text{ pc}^{-2}$.

The radial distribution of these gas components is much less certain. The radial surface density of HI is shown in the left panel of Figure 1.13. Thus, the density of HI is almost constant within 10 kpc or at most displays a shallow decline; outside

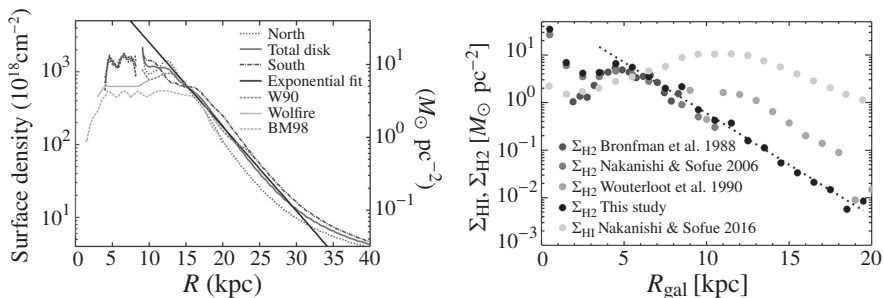


Figure 1.13: The surface density of neutral hydrogen (Kalberla & Kerp 2009; left panel) and of molecular hydrogen (Miville-Deschênes et al. 2017; right panel) in the Milky Way.

of 10 kpc it falls off exponentially. But note that there are significant asymmetries between different parts of the distribution.

A smaller fraction of the ISM is contained in molecular gas (about 10% near the Sun). The radial profile of the molecular gas is similar to that of the HI: it is approximately constant in the inner Galaxy and drops off exponentially at larger radii, as shown in the right panel of Figure 1.13. But the molecular gas has a shorter scale length than the HI, so very little of it is found outside of 15 kpc. Almost all of the molecular gas is contained in molecular clouds.

1.2.3 The distribution of mass in galaxies

Now that we have a good sense of the stellar and gas distribution in a typical disk galaxy, let's take a look at the largest contributor to the galactic mass distribution: the dark-matter halo. Dark matter is a largely unknown constituent of the Universe, likely a new particle, that overall is about 5 times more abundant (e.g., Planck Collaboration et al. 2016) than ordinary matter (sometimes called “baryonic matter” although this misses the ordinary, fermionic matter that is, however, negligible in mass compared to its baryonic counterpart). In galaxies such as the Milky Way, there is more than ten times more dark matter than ordinary matter, with the remaining baryonic matter likely in warm and hot phases in the interstellar and intergalactic medium (e.g., Shull et al. 2012). We know about the presence of dark matter solely through its gravitational influence and this book will cover how we have learned and are continuing to investigate the distribution of dark matter in galaxies.

To get a sense of the distribution of dark matter, we will plot the density and enclosed mass of different components of the Milky Way. We use the mass model for the Milky Way from McMillan (2017). This model has a bulge, disk, and dark-matter halo potential (ignoring the contributions from the stellar halo and including those from the gas together with the stellar disk). In the top panel of Figure 1.14, we look at the density of the different components in the mid-plane of the Milky Way

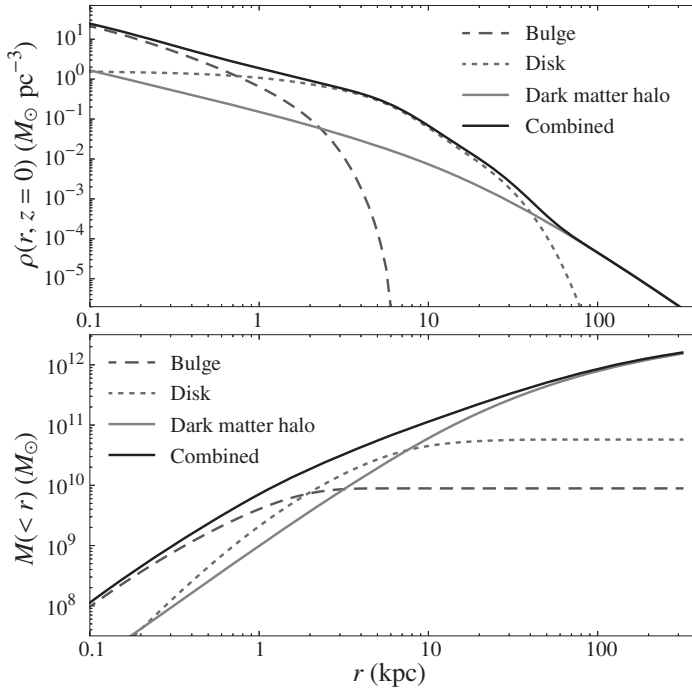


Figure 1.14: The mass profile of the Milky Way.

(at $z = 0$) as well as the total density. Much like in M31 above, the bulge component dominates the density within 1 kpc, but becomes less important after that. The stellar disk dominates the density between 1 kpc and about 30 kpc, after which the dark-matter halo dominates.

The total density near the Sun (at ≈ 8 kpc from the center) is about $0.1 M_{\odot} \text{pc}^{-3}$, with only about 1/10th of that contributed by dark matter. The bottom panel of Figure 1.14 shows the total mass enclosed within a given radius for the different components and for the total mass. The total mass in the bulge+disk is approximately equal to the total mass in the dark-matter halo within 10 kpc from the center. Outside of this, the dark-matter halo dominates the mass and at the edge of the Milky Way, around 250 kpc from the center, there is more than ten times as much dark matter as there is ordinary matter. We will have much more to say about the distribution of dark matter in later chapters.

(continued...)

Index

- 4000 Å break, 594
- A , 210
- B , 210
- $D4000$, 594
- H , 541
- H_0 , 542
- M_{vir} , 35
- R_{25} , 194
- $T_{0,\text{CMB}}$, 560
- X_{CO} factor, 644
- $[\text{Fe}/\text{H}]$, 314
- $[\text{O}/\text{Fe}]$, 314
- $[\text{O II}]$ line, 591
- $[\text{X}/\text{H}]$, 314
- Δ_V , 35, 567
- $\Omega_{0,\Lambda}$, 542
- $\Omega_{0,b}$, 542
- $\Omega_{0,k}$, 542
- $\Omega_{0,m}$, 542
- $\Omega_{0,r}$, 542
- α elements, 286
- α enhanced, 305
- $\dot{M}_*$, 290
- γ models, *see* Gamma models
- $\mathcal{R} = R_{\text{CR}}/R_b$, 750
- ρ_c , 542
- r_{max} , 36
- r_{vir} , 35, 567
- t_{dyn} , 27
- v_{max} , 36
- v_c , 26
- $[\text{NII}]$, 185
- Abel integral equation, 823
- Abel inversion, 823
- Abundance ratios
 - definition of $[\text{X}/\text{H}]$, 314
 - evolution, 305
 - evolution of $[\text{O}/\text{Fe}]$ vs. $[\text{Fe}/\text{H}]$, 315
 - solar abundances, 315
- Accreting box model, 301
 - metallicity distribution, 303
 - with outflows and recycling, 303
 - with type Ia delays, 317
- Action diamond, 241
- Action-angle coordinates, 57
 - action diamond, 241
 - adiabatic approximation, 233
 - adiabatic invariance, 58
 - convergent method, 240
 - Staeckel approximation, 237
- Active galactic nuclei
 - BL Lac objects, 631
 - blazars, 631
 - duty cycle, 632
 - LINERs, 630
 - optically violent variables, 631
 - quasars, 631
 - radio galaxies, 630
 - Seyfert galaxies, 630
 - unified model, 632
- Active galactic nucleus, 630
- Active galaxies, 630
- Adiabatic approximation, 233
- Adiabatic contraction, 409
- Adiabatic initial density perturbations, 556
- Adiabatic invariance, 58, 577
- Age–metallicity relation, 791
- Age–velocity–dispersion relation, 786
- AGN feedback, *see* Feedback
- Alpha elements, 286
- Alpha enhanced, 305
- Andromeda galaxy, 10
- Angle-action coordinates, *see* Action-angle coordinates
- Angular diameter distance, 454, 856
- Angular momentum, 44, 61
 - z component, 218
 - conservation of, 218
 - conservation of, 61
 - specific, 61
- Angular momentum barrier, 62, 219
- Anisotropic distribution function, 118
 - stability, 575

- Anisotropy
 - degeneracy with mass, 107, 137
 - orbital, 106
- Antennae galaxies, 651
- Anti-spiral theorem, 778
- Antonov's second law, 575
- Apocenter, 63, 220
- Arp 271, 651
- Associated Legendre polynomials, 354, 821
 - recurrence, 822
- Astroid caustic, 487
- Asymmetric drift, 250, 256
 - solar neighborhood, 261
- Atomic gas
 - radial profile, 12
- Attenuation, 605
- Augmented density, 119
- Axisymmetric Jeans equations, *see also*
 - Jeans equations
 - asymmetric drift, 250
 - azimuthal, 249
 - azimuthal, razor-thin, time-dependent, 723
 - decoupled vertical, 250
 - radial, 249
 - radial, razor-thin, time-dependent, 723
 - separable, 250
 - vertical, 249, 270
 - in terms of surface density, 271
- Azimuthal action, 58, 79
- Azimuthal frequency, 63
 - in the epicycle approximation, 227
- Azimuthal period, 63
- Banana orbit, 406
- Bar instability, 733
- Barnes–Hut tree, 655
- Bars
 - connection to spirals and rings, 750
 - dissolution, 770
 - fast and slow, 750
 - gas flows, 769
 - pattern speeds, 749
 - peanut shape, 748
 - radial profile, 747
 - shape, 747
 - strong and weak, 747
 - Tremaine–Weinberg method, 749
 - vertical profile, 747
- Baryon acoustic oscillations, 560
- Baryonic feedback, *see* Feedback
- Baryonic Tully–Fisher relation, 636
- Basis-function expansion, 352
- Bessel functions, 814
 - Fourier–Bessel theorem, 815
 - modified Bessel functions, 815
 - numerical integration, 816
 - of the first kind, 814
 - spherical Bessel functions, 816
- Big Bang nucleosynthesis, 285
- Binding energy, 111
- Birkhoff's theorem, 562
- BL Lac objects, 631
- Black-hole–galaxy scaling relations, 508
- Blazars, 631
- Blue cloud, 628
- Bottom-up structure formation, 580
- Bound-bound emission, 675
- Bound-free emission, 675
- Box orbit, 386
 - parent, 387
- Boxlet, 403, 406, 755
- Boxy isophote, 335
- Bremsstrahlung, 675
- Buckling instability, 769
- Bulge shock, 707, 709
- Canonical momentum, 49
- Canonical transformation, 53
- Caustic (lensing), 485
 - fold and cusp, 488
- Cen A, 1
- Centrifugal force, 699
- Centrophilic orbit, 403
- Centrophobic orbit, 403
- CGM, *see* Circumgalactic medium
- Chaotic orbit, 389
- Characteristic curve, 758
- Chemical enrichment
 - r-process, 286
 - s-process, 286
 - time scales, 287
 - type Ia supernovae, 286
 - type II supernovae, 285
 - yields, 288
- Chemical evolution
 - abundance ratios, 305

- accreting-box model, *see* Accreting box model
- closed-box model, *see* Closed box model
- instantaneous recycling
 - approximation, 289
- leaky-box model, *see* Leaky box model
- pre-enrichment, 297
- recycling fraction, 300
- Christoffel connection, 834
- Churning, 784
- Circular frequency, 227
- Circular velocity, *see* Rotation curve
- Circumgalactic medium, 687
- Classical dwarf spheroidal, 139
- Closed box model, 290
 - G-dwarf problem, 296
 - metallicity distribution, 291
 - with type Ia delays, 310
- Closed long-axis orbit, 387
- Closed short-axis orbit, 388
- Cold-mode accretion, 674
- Collisional excitation cooling, 675
- Collisional ionization cooling, 675
- Collisional ionization equilibrium, 676
- Collisionless Boltzmann equation, 100
 - canonical coordinates, 101
 - Cartesian coordinates, 101
 - equilibrium, 102
 - Liouville's theorem, 102
 - one-dimensional, 269
- Collisionless dynamics, 94
 - disk galaxies, 96
 - versus collisional, 93
- Comoving coordinates, 544
- Comoving distance, 544, 852
- Compton heating, 677
- Concentration
 - King profile, 117
 - NFW profile, 35
- Conditional halo mass function, 668
- Configuration space, 51
- Conjugate momentum, 49
- Conservative force, 45
- Constant anisotropy distribution
 - function, 120
 - sample, 122
 - stability, 575
- Continuity equation
 - axisymmetric, razor-thin disk, 723
 - density, 104
 - phase-space density, 101
- Convergence, 457
 - elliptical power law, 459
- Cooling function, 675
- Cooling time, 680
- Core collapse (globular clusters), 720
- Core galaxies, 332
- Core-collapse supernova enrichment, 285
- Cored logarithmic potential, 193
- Coriolis force, 699
- Corotation radius, 737
- Cosmic noon, 642
- Cosmic ray spallation, 285
- Cosmic shear, 450
- Cosmic star-formation history, 642
- Cosmological constant, 540, 839
- Counter-rotating disk, 512
- Covariant derivative, 832
- Critical curve, 485
- Critical density, 542
- Critical surface density, 457
- Critical wave number (stability), 725
- Critical wavelength (stability), 725
- Crossing time, 27
- Curl, 807
- Curvature potential, 844
- Cusp caustic, 488
- Cut (lensing), 492
- Cyclic coordinate, 49
- D4000 index, 594
- Dark energy, 854
- Dark matter, 13
 - cusp vs. core, 147
 - density near the Sun, 278
 - disk rotation curves, 201
 - subhalos, 329, 712
- Dark matter halos
 - radial profile, 34
 - shape, 336, 576
 - subhalos, 329, 712
- Dark-energy domination, 541
- de Vaucouleurs profile, 10, 330
 - approximation with $\gamma=3/2$ model, 332
- Decaying mode, 553
- Declination, 803

- Deflection angle
 - axially-symmetric lens, 452
 - GR calculation, 847
 - logarithmic potential, 452
 - NFW potential, 452
 - point mass, 451
 - reduced, 454
 - singular isothermal sphere, 452
- Deflection potential, *see* Lensing potential
- Dehnen distribution function, 259
 - solar neighborhood, 266
- Delay time distribution, *see* Type Ia supernovae delay time distribution
- Delta function, 810
- Density parameter, 855
- Depletion time scale, 316
- Differential rotation, 209
 - Oort constants, 211
- Dirac delta function, *see* Delta function
- Direct summation, 365
- Disk shock, 707
- Disk-halo conspiracy, 526
- Disky isophote, 335
- Disky potential, 232
- Dispersion criterion
 - multi-component disk, 731
- Dispersion relation, 724
 - cold stellar disk, 724
 - forbidden region, 739
 - gas disk, 730
 - non-axisymmetric perturbations, 737
 - warm stellar disk, 726
- Distant-tide approximation, 701
- Distribution function, 100
 - anisotropic, 118
 - stability, 575
 - augmented density, 119
 - constant anisotropy, 120
 - stability, 575
 - Dehnen, 259
 - Eddington formula, 112
 - ergodic, 111
 - stability, 575
 - general, one-dimensional density, 272
 - isothermal, 115
 - vertical, 272
 - Osipkov–Merritt anisotropy, 124
 - stability, 575
 - sampling an anisotropic spherical distribution function
 - constant anisotropy, 122
 - Osipkov–Merritt anisotropy, 127
 - sampling an isotropic spherical distribution function, 112
 - Schwarzschild, 254
 - Shu, 258
- Divergence, 807
- Divergence theorem, 807
- Doremus–Feix–Baumann theorem, 575
- Dormand–Prince method, 85
- Double-exponential disk, 159, 178
 - approximate circular velocity, 179
- Downsizing, 641, 643
- Dust attenuation, 605
- Dust extinction, 604
- Dwarf elliptical galaxy, 626
- Dwarf irregular galaxy, 626
- Dwarf spheroidal galaxy, 2, 138, 627
 - inner dark-matter profile, 147
 - mass-to-light ratio, 141
 - ultra-faint, 627
- Dwarf spiral galaxy, 626
- Dynamical friction, 654
 - bars, 770
 - deceleration formula, 691
 - isotropic velocity distribution, 692
 - time scale, 696
 - time scale with mass loss, 697
 - Tremaine–Weinberg formula, 771
- Dynamical time, 27
- Dynamical-friction wake, 694
- Early-type galaxy, 511, 625
- Eccentric anomaly, 69
- Eccentricity
 - ellipse, 65
 - orbital, 63, 220
 - two-dimensional, 476
- Eddington formula, 112
- Effective potential, 62, 218
 - rotating frame, 700
- Effective radius, 330
- Effective yield, 300
- Einstein angle, 425
 - point mass, 455

- singular isothermal sphere, 455
- Einstein field equations, 839
- Einstein radius, 455, 465
- Einstein ring, 449, 455, 465
- Einstein summation convention, 830
 - contraction, 830
- Einstein tensor, 836
- Einstein–de Sitter model, 543, 554
- Ellipsoidal coordinate, 397
- Ellipsoidal coordinates, 345
- Elliptic integral, 813
- Ellipticity, 334, 476, 623
- Energy, 45
 - binding, 111
 - conservation of, 45
 - kinetic, 44
 - potential, 45
 - radial, 62
 - relative, 111
 - specific, 45
 - vertical, 226, 269
- Energy density of the Universe, 540
- Energy-momentum tensor, *see*
 - Stress-energy tensor
- Epicyle approximation, 227
 - azimuthal frequency, 227
 - epicycle frequency, 227
 - solution, 228, 229
 - velocities, 230
 - vertical frequency, 227
- Epicyle frequency, 227
- Equation of state, 542
- Equations of motion
 - cylindrical coordinates, 48
 - epicycle approximation, 227
 - meridional plane, 218
 - rotating frame, 699
 - spherical mass distribution, 61
- Equatorial coordinates, 803
- Equivalence principle, 827
- Ergodic distribution function, 111
 - Eddington formula, 112
 - sample, 112
 - stability, 575
- Ergodic orbit, 389
- Error function, 809
- Escape velocity, 45
 - mass of the Milky Way, 46, 132
- Euler force, 699
- Euler method, 83
- Euler–Lagrange equation, 47, 808
- Excursion-set formalism, *see* Extended Press–Schechter formalism
- Exponential disk, 7, 156
 - double-exponential disk, 159
 - razor-thin, 175
 - razor-thin, circular velocity, 175
 - razor-thin, peak circular velocity, 175
 - scale length/height, 158
 - type I, II, or II profile, 156
- Extended Press–Schechter formalism, 660
- External convergence, 474
- External shear, 474
- Extinction, 604
- Extinction curve, 604
- Faber–Jackson relation, 637
- Factorial, 813
- Fast rotators, 335, 521
- Feedback, 149, 411, 657, 684, 685
 - AGN feedback, 686
 - early-stellar feedback, 686
 - quasar mode, 686
 - radio mode, 686
 - stellar-mass–halo-mass relation, 621
 - sub-grid physics, 687
 - supernova feedback, 149, 685
- Fermat potential, 460
- Fictitious force, 43, 699
- First-order method, 84
- Fish orbit, 406
- Flattened logarithmic potential, 168
- Flattening, 168
 - potential vs. density, 169
- Flocculent spiral, 772
- Flux, 589
- Flux density, 469, 589
- Flux ratio anomalies, 490
- Fold caustic, 488
- Forbidden region (dispersion relation), 739
- Fornax, 2
- Fourier mode, 551
- Fourier transform, 170
- Fourier–Bessel theorem, 815
- Free-bound emission, 675
- Free-fall time, 681

- Free-free emission, 675
- Freeman's law, 627
- Frequency
 - azimuthal, 63
 - in the epicycle approximation, 227
 - circular, 227
 - epicycle, 227
 - orbital, 57
 - radial, 63
 - in the epicycle approximation, 227
 - vertical, *see also* Vertical frequency
 - in the epicycle approximation, 227
- Friedmann equations, 854
- Friedmann–Lemaître–Robertson–Walker metric, 852
- Fundamental plane, 515, 638
 - tilt, 515, 528, 639
- G-dwarf problem, 296
- Gaia* mission, 265
- Gaia* phase-space spiral, 268
- Galactic coordinates, 803
- Galactocentric coordinates, 803
- Galaxy luminosity function, 609
 - $1/V_{\max}$ method, 610
 - luminosity density, 614
 - Schechter function, 613
 - ultraviolet, 613
- Galaxy morphology, 623
- Galaxy quenching, 617
- Galaxy stellar mass function, 615
 - stellar-mass density, 615
- Galaxy-galaxy lensing, 478
- Gamma function, 812
- Gamma models, 33
 - as an approximation of the de Vaucouleurs profile, 331
- Gauss's law, 22
- Gaussian distribution, 809
- Gaussian hypergeometric functions, 822
- Gegenbauer polynomials, 361, 818
 - differential equation, 820
 - recurrence relation, 821
- Generalized coordinates, 48
- Generalized momentum, 49
- Generating function, 53
- Generating function of the first kind, 54
- Generating function of the third kind, 235
- Geodesic
 - equation, 835
 - null, 846
- Geodesic equation, 835
- Globular clusters, 5
 - core collapse, 720
- Gradient, 806
- Gradient theorem, 808
- Gram–Schmidt orthogonalization, 353, 818
- Grand-design spiral, 1, 772
- Gravitational constant, 22
- Gravitational field, 21
- Gravitational instability, *see also* Jeans instability, 572
- Gravitational potential, 18
- Gravitational softening, 364
- Gravo-thermal catastrophe, 720
- Green valley, 629
- Green's function, 19
- Green's theorem, 808
- Growing mode, 553
- Growth factor, 554
- Guiding-center radius, 220
- $H\alpha$, 185
- $H\alpha$ line, 591
 - SFR, 592
- $H\beta$ line, 591
- Half-mass radius, 143
- Half-normal distribution, *see also* normal distribution, 810
- Halo mass function, 579
 - conditional, 668
 - subhalo mass function, 620
- Hamilton–Jacobi equation, 55
 - and the perfect ellipsoid, 350
 - spherical potential, 78
- Hamilton's characteristic function, 56
- Hamilton's equations, 50
- Hamilton's principal function, 55
- Hamilton's principle, 47
- Hamiltonian, 50
 - cylindrical coordinates, 51
 - polar coordinates, 51
 - spherical coordinates, 105
- Hamiltonian mechanics, 49
 - action-angle coordinates, 57
 - angle-action coordinates, 57
 - canonical momentum, 49

- canonical transformation, 53
- generating function, 53
- generating function of the first kind, 54
- generating function of the third kind, 235
- Hamilton–Jacobi equation, 55
- Hamilton’s characteristic function, 56
- Hamilton’s equations, 50
- Hamilton’s principal function, 55
- Modified Hamilton’s principle, 52
- phase space, 51
- phase-space volume conservation, 55
- point transformation, 54
- Heating (kinematical), 787
- Heaviside function, 812
- Helmholtz equation, 353
- Henon–Heiles potential, 389
- Hermite integrators, 375
- Hermitian operator, 352
- Hernquist profile, 33
 - as an approximation of the de Vaucouleurs profile, 331
- constant-anisotropy distribution function, 121, 122
- enclosed mass, 33
- ergodic distribution function, 113
- Osipkov–Merritt distribution function, 127
- potential, 33
- Hierarchical structure formation, 580
- Hill radius, *see* (idal radius)704
- Hipparcos mission, 261
- Homogeneous sphere
 - orbits, 64
 - potential, 29
- Horizon, 544
- Horseshoe orbit, 784
- Hot-mode accretion, 674
- Hubble classification, 623
 - elliptical galaxies, 334
 - Hubble type, 623
- Hubble constant, 542, 855
- Hubble drag, 572
- Hubble parameter, 541
- Hypergeometric functions, 822
- Impact parameter, 94
- Impulse approximation, 94
- Inclination, 189
- deprojection, 440
- disk galaxy, 194
 - of an orbit in a spherical potential, 64
- Incomplete elliptic integral, 813
- Inertial reference frame, 42
- Inflation, 544
- Initial mass function, 595
 - Chabrier, Kroupa, Salpeter, 595
- Inner Lindblad resonance, 737
- Instability
 - bar, 733
 - buckling, 769
 - Jeans, 572
 - radial-orbit, 575
- Instantaneous recycling approximation, 289
- Integrable potential, 394
- Integral of motion, 75, 223
 - independent, 76
 - isolating, 75, 223
 - non-classical, 382
 - third, *see also* Third integral, 224
- Integral-field spectroscopy, 511
- Interstellar medium
 - evolution of elemental abundances, 284
 - mass, 11
- Intracluster light, 657
- Ionization parameter, 606
- Isochrone
 - orbits, 72, 81
 - potential, 30
- Isochrone (stellar), 596
- Isolating integral of motion, 75, 223
- Isophotal twists, 335
- Isothermal sheet, 272
 - distribution function, 272
- Jacobi integral, 700
- Jacobi radius, *see* (idal radius)704
- Jaffe profile, 34
 - as an approximation of the de Vaucouleurs profile, 331
- Jansky, 590
- Jeans equations, *see also* Spherical Jeans equation, 103, *see also* Axisymmetric Jeans equations
 - closure, 104, 516
 - general, 103

- Jeans instability, 572, 679, 680
- Jeans length, 552, 572, 679
- Jeans mass, 679
- Jeans swindle, 574
- Jeans theorem, 109
- Jeans wavenumber, *see also* Jeans length, 572
- Jerk, 128

- K correction, 603
- Kennicutt–Schmidt law, *see* Schmidt law
- Kepler
 - eccentric anomaly, 69
 - first law, 69
 - Kepler’s equation, 70
 - mean anomaly, 70
 - orbits, 67
 - potential, 28
 - second law, 62
 - third law, 69
 - true anomaly, 68
- Kepler’s equation, 70
- Keplerian, 28
- Kinematic major axis, 186, 511
- Kinematic position angle, 518
- Kinematical heating, 787
- Kinematically-decoupled core, 512
- Kinometry, 518
- Kinetic energy, 44
- King profile, 116
 - concentration, 117
 - elliptical galaxies, 332
 - radius, 117
- Kuzmin model
 - circular velocity, 163
 - potential, 162

- Lagrange equations, 48
- Lagrange points, 703
 - bar, 752
 - two orbiting point masses, 703
- Lagrangian, 47
- Lagrangian mechanics, 47
 - generalized coordinates, 48
 - generalized momentum, 49
 - Hamilton’s principle, 47
- Landau damping, 573
- Lane–Emden equation, 716
- Laplacian, 807

- Large Magellanic Cloud, 2
- Late-type galaxy, 625
- Latitudinal action, 79
- Leaky box model, 299
 - effective yield, 300
 - outflow efficiency, 299
 - recycling fraction, 300
 - with type Ia delays, 316
- Leapfrog method, 87
- Legendre polynomials, 818
 - associated, 354, *see also* Associated Legendre polynomials, 821
 - differential equation, 820
- Legendre transformation, 50
- Lensing degeneracies, 467
- Lensing equation, 454
 - axially-symmetric lens, 455
 - multi-plane, 454
- Lensing potential (in GR), *see* Curvature potential
- Lensing potential (in gravitational lensing), 457
- Lenticular galaxy, 624
- Lin–Shu hypothesis, 795
- Lindblad diagram, 783
- Lindblad resonance, 737
 - vertical, 780
- Line of nodes, 189
- Linear matter power spectrum, 561
- LINERs, 630
- Liouville’s theorem, 102
- Little h , 542
- Local Group, 4
 - mass from timing argument, 136
- Local Group timing argument, 134
- Local standard of rest, 205
- Logarithmic potential, 33
 - cored, 193
 - deflection angle, 452
 - flattened, 168
 - non-axisymmetric, 384
 - non-axisymmetric, rotating, 752
- Logarithmic spiral, 778
- Long & Murali bar potential, 759
- Long-axis tube orbit, 400
- Long-wave branch, 739
- Longitude of the ascending node, 64
- Longitude-velocity (l, v) diagram, 203
 - and circular rotation, 205

- Loop orbit, 386
 - parent, 386
- Lorentz transformation, 829
- Low-surface brightness galaxy, 626
- Lower incomplete gamma function, 813
- Luminosity density, 614
- Luminosity distance, 857
- Luminosity function (galaxy), *see* Galaxy
 - luminosity function
- Luminous infrared galaxy, 643
- Lyman break, 594

- M–sigma relation, 508
- M100, 773
- M101, 17
- M11, 5
- M31, 10
- M33, 774
- M51, 1, 653
- M80, 5
- M81, 4
- Magnification, 469
- Magnification matrix, 469
- Magnitude (flux), 590
- Main sequence (stars), 596
- Major axis
 - kinematic, 186, 511
 - photometric, 186, 511
- Major merger, 657
- Mass loading factor, *see* Outflow
 - efficiency
- Mass-anisotropy degeneracy, 107
- Mass-sheet degeneracy, 467
- Mass-to-light ratio, 9, 158
 - and dark matter, 201
 - disk galaxies, 197
 - from stellar-population synthesis
 - modeling, 592
 - maximum-disk fit, 197
- Matter domination, 541
- Matter power spectrum, 561
- Matter-radiation equality, 541
- Maximum height above the plane z_{max} , 220
- Maximum-bulge fits, 526
- Maximum-disk fit, 197
- Mean anomaly, 70
- Mechanical work, 44
- Merger rate, 657
- Meridional plane, 218
- Metal-line blanketing, 295
- metric, 828
 - tensor, 830
- Michie model, 118
- Mid-plane, 8
- Milky Way
 - bar, 747, 767
 - circular velocity, 26
 - dark matter, 13
 - disk stability, 731
 - gas mass, 12
 - local density, 14
 - local ISM density, 12
 - local stellar density, 12
 - mass distribution, 13
 - mass from escape velocity, 134
 - mass from halo Jeans analysis, 138
 - mass from virial theorem, 99
 - spiral structure, 773
 - stellar mass, 12
- Minimal coupling, 834
- Minkowski metric, 829
- Minor merger, 657
- Minor-axis rotation
 - in elliptical galaxies, 335
 - in triaxial mass distributions, 401
- Miyamoto–Nagai model, 165
- Mode (Fourier), 551
- Modified Euler method, 87
- Modified Hamilton’s principle, 52
- Modified Newtonian Dynamics, 637
- Molecular gas
 - radial profile, 13
- Momentum, 42
 - specific, 60
- Moving groups, 268
- Multi-plane lensing, 454
- Multiplicity function (Press–Schechter), 582
- Multipole expansion, 357
- MWPotential2014, 217

- Naked cusp, 488
- nebular emission, 604
- nebular emission, 605
- Newton
 - first shell theorem, 22
 - laws of motion, 42

- Newton (*continued*)
 - in the general theory of relativity, 835
 - second shell theorem, 22
 - third shell theorem, 378
- NFW profile, 33
 - circular velocity
 - oblate or prolate, 348
 - spherical, 36
 - concentration, 35
 - enclosed mass, 36
 - radial velocity dispersion, 107
 - r_{max} , 36
 - triaxial
 - orbits, 401
 - virial mass, 35
 - v_{max} , 36
- NGC 1291, 751
- NGC 1300, 3
- NGC 147, 626
- NGC 185, 626
- NGC 2336, 750
- NGC 2623, 654
- NGC 2974, 513
- NGC 2998, 198
- NGC 3198, 187, 201
- NGC 3315, 330
- NGC 3379, 330
- NGC 3923, 4
- NGC 4150, 329
- NGC 4244, 8
- NGC 4314, 719
- NGC 4342, 439, 531
- NGC 4365, 512
- NGC 4414, 774
- NGC 4535, 634
- NGC 4549, 330
- NGC 4565, 6
- NGC 4660, 336
- NGC 4676, 651
- NGC 5322, 336
- NGC 6066, 601
- NGC 660, 17
- NGC 753, 197
- NGC 801, 197, 198
- Noether's theorem, 49
 - integral of motion, 77
- Non-homology of the orbit distribution, 640
- Non-inertial reference frame, 43
- Non-regular orbit, 392
- Normal distribution, 809
- Normal mode, 777
- Nuclear star cluster, 7
- Nuker profile, 333
- Null geodesic, 846
- Oblate spheroid, 339
- Oblate spheroidal coordinates, 339
 - gradient, 341
 - Laplacian, 341
- Oct-tree, 367
- One form, 830
 - general manifold, 832
- Oort constants, 210
 - and the epicycle approximation, 229
 - and the local velocity dispersion tensor, 231
 - and the local velocity distribution, 260
 - and the Poisson equation, 270
 - azimuthal shear, 211
 - rotation curve, 213
- Open cluster, 5
- Opening angle, 368
- Optical radius, 194
- Orbit
 - x_1 , 753
 - x_2 , 755
 - x_3 , 755
 - x_4 , 753
 - banana, 406
 - box, 386
 - boxlet, 403, 406, 755
 - centrophilic, 403
 - centrophobic, 403
 - chaotic, 389
 - ergodic, 389
 - fish, 406
 - horseshoe, 784
 - long-axis tube, 400
 - loop, 386
 - parent, 383
 - polar, 380
 - pretzel, 406
 - regular, 58, 389
 - resonant, 383
 - short-axis tube, 400
 - tube, 400

- Orbital anisotropy, 106
 - kinetic energy tensor, 520
 - meridional plane, 516
- Orbital eccentricity, 63
- Orbital frequencies, 57
- Orbital non-homology, 640
- Orbital torus, 58
- Ordinary differential equation
 - homogeneous, 824
 - inhomogeneous, 824
 - particular solution, 824
 - linear, 824
 - linear and homogeneous, 824
 - characteristic equation, 824
- Ordinary hypergeometric functions, 822
- Orthogonal polynomials, 818
 - Rodrigues' formula, 820
 - three-term recurrence formulae, 820
- Osipkov–Merritt anisotropic distribution
 - function, 124
 - sample, 127
 - stability, 575
- Ostriker & Peebles stability criterion, 734
- Outer Lindblad resonance, 737
- Outflow efficiency, 299
- Over-cooling problem, 683

- PAHs, 605
- Pal 5
 - stream, 711
 - tidal field, 701, 707
- Parent orbit, 383
- Particle horizon, 544
- Particle-spray technique, 717
- Peak height, 582
- Perfect ellipsoid, 350
 - circular velocity, 350
- Perfect fluid, 837
- Pericenter, 62, 220
- Period
 - azimuthal, 63
 - radial, 63
- Phase space, 51
- Phase-space spiral, 268
- Photo-ionization, 677
- Photometric major axis, 186, 511
- Photometry, 589
- Pitch angle
 - and the shape function, 777
 - definition, 775
- Hubble classification, 624
- Plummer model, 30
 - surface density, 140
- Poincaré invariant theorem, 52
- Point transformation, 54
- Point-mass potential
 - deflection angle, 451
- Poisson equation, 18
 - cylindrical coordinates, 228, 269
 - for the surface density, 271
 - in a galactic disk, 228, 269
 - in terms of the relative potential, 111
 - in two dimensions, 457
 - oblate spheroidal coordinates, 341
 - phantom density, 270
- Polar orbit, 380
- Polycyclic aromatic hydrocarbons (PAHs), 605
- Position angle
 - kinematic, 518
- position angle, 440
- Potential
 - arbitrary oblate spheroid, 345
 - arbitrary prolate spheroid, 346
 - arbitrary razor-thin disk, 177
 - arbitrary spherical density, 24
 - arbitrary triaxial ellipsoid, 346
 - arbitrary, disky, 181
 - bar (Long & Murali), 759
 - basis-function expansion, 352, 359
 - cored logarithmic, 193
 - Cox & Gomez spiral structure, 781
 - disky, 232
 - double-exponential disk, 178
 - effective, 62, 218
 - gamma models, 33
 - Henon–Heiles, 389
 - Hernquist, 33
 - homogeneous sphere, 29
 - in the general theory of relativity, 844
 - integrable, 394
 - Isochrone, 60
 - Kepler, 28
 - Kuzmin, 162
 - logarithmic, 33
 - flattened, 168
 - non-axisymmetric, 384
 - non-axisymmetric, rotating, 752

- Potential (*continued*)
 - Miyamoto–Nagai, 165
 - multipole expansion, 357
 - NFW, 33
 - perfect ellipsoid, 350
 - plane wave, 722
 - Plummer, 30
 - point mass, 28
 - power-law, 32
 - relative, 111
 - self-consistent field method, 361
 - thick disk, 178
 - thin oblate shell, 342, 343
 - thin ring, 159, 176
 - thin spherical shell, 24
 - arbitrary surface density, 357
 - non-uniform surface density, 356
 - tightly-wound perturbation, 722
 - two-power density, 33
- Potential energy, 45
- Potential-density pair, 20
- Power-law galaxies, 332
- Pre-enrichment, 297
- Press–Schechter formalism, 581
 - Extended Press–Schechter formalism, 660
- Press–Schechter halo mass function, 582
 - in the extended Press–Schechter formalism, 663
 - peak height, 582
- Pressureless dust, 838
- Pretzel orbit, 406
- Prolate spheroid, 339
- Prolate spheroidal coordinates, 235, 342
 - gradient, 378
 - Laplacian, 378
- Proper distance, 852
- Proper motion, 804
- Proper time, 829

- Q, 727
- QSO 0957+561, 448, 450
- QSOs, *see* Quasars
- Quad lens, 456
- Quasars, 631
- Quasi-equilibrium state, 93, 718
- Quenching, 617

- r-process enrichment, 286

- Radial acceleration relation, 637
- Radial action, 58, 79
- Radial frequency, 63
 - in the epicycle approximation, 227
- Radial Jeans equation, *see also*
 - Axisymmetric Jeans equations, 249
- Radial migration, 792
- Radial period, 63
- Radial-orbit instability, 575
- Radiation domination, 541
- Radio galaxies, 630
- Razor-thin disk, 162
 - distribution function, 251
 - cold, 253
- Razor-thin exponential disk, 175
 - circular velocity, 175
 - peak circular velocity, 175
- Recombination cooling, 675
- Rectangular function, 811
- Recycling fraction, 300
- Red sequence, 628
- Reduced deflection angle, 454
- Reduced mass, 89
- Reduced shear, 476
- Reduction factor, 725
- Regular orbit, 58, 389
 - actions, 58
- Reheating, 556
- Relative energy, 111
- Relative potential, 111
- Relaxation time, 94
 - near an SMBH, 503
- Resolution (spectrograph), 590
 - convolution, 602
- Resonant orbit, 383
- Restricted three-body approximation, 653
- Ricci scalar, 836
- Ricci tensor, 836
- Riemann tensor, 835
- Right ascension, 803
- Robertson–Walker metric, 852
- Roche radius, *see* (idal radius)704
- Rodrigues’ formula, 820
- Rosenbluth potential, 691
- Rosette, 73, 221
- Rotation curve, 27, 187
 - and the (l, v) diagram, 205
 - arbitrary razor-thin disk, 196

- arbitrary spherical density, 196
- circular velocity at the Sun, 207
- disk potential, 163
- effect of flattening, 348
- from spherically-symmetric surface
 - brightness, 196
- maximum-disk fit, 197
- Oort constants, 211
- spherical mass distribution, 27
- spheroidal mass distribution, 347
- tangent-point method, 208
- terminal velocity curve, 208
- Rotational frequency, 63
 - in the epicycle approximation, 227
- Rotational period, 63
- Runge–Kutta method, 84
- RXJ1131-1231, 448

- s-process enrichment, 286
- S0 galaxy, 1, 624
- SB(r) galaxy, 750
- SB(s) galaxy, 750
- Scale factor, 852
- Scale height, 9, 158
 - ratio with scale length, 9
- Scale length, 9, 156, 158
- Schechter function, 613
- Schmidt law, 643
- Schwarzschild distribution function, 254
- Schwarzschild radius, 502
- Sech² profile, 8, 158
- Secular evolution, 718
- Seeing, 502
- Self-consistent field method, 361
- Self-gravitating isothermal sheet, *see also* Isothermal sheet, 272
- Semi-analytic galaxy formation, 660
- Sérsic profile, 9, 331
- Seyfert galaxies, 630
- Shape
 - ellipticity, 334, 623
 - triaxiality parameter, 337
- Shapiro delay, 462, 849
- Shear, 472
- Shearing sheet, 246, 739
- Shell crossing, 576
- Shell galaxy, 4
- Short-axis tube orbit, 400
- Short-wave branch, 739

- Shu distribution function, 258
- Single stellar populations, 594
- Singular isothermal sphere, 115, *see also*
 - Logarithmic Potential
- convergence and shear, 473
- deflection angle, 452
- Einstein angle, 455
- isotropic distribution function, 115
- lensing equation, 455
- magnification, 469
- potential, 33
- Slow rotators, 335, 521
- Softening, 364
- Softening length, 365
- Solar abundances, 285
- Solar motion, 262, 264
- Solar neighborhood
 - abundance ratios, 316
 - age–metallicity relation, 791
 - age–velocity–dispersion relation, 786
 - asymmetric drift, 261
 - epicycle amplitude, 231
 - epicycle frequency, 230
 - ISM density, 12
 - local dark matter density, 278
 - metallicity distribution, 296
 - moving groups, 268
 - Oort constants, 213
 - phase-space spiral, 268
 - stellar density, 12
 - total density of matter, 276
 - total surface density of matter, 277
 - velocity distribution, 267
- Solar system barycenter, 260
- Solid-body rotation, 29
- Soltan argument, 509
- Source plane, 454
- Specific intensity, 470
- Specific star formation rate, 640
- Spectra, 589
 - typical elliptical, spiral, and irregular galaxy spectra, 593
- Spectral energy distribution, 590
- Spectral library, 598
- Spectroscopy, 589
- Sphere of influence, 402, 502
- Spherical collapse, 563
- Spherical harmonics, 355

- Spherical Jeans equation, *see also* Jeans equations, 106
 - in terms of line-of-sight velocity dispersion, 140
 - in terms of the circular velocity, 136
 - in terms of the enclosed mass, 106
 - in terms of the gravitational potential, 106
 - solution, arbitrary $\beta(r)$, 107
 - solution, constant β , 107
- Spheroid, 339
- Spiral structure
 - angular-momentum transport, 794
 - anti-spiral theorem, 778
 - churning, 784
 - grand design, 1
 - grand-design vs. flocculent, 772
 - Lin–Shu hypothesis, 795
 - logarithmic, 778
 - material arms, 777
 - pitch angle, 775
 - trailing vs. leading, 774
 - winding problem, 777
- Splashback radius, 569
- sSFR, 640
- Stability criterion
 - gas disk, 730
 - multi-component disk, 731
 - Ostriker & Peebles, 734
 - stellar disk, 727
 - thick disk, 728
- Staeckel approximation, 237
- Staeckel fudge, 237
- Staeckel potential, 236, 397
- Star formation efficiency, 289, 293, 644
- Star formation rate, 293
 - delay-time-distribution-averaged, 311
 - from emission lines, 591
 - specific, 640
- Stellar evolutionary tracks, 595
- Stellar halo, 10
- Stellar isochrone, 596
- Stellar mass function (galaxy), *see* Galaxy stellar mass function
- Stellar population synthesis, 592
- Stellar-mass density, 615
- Stellar-mass–halo-mass relation, 618
- Stellar-population synthesis, 527
- Step function, 812
- Stephan’s Quintet, 651
- Stress-energy tensor, 837
- Stromberg asymmetric drift, *see* Asymmetric drift
- Strong gravitational lensing, 465
- Strong lensing, 450
- Sturm–Liouville theory, 353
- Sub-grid physics, 687
- Subhalo mass function, 620
- Subhalos, 329
- Supernova feedback, *see* Feedback
- Surface brightness, 470
- Surface density
 - deprojection assuming spherical symmetry, 140
 - in terms of potential for a razor-thin disk, 163
- Surface of section, 381
- Swing amplification, 735
 - bars, 768
 - criterion, 743
 - feedback loop, 745
- Tangent-point method, *see* Terminal velocity curve
- Temperature, of a self-gravitating system, 719
- Tensor, 830
 - general manifold, 832
- Terminal velocity curve, 208
- Test particle, 60
- Thin screen approximation, 453
- Third integral, 224
 - surface of section, 382
- Tidal bridge, 650, 652
- Tidal radius, 704
 - King profile, 117
- Tidal stream, 710
 - particle-spray technique, 717
- Tidal tail, 650, 652
- Tight-winding approximation, 721
- Tilted-ring model, 194
- Time delay in lensing, 462
 - geometric, 461
 - Shapiro, 462
- Time-delay distance, 462
- Toomre Q , 727
 - thick disk, 728
- Toomre stability criterion, 727

- Top-down structure formation, 580
- Torque, 44
- Torus, 58
- Transfer function, 558
- Tremaine–Weinberg method, 749
 - bars, 749
 - spirals, 776
- Triangulum galaxy, 774
- Triaxiality parameter, 337
- True anomaly, 68
- Tube orbit, 400, 513
- Tully–Fisher relation, 634
 - baryonic form, 636
- Turn-around time, 566
- Turn-off mass, 596
- Twin quasar QSO 0957+561, 448, 450
- Two-body encounters, 689
- Two-body relaxation, 95, 720
- Type I, II, or III disk profile, 156
- Type Ia supernovae
 - Chandrasekhar mass, 306
 - enrichment, 286
 - progenitor systems, 306
 - total yield of iron, 309
- Type Ia supernovae delay time
 - distribution, 307
 - expected from progenitor models, 307
 - normalization, 308
 - observed distribution, 308
- Type Ia supernovae
 - delay-time-distribution-averaged star-formation history, 311
- Type II supernovae
 - enrichment, 285
 - progenitor systems, 306
- ULIRG, 643, 656
- Ultra-diffuse galaxy, 144, 627
- Ultra-faint dwarf spheroidal, 139
- Ultra-faint dwarf spheroidal galaxy, 627
- Ultra-luminous infrared galaxies, 656
- Ultra-luminous infrared galaxy, 643
- Ultraviolet background, 677
- Unified Model of AGN, 632
- Upper incomplete gamma function, 813
- UV excess, 295
- Vector
 - general manifold, 832
- Vector calculus
 - curl, 807
 - divergence, 807
 - divergence theorem, 807
 - gradient, 806
 - gradient theorem, 808
 - Green’s theorem, 808
 - Laplacian, 807
- Velocity ellipsoid, 266
 - tilt, 270
 - vertex deviation, 267
- Velocity field, 185
 - axisymmetric model, 190
 - flat rotation curve, 191
 - observations, 186
 - peaked rotation curve, 191
 - sky and galaxy coordinate systems, 189
 - solid-body rotation, 190
 - tilted-ring model, 194
- Vertex deviation, 267
- Vertical action, 58
- Vertical frequency
 - in terms of the local density, 228
 - in the epicycle approximation, 227
- Vertical Jeans equation, *see also* Axisymmetric Jeans equations, 271
- Vertical Lindblad resonances, 780
- Violent relaxation, 576
- Virial mass, 35, 567
 - Milky Way, 35
- Virial overdensity, 567
- Virial quantity, 97
- Virial radius, 35, 567
 - Milky Way, 35
- Virial temperature, 680
- Virial theorem, 97
 - external point-mass potential, 97
 - scalar, 99
 - self-gravitating system, 98
- Wake (dynamical friction), 694
- Wave propagation (stellar disks), 738
- Weak equivalence principle, 21, 827
- Weak lensing, 450, 475
- Winding problem, 777
 - kinematic waves, 779
- Wolf mass estimator, 141

- Work, 44
- X (swing amplification), 743
- x_1 orbits, 753
- x_2 orbits, 755
- x_3 orbits, 755
- x_4 orbits, 753
- Yield, 288
 - effective yield, 300
 - of iron, 309
 - of oxygen, 314
 - population, 288
 - total yield of all elements, 288
- Zero-velocity curve, 219, 223, 700
- Zoom-in simulations, 377