

Contents

Introduction xxi

PART I Introduction to the Solid Earth System 1

1	AN EXPLORATION OF BASIC SOLID EARTH STRUCTURE AND DYNAMICS	3
1.1	Topography	3
1.1.1	Isostasy	6
1.2	Geopotentials: Shape, spin, and geoid	10
1.2.1	Gravitational potential	10
	Expanded details 1: Moments of inertia and geopotentials of an ellipsoidal Earth	12
1.2.2	Reference geoid, spin, and the Earth-Moon system	16
1.2.3	Geoid and gravity anomalies	17
	Expanded details 2: Geoid, spherical harmonics, and gravity anomalies	18
1.3	Internal structure, temperature, and composition of Earth	22
1.3.1	1-D structure of Earth as seen from seismology	22
1.3.2	Pressure and mass within the Earth	24
1.3.3	Complexities in the 1-D model	25
1.3.3.1	The lithosphere and its topography	25
1.3.3.2	The asthenosphere	27
1.3.4	Thermal background state of the mantle	28
	Expanded details 3: Thermodynamics of the adiabatic mantle temperature gradient	29
1.3.5	Composition and mantle phase transitions	32
1.4	Plate motions in the past and at present	34
1.4.1	Seafloor spreading and seafloor age	35
	Expanded details 4: Rikitake dynamo, magnetic field creation, and chaos	38
1.4.2	Current plate motions	42
1.4.2.1	Absolute reference frames and net rotation	42
1.4.2.2	Poloidal and toroidal flow	45
1.4.3	Plate motion reconstructions	47
1.5	Seismic tomography—3-D mantle structure	50
	Review questions and discussion topics 1	54

PART II Fundamental Physics 57

2 CONTINUUM MECHANICS 59

- 2.1 Concept of a continuum 59
- 2.2 Displacements, velocities, and reference frames 61
 - 2.2.1 Reference frames for considering deformation of a continuum 63
 - 2.2.1.1 Lagrangian reference frame 63
 - 2.2.1.2 Eulerian reference frame 63
- 2.3 Strain 64
 - 2.3.1 Displacement gradient tensor 68
 - 2.3.2 Strain rates 69
 - 2.3.3 Finite strain 70
- 2.4 Forces and stress 72
 - 2.4.1 Sign convention for stress 72
 - 2.4.2 Cauchy stress tensor 72
 - 2.4.3 Static force equilibrium and stress tensor symmetry 73
 - 2.4.4 Principal axes of the stress tensor 75
 - 2.4.5 Pressure and deviatoric stress 76

Expanded details 5: Tensor invariants and measures of stress 79

 - 2.4.6 Stress in two dimensions 81
 - 2.4.7 Pure shear and simple shear 82
 - 2.4.8 Continuity of stress tensor components 83

Exercise 1: Strain, strain rates, and stress 84

- 2.5 Constraining crustal deformation and stress 84
 - 2.5.1 Moment tensors and the gCMT catalog 84

Expanded details 6: Non-double-couple moment tensor deformation 87

 - 2.5.2 Seismic coupling and seismotectonics 89
 - 2.5.3 Crustal strain-rate fields from geodesy 91
 - 2.5.4 Constraints on stress within the lithosphere 92
 - 2.5.5 Stress vs. strain from focal mechanisms 93

Review questions and discussion topics 2 94

3 ELASTIC AND BRITTLE BEHAVIOR OF ROCKS 97

- 3.1 Constitutive laws 98
- 3.2 Elasticity 100
 - 3.2.1 2-D elasticity, plane strain/stress 104

Expanded details 7: Isotropic and anisotropic elasticity 105

Exercise 2: Stresses in the lithosphere 107

 - 3.2.2 Elastic flexure 110
 - 3.2.2.1 Flexural profiles 110
 - 3.2.2.2 Flexural vs. isostatic compensation 112
 - 3.2.2.3 Gravity-topography ratios 114
- 3.3 Brittle failure 116
 - 3.3.1 Fracturing and faulting 117
 - 3.3.1.1 Griffith criterion and linear elastic fracture mechanics 117

3.3.2	Mohr-Coulomb failure and friction	119
3.3.2.1	Byerlee's rule (or "law")	120
3.3.2.2	Mohr's circle	122
Exercise 3: Byerlee's law applied to the lithosphere		124
3.3.2.3	Griffith cracks and fluid effects	125
3.3.2.4	Shear cracks	126
3.3.3	Fault zone structure	126
3.4	Earthquakes and friction	130
3.4.1	Earthquake statistics	132
3.4.1.1	Frequency-magnitude (Gutenberg-Richter) relationship	134
3.4.1.2	Clustering: Omori's and Båth's laws	137
3.4.1.3	Some implications of clustered seismicity	139
3.4.2	Static and dynamic friction and the seismic cycle	139
3.4.2.1	Elastic rebound and stick-slip cycles	140
3.4.2.2	How regular are large earthquakes?	141
3.4.2.3	Earthquake predictability	142
3.4.3	Modeling and constraining coseismic deformation	144
3.4.3.1	Screw dislocation in an elastic half-space	145
3.4.3.2	Postseismic relaxation in a confined depth region	147
3.4.3.3	Fault solution for two elastic layers	147
3.4.3.4	Dislocation models for finite faults	148
3.4.3.5	Tectonic space geodesy	152
3.4.4	Earthquake source mechanics	153
3.4.4.1	Constant stress drop scalings	155
Expanded details 8: Rupture dynamics energy considerations and stress drop		156
3.4.5	Rate and state friction as a constitutive law	159
Expanded details 9: Physical processes behind rate-state friction		163
3.4.5.1	Stability and limit cycles of an RSF spring slider	164
Expanded details 10: Numerical implementation of rate-state friction equations		166
3.4.5.2	Frictional stability in the lab and in nature	167
3.4.5.3	Complex seismic cycles on simple faults	169
3.4.6	Fault strength, stress, and heterogeneity	172
Review questions and discussion topics 3		174
4	VISCOUS, PLASTIC, AND TRANSIENT BEHAVIOR	175
4.1	Newtonian fluids	175
4.1.1	Bulk viscosity and partial melting	177
4.1.2	Viscous dissipation	179
4.2	Non-Newtonian fluids	179
4.2.1	Work hardening and work softening	180
4.2.2	Plastic behavior	180
4.2.2.1	Von Mises yield criterion	181
4.2.2.2	Drucker-Prager criterion	182
4.2.3	Strain and strain-rate-dependent weakening	183
4.3	Combined material behavior and transient viscoelasticity	184
4.3.1	Bingham viscoplasticity	184
4.3.2	Two viscous dashpots in series	185

4.3.3	Viscoelastic behavior: The Maxwell body	185
	Expanded details 11: Derivation of the viscoelastic Maxwell body response	186
	Expanded details 12: Other viscoelastic transient models	189
4.4	Attenuation and frequency-dependent moduli	192
4.4.1	Attenuation for the standard linear solid	193
4.4.2	Q within the Earth	195
4.4.3	Temperature derivatives of seismic velocities	196
4.5	Transient deformation throughout the seismic cycle	199
4.5.1	Relaxation of an elastic plate over a viscous layer	200
4.5.2	Dislocation solution for a viscoelastic half-space	200
4.5.3	Earthquake cycle solution for a viscoelastic half-space	202
4.5.4	Postseismic response after earthquakes	202
	Review questions and discussion topics 4	206
5	LITHOSPHERIC STRENGTH FROM COMPLEX ROCK RHEOLOGY	207
5.1	Microphysical mechanisms for viscous creep	207
5.1.1	Pressure solution	207
5.1.2	Temperature dependence: The Arrhenius law	207
5.1.3	Diffusion creep	209
5.1.4	Dislocation creep	209
5.1.5	Mapping ductile crustal deformation	211
5.1.6	Laboratory-derived creep laws	213
5.1.7	Peierls creep/plasticity	216
	Expanded details 13: Fugacity and volatile concentration	217
	Exercise 4: Laboratory-derived creep laws	218
5.2	Piezometers and grain size evolution	221
	Expanded details 14: Grain size evolution laws	222
5.3	Averaging of heterogeneous media	224
5.3.1	Harmonic (weak) mean	224
5.3.2	Arithmetic (strong) mean	224
5.3.3	Geometric (intermediate) mean	225
5.3.4	Self-consistent averaging	225
5.4	Deformation maps	226
5.5	Strength of the lithosphere and upper mantle	228
5.5.1	Byerlee's law as a function of pressure	228
5.5.2	Brittle-ductile behavior	229
5.5.2.1	Oceanic plates	229
5.5.2.2	Continental plates	231
	Review questions and discussion topics 5	232
6	TRANSFER OF MOMENTUM: FLUID DYNAMICS	233
6.1	Nondimensional numbers and flow regimes	233
	Expanded details 15: Coriolis forces and geodynamo dynamics	235
	Exercise 5: Scaling analysis and nondimensional numbers	237

6.2	Steady, unidirectional flow and the 1-D Stokes equation	240
	Expanded details 16: Stokes (force balance) equation in 3-D	241
6.3	Shear and pressure-driven flow	243
6.3.1	Couette flow	243
6.3.2	Hagen-Poiseuille flow	244
6.3.3	Gravity-driven flow down a plane	245
6.4	Density-driven flow: The Stokes sinker	246
6.4.1	Stokes velocity	246
6.4.2	Rayleigh number from Stokes velocity and Peclet number	249
6.4.3	Dynamic topography of the Stokes sinker	249
	Expanded details 17: Role of viscosity variations for dynamic topography	251
6.5	Stream function solutions	252
6.5.1	Corner flow	253
6.5.2	Parabolic flow: Source in a uniform stream	254
6.5.3	Hale-Shaw flow and superposition	254
6.6	Thin viscous sheets and gravitational potential energy	255
6.7	Postglacial rebound and glacial isostatic adjustment	259
6.7.1	The Haskell constraint on mantle viscosity	260
6.7.1.1	Wavelength dependence of viscoelastic relaxation	262
	Expanded details 18: Glacial isostatic adjustment and sea level	266
	Review questions and discussion topics 6	267
7	TRANSFER OF ENERGY: HEAT TRANSPORT	269
7.1	Conduction of heat and budgets	270
7.1.1	Heat budgets and fractionation	272
7.1.1.1	Fractionation and rare earth elements	274
7.1.1.2	Total radiogenic budget of the mantle	275
7.1.1.3	Radioactive decay and time dependence of internal heating	276
7.2	Diffusion of heat and the conduction equation	277
7.2.1	Steady-state conduction solutions: Geotherms	279
7.2.1.1	Fixed surface temperature and heat flux	279
7.2.1.2	Fixed surface and base temperatures	280
7.2.1.3	Fixed surface temperature, constant flux at base	280
7.2.1.4	Fixed surface temperature, fixed deep flux	281
7.3	Time-dependent solutions of the conduction equation	282
7.3.1	Half-space cooling (HSC)	283
	Expanded details 19: Half-space cooling solution by separation of variables	284
	Expanded details 20: Periodic heating of a semi-infinite half-space	287
	Exercise 6: Finite difference solution of 1-D heat equation	289
7.3.2	Application of half-space cooling to the oceanic lithosphere	293
7.3.2.1	Oceanic plate bathymetry from HSC	294
	Exercise 7: Plate driving forces from half-space cooling	297
7.4	Convection	302
7.4.1	Instability analysis for the Rayleigh-Bénard problem	303

	Expanded details 21: Linear instability analysis for Rayleigh-Bénard convection	304
7.4.1.1	Internal heating	308
7.4.2	Earth's mantle is convecting vigorously	309
7.4.3	Finite amplitude boundary layer model	310
	Expanded details 22: Modified boundary layer models	312
7.4.3.1	Scaling laws for mantle convection	314
7.4.3.2	Layer thickness independence of heat flux in convection	314
	Expanded details 23: Boundary layer instability and local Rayleigh number	315
7.4.4	Nondimensional equations and the Rayleigh number	316
Review questions and discussion topics 7 317		
8	MANTLE CONVECTION	319
8.1	Numerical Rayleigh-Bénard convection experiments	319
8.1.1	Role of internal heating	321
	Expanded details 24: Compressibility and viscous heating	323
8.1.1.1	Top down or bottom up?	324
8.1.2	Earth's thermal initial condition	325
8.1.2.1	Moon-forming impact	325
8.2	Temperature and stress dependence of viscosity	326
8.2.1	Approximations of rheological descriptions	326
8.2.1.1	Partial melt parameterization	327
8.2.2	Temperature dependence and the stagnant lid	328
8.2.3	Non-Newtonian flow	331
	Expanded details 25: Power-law based Rayleigh numbers	332
8.3	Effect of phase transitions	333
8.3.1	Kinetics, metastability, and multiminerality	336
8.4	Effects of 3-D spherical geometry and toroidal flow	337
8.4.1	Generation of toroidal flow	338
	Expanded details 26: Generation of toroidal flow	341
8.5	Plate-generating convection models	342
8.5.1	Viscoplasticity to break the stagnant lid	342
8.5.1.1	The low-yield stress problem	344
8.5.1.2	Hysteresis of heat transport state?	346
8.5.2	The role of the asthenosphere	347
8.5.3	Effects of continents and the supercontinental cycle	347
8.5.4	Localization and memory: Rheological hysteresis	350
Review questions and discussion topics 8 354		

PART III Tectonics and Mantle Dynamics 355

9 GLOBAL MANTLE CIRCULATION 357

9.1	The mantle wind: Predicting mantle flow	357
9.2	Surface topography	360
9.2.1	Residual and dynamic topography	361

9.2.2	Estimates of dynamic topography	362
9.2.2.1	Fréchet kernels	363
9.2.3	Residual vs. dynamic surface topography	365
9.3	Time reversal of mantle convection	367
9.4	The geoid constraint on viscosity	368
9.4.1	Venus, volatile storage capacity, and geoid-topography ratios	370
9.4.2	Typical viscosity profiles and nonuniqueness	371
9.5	Plate driving and crustal deformation studies	372
9.5.1	Plate driving forces	373
Exercise 8: Modeling global mantle circulation with SEATREE		377
9.5.2	Crustal stress and lithospheric deformation	380
9.5.3	Opportunities for model refinement	381
9.6	Seismic anisotropy as a constraint for mantle flow	382
9.6.1	Origin of upper mantle seismic anisotropy	385
9.6.2	Development of CPO anisotropy in upper mantle flow	386
9.6.2.1	Mechanical anisotropy	388
9.6.2.2	Absolute plate motion and net rotations	389
9.6.2.3	Memory of deformation	391
Review questions and discussion topics 9		393

10 DIVERGENT MOTIONS AND CREATION OF THE LITHOSPHERE 395

10.1	The role of plumes in mantle convection	395
10.1.1	Hotspots and plumes	395
10.1.2	Large igneous provinces	397
10.1.3	Plumes and the evolution of tectonics	399
10.1.4	Hotspot swells and plume heat transport	401
10.1.5	Hotspot reference frames and moving plumes	406
10.1.6	Thermal anomaly underneath hotspots and imaging plumes	407
10.1.7	Hotspots as probes of deep mantle reservoirs	408
10.1.7.1	Mixing of chemical heterogeneity in convection	410
10.1.7.2	Origin and geometry of geochemical reservoirs	412
10.1.8	Flood basalts and mass extinctions	413
10.2	Rifting: Breaking continents apart	415
10.2.1	Anatomy of a rift zone	416
10.2.2	Style and mechanics of rifting	421
10.2.3	Dynamics of rifting	426
10.2.4	The East African rift (EAR)	431
10.2.5	The Basin and Range	436
10.3	Oceanic spreading: Generating lithosphere	439
10.3.1	Structure of the oceanic crust	446
10.3.2	Slow to fast spreading and melt generation	448
10.3.3	Hydrothermal circulation	450
10.3.4	Partial melting	451
10.3.4.1	Two-phase flow	452
10.3.4.2	Melt parameterizations	453
10.3.5	Deviations from half-space cooling at old ages	455
10.3.5.1	Plate model	455
10.3.5.2	Chablis model	457

	10.3.5.3	Physical processes for deviations from HSC	458
	10.3.5.4	Regional deviations from HSC and reheating	459
10.4		Plates as thermochemical boundary layers	460
	10.4.1	Compositional density anomalies in the oceanic lithosphere	460
	10.4.2	The continental tectosphere	462
	10.4.2.1	Continental keels	464
	10.4.2.2	Midlithospheric discontinuities	465
		Review questions and discussion topics 10	466
11		LATERAL MOTION OF THE LITHOSPHERE: STRIKE SLIP	467
11.1		Introduction	467
	11.1.1	Hazard and risk	469
11.2		Anatomy of a strike-slip fault zone	472
11.3		Tectonic context of strike-slip systems	473
	11.3.1	Oceanic transforms	475
	11.3.1.1	Origin of oceanic transform faults	477
	11.3.2	Continental transforms	479
	11.3.2.1	Strike slip in collisional settings	479
	11.3.2.2	Tibetan collision	481
	11.3.2.3	Anatolian system	482
	11.3.3	Strike-slip systems in an active subduction setting	484
11.4		Continental transforms and earthquake dynamics	487
	11.4.1	Strain localization underneath faults	487
	11.4.2	Seismic hazard assessment and fault systems	489
	11.4.2.1	Time-dependent hazard estimates	489
	11.4.2.2	Constant fault geometry seismicity dynamics	490
	11.4.2.3	Fault system evolution	492
	11.4.3	Off-fault deformation	493
		Review questions and discussion topics 11	494
12		RECYCLING THE LITHOSPHERE: SUBDUCTION	495
12.1		Introduction	495
12.2		Anatomy of a subduction zone	498
12.3		Trench forearc and sedimentary systems	499
12.4		Temperature, magmatism, and metamorphism	504
	12.4.1	Thermal parameter and shear heating	504
	12.4.2	Volatile fluxes	509
	12.4.3	Petrological constraints	510
12.5		Arc volcanism	515
	12.5.1	Slab-associated, off-arc volcanism	519
12.6		Back-arc systems	519
12.7		Slab and trench kinematics: Rollback	520
12.8		Flat slabs	525
12.9		Slab dynamics from force balance considerations	526
	12.9.1	Viscous force balance	528
	12.9.1.1	Effective strength of the subducting lithosphere	530

12.10	Subduction zone seismicity	533
12.10.1	The megathrust interface	533
12.10.1.1	Megathrust deformation cycles	536
12.10.1.2	The spectrum of fault slip	538
12.10.1.3	Megathrust evolution and tectonics	540
12.10.2	Bulk deep slab deformation from moment release rates and style	541
12.10.3	Deep earthquake mechanisms	542
12.11	Tectonics and dynamics of subduction evolution	543
12.11.1	Subduction initiation	543
12.11.2	Subduction systems of the Pacific	546
12.11.2.1	The motion of the Pacific and back-arc extension	546
12.11.2.2	Izu-Bonin-Mariana and slab-slab interactions	549
12.11.2.3	Double slab systems	550
12.11.3	Slab rollback in the Mediterranean	552
12.12	Slab transport through the transition zone	557
12.12.1	Controls on regional slab and transition zone dynamics	561
12.12.2	Surface geology record of slab penetration	563
	Review questions and discussion topics 12	565

13 OROGENY: MAKING MOUNTAINS 567

13.1	Introduction	567
13.2	Anatomy of orogenic belts	570
13.3	The kinematics of mountain building	576
13.4	Mechanics of mountain building	578
13.5	The elevation of mountain belts	582
13.5.1	Wedge dynamics	587
13.5.1.1	Critical taper and Coulomb wedge theory	587
	Expanded details 27: Coulomb wedge equations	588
13.6	Tectonics, climate, and erosion	591
13.6.1	What is uplifting relative to what?	591
13.6.2	Uplift rates and surface transport	594
	Expanded details 28: Stream power and surface transport laws	595
13.6.3	Fingerprinting topography	598
13.7	Exhumation of deep-seated rock units	601
13.8	Dynamics of mountain building	607
13.9	Regional case histories	609
13.9.1	Cordilleran orogeny: The Andes	610
13.9.2	Accretionary orogeny: The Mediterranean	618
13.9.3	Collisional orogeny: Himalaya-Tibet	621
13.10	Orogeny, the Wilson cycle, and supercontinental assembly	630
	Review questions and discussion topics 13	635

14 PLATE TECTONICS AND PLANETARY EVOLUTION 637

14.1	Onset of plate tectonics	637
14.2	Different modes of early Earth heat transport	638

- 14.3 Parameterized thermal and volatile evolution models 639
- 14.4 Plate tectonics on other planets 644
 - 14.4.1 Should exoplanets have plate tectonics? 645

OUTLOOK 647

PART IV Appendixes 649

A REFERENCE DATA AND ADDITIONAL FIGURES 651

- A.1 Important constants and typical parameter values 651
- A.2 Geological timescale and magnetic reversals 651
- A.3 Phase diagrams 651
- A.4 Gravitational potentials for terrestrial planets 655
 - A.4.1 Comparison to Venus and Mars: Aphroditoid and areoid 657
- A.5 Plate geometry, names, and motion statistics 659

B ADDITIONAL TOPICS IN CONTINUUM MECHANICS 663

- B.1 General conservation laws: One law to derive them all 663
- B.2 Strain compatibility equations 666
- B.3 Finite strain 668
- B.4 Relationship between moment tensors and fault plane solutions 670
- B.5 Rayleigh-Taylor instabilities 670

C MATH AND STATISTICS NOTES 675

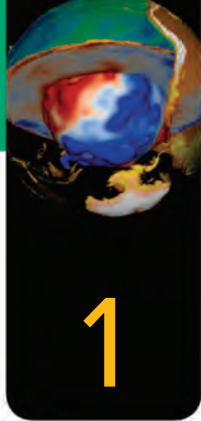
- C.1 Useful conventions and basic mathematical functions 675
 - C.1.1 Scientific notation, unit prefixes, and messing with exponents 675
 - C.1.2 Logarithms 675
 - C.1.3 Oscillations and trigonometric (or harmonic) functions 676
 - C.1.3.1 Hyperbolic functions 678
 - C.1.4 Complex numbers and trigonometric functions 678
 - C.1.4.1 Quadratic equation 679
- C.2 Calculus concepts 679
 - C.2.1 Full and partial derivatives 679
 - C.2.2 Taking derivatives 680
 - C.2.3 Series approximations 682
 - C.2.4 Integrals 683
 - C.2.5 Singularities 685
- C.3 Linear algebra 685
 - C.3.1 Vectors 685
 - C.3.1.1 Norms 686
 - C.3.2 Matrices 687
 - C.3.2.1 Operations on square matrices 687
 - C.3.3 Adding and multiplying vectors and matrices 688
 - C.3.3.1 Addition 688
 - C.3.3.2 Dot vector product, δ_{ij} , and ε_{ijk} 688

	C.3.3.3	Cross vector product	689
	C.3.3.4	Dyadic vector product	690
	C.3.3.5	Multiplication of a matrix with a scalar	690
	C.3.3.6	Multiplication of a matrix with a vector	690
	C.3.3.7	Multiplication of two matrices	691
	C.3.3.8	Inner product of two tensors	691
C.3.4		Matrix flavors and additional operations	691
	C.3.4.1	Identity matrix	691
	C.3.4.2	Matrix inverse	691
	C.3.4.3	Orthogonal or rotation matrices	692
	C.3.4.4	Euler angles and rotation matrices	692
	C.3.4.5	Matrix decomposition	693
	C.3.4.6	Eigenvalues and eigenvectors	693
	C.3.4.7	Linear inverse problems	694
C.4		Coordinate systems and spherical trigonometry	694
	C.4.1	Cartesian and spherical coordinate systems	694
	C.4.2	Useful formulas for spherical trigonometry	696
C.5		Vector calculus: Divergence, curl, and tensors	698
	C.5.1	The Gauss and Stokes theorems	701
	C.5.2	Tensors	701
	C.5.2.1	The wedge operator	702
C.6		Spectral analysis	702
	C.6.1	Fourier series	702
	C.6.2	Spherical harmonics	703
	C.6.2.1	Helmholtz decomposition of vector fields	706
C.7		Simple statistics and curve fitting	707
	C.7.1	Means, variance, and standard deviation	707
	C.7.1.1	Averaging of directional and orientational data	708
	C.7.2	Distributions and moments	709
	C.7.3	Linear regression	710
	C.7.4	Correlation	711
	C.7.5	Estimates of uncertainties	711

Bibliography 713

Acronyms 833

Index 835



An Exploration of Basic Solid Earth Structure and Dynamics

The Earth sciences have always been interdisciplinary and are founded on interpreting the rich geological record of our planet. Much of the dramatic progress over the last century has been driven by discoveries based on observations in solid Earth geophysics, from gravity, to magnetic field measurements, to global seismology, and more recently to space geodetic techniques. However, the biggest advances usually arise when diverse observations from different disciplines are combined in mechanically consistent ways to establish a comprehensive theory for why the Earth works this way.

The establishment of the integrative model of plate tectonics is such an occasion and *the* big story in the solid Earth geosciences. As conceived originally, plate tectonics is a kinematic description for how the surface of the Earth deforms and moves horizontally, without specifying anything about the driving forces (*McKenzie & Parker, 1967; Morgan, 1968; Le Pichon, 1968*). However, those forces were quickly reconnected to the fact that plate tectonics is the surface expression of mantle convection, as had been suggested roughly 40 years earlier (*Holmes, 1931*).

Because of sustained radiogenic heat production within the mantle and the sluggish nature of mantle convection, our planet is still cooling and its surface continues to be shaped and reworked by its internal dynamics. Mantle convection controls the evolution of the solid systems of our planet, encompassing nearly all tectonic and magmatic activity, including the earthquakes and volcanic eruptions that affect society in sometimes catastrophic ways. Moreover, mantle convection and plate tectonics also interact with the exosphere, comprising the ocean and atmospheric layers, and with that the climate and the biosphere we are part of. However, there are still many open questions as to exactly how those interactions occur, and there are a number of important observations, such as those relating to tectonic transients, continental dynamics, and plate boundary evolution, that have so far resisted integration into a proper, comprehensive dynamic theory.

We proceed to review some of the fundamental geophysical and geological observations for Earth, with a focus on global scales, as if we were exploring an alien planet remotely. We then discuss the Earth's thermochemical structure, often explored jointly with seismological constraints, and then turn to an overview of a dynamic description of tectonic activities, which we follow up on throughout the remainder of the book.

1.1 Topography

When viewed from space, the outer terrestrial planets are strikingly different. The famous, Earth-as-a-blue-marble photo taken during the Apollo 17 mission emphasized our special, perhaps precarious, state with water oceans covering much of the planet, and a temperate atmosphere with clouds and prevailing wind currents generated by the interaction of solar heat influx, rotation, and the hydrological cycle, along with the atmospheric signatures of life (*Sagan et al., 1993*). There

is evidence for water having been available in fluid form on Earth's surface over billions of years (Mojzsis *et al.*, 2001), yet why and how this stable condition was maintained is not entirely clear.

The early Sun was much fainter than at present day, such that our oceans would have likely been permanently frozen in the Archean (Feulner, 2012). While there were periods of near-global glaciation in Earth's history (Hoffman *et al.*, 2017), those episodes were temporarily restricted, and for the most part, a self-regulatory climate feedback mechanism appears to have been at work, thanks in part to plate tectonics (Walker *et al.*, 1981): If the atmosphere gets too hot, rock is more readily eroded by surface processes. Weathering transports carbon into the oceans, where it forms sediments that are subducted back into the mantle. This pathway draws down atmospheric CO₂, and so reduces the atmospheric greenhouse effect. This reduces temperatures, and with it the strength of weathering, and as a consequence, less carbon gets transported into the mantle. Due to the sustained convective and tectonic outflux of CO₂ from the interior of the Earth, e.g., from volcanism, carbon dioxide concentrations increase in the atmosphere, leading to warming.

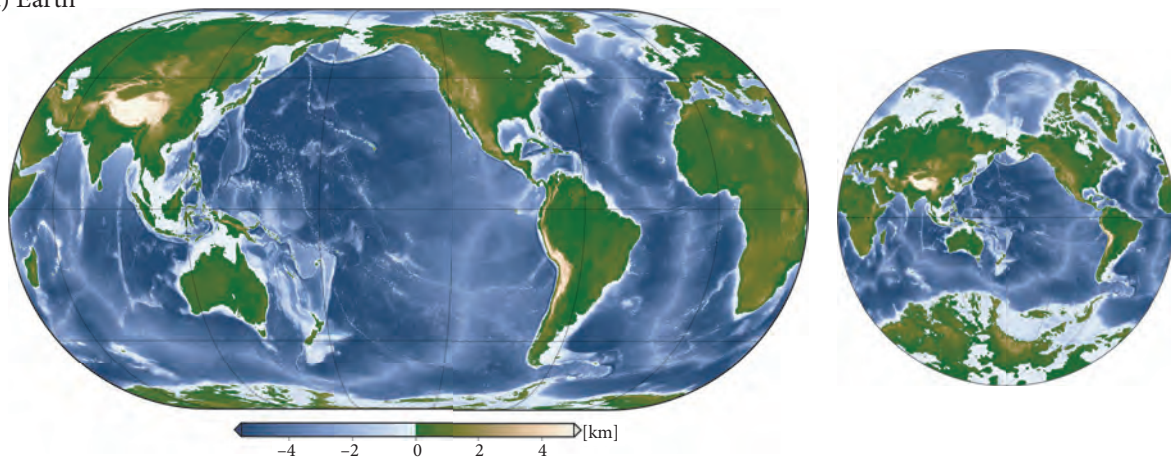
Such long-term climate regulation, and the existence of oxygen or any other atmospheres, are not guaranteed for terrestrial planets, as our sister planet Venus shows. The atmosphere of Venus is ~340 K hotter than Earth's, thanks to runaway greenhouse effects due a high CO₂ and sulfur-rich atmosphere that also leads to surface pressures ≈ 92 times those of Earth. There are no oceans or other surface water on Venus, and plate tectonics is absent, perhaps because of rheological effects due to the absence of water, the high surface temperatures, or both. Heat from Venus's mantle is transported mainly through a more or less stagnant surface that exhibits volcanism and appears to show episodic reworking. Mars has a more temperate atmosphere but is likewise frozen tectonically, indicating that Earth's style of tectonics is unique in our solar system (§14.4). Surface conditions in terms of temperature, availability of water, and the carbon cycle are thus linked with the internal evolution of a planet, and somehow interact to lead to habitable conditions, at least for Earth.

If a spacecraft remotely exploring planets in our solar system were to use a radar instrument, an explorer could then peek through the atmosphere to reveal the planets' surface elevation with respect to some reference. Figure 1.1 shows topography for the three Earthlike planets using the same type of colorscale, centered on the reference (sea level for Earth). Thanks to space exploration, we have learned much about the surface geology of Mars, which shows the remnants of fluid water and wind-driven transport processes, but is at present characterized by a hemispheric dichotomy with southern highlands and northern lowlands. This feature is related to the internal dynamics of Mars's mantle, and there are remnants of extensive volcanism, including Olympus Mons, the highest topographic feature of all terrestrial planets. We know that Mars's surface is relatively static because many large-impact craters are preserved, and the relative decrease in typical impact size over solar system evolution can be used for dating.

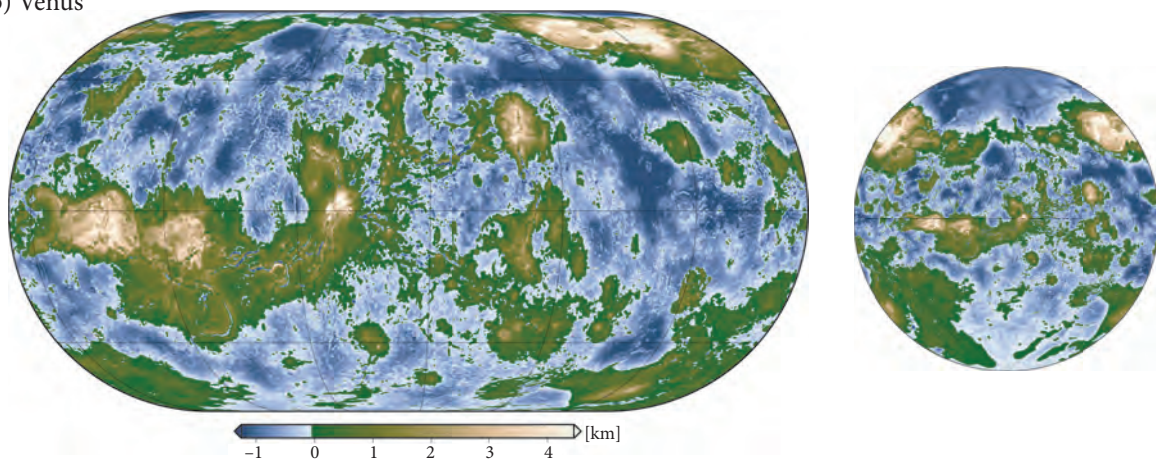
Venus's surface is comparatively younger and shows features of large-scale reorganizations, in particular near-circular features, *coronas*, as well as indications of active volcanism. Earth shares the general planetary age of ~4.6 Gyr with the terrestrial planets that formed from silicic- and iron-type materials in similar fashion during the formative eon of the solar system. The last planet-shattering event for Earth was the impact of a Mars-sized object that led to the formation of Earth's moon (§8.1.2.1). However, compared to Mars, Earth's surface is much younger. While there are rocks of ~3.8 Gyr age preserved in Earth's continents, there are few signs of impact craters, indicating tectonic reworking and active surface processes. In particular, Earth's oceans are underlain by seafloor which, for the most part, is at present nowhere older than ~200 Ma.

One way to analyze topography is by computation of fractional or cumulative topography histograms, *hypsometric curves* (Fig. 1.2). Such analysis brings out the aforementioned global features; for example, Venus has a slightly skewed, but overall nearly normally distributed topography. Earth shows clear peaks at the average continental and oceanic elevations of ~100 m and ~ -4500 m,

a) Earth



b) Venus



c) Mars

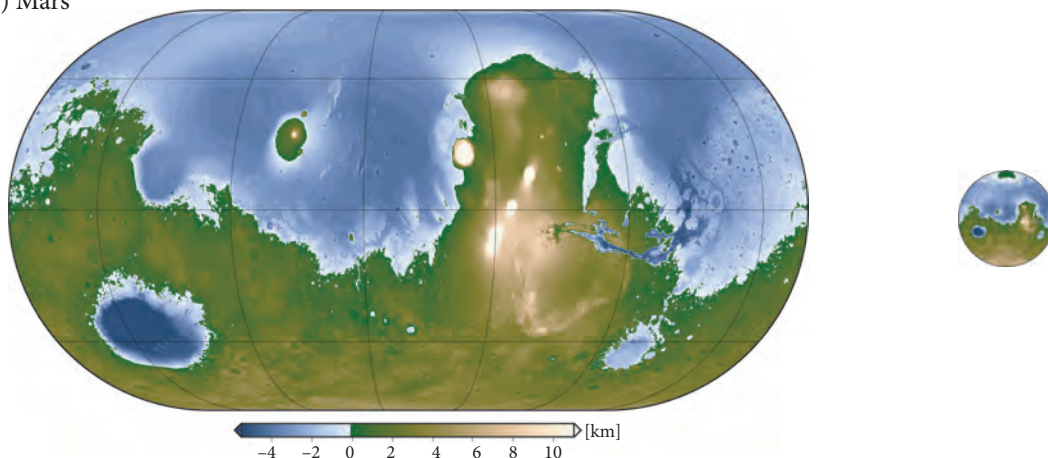


Figure 1.1: Topography of Earth (a; bedrock elevation, removing ice for Greenland and Antarctica; *Amante & Eakins, 2009*), Venus (b; Magellan, Pioneer Venus, and Venera missions), Mars (c; Mars Global Surveyor), where datasets for b) and c) were expanded from spherical harmonics (*Wieczorek, 2015*) (up to degree 720, when available). On left, Eckert-IV projection; on right, van der Grinten projection, scaled with the square of the object radius. See Fig. A.5 for power spectra and geoid anomalies.

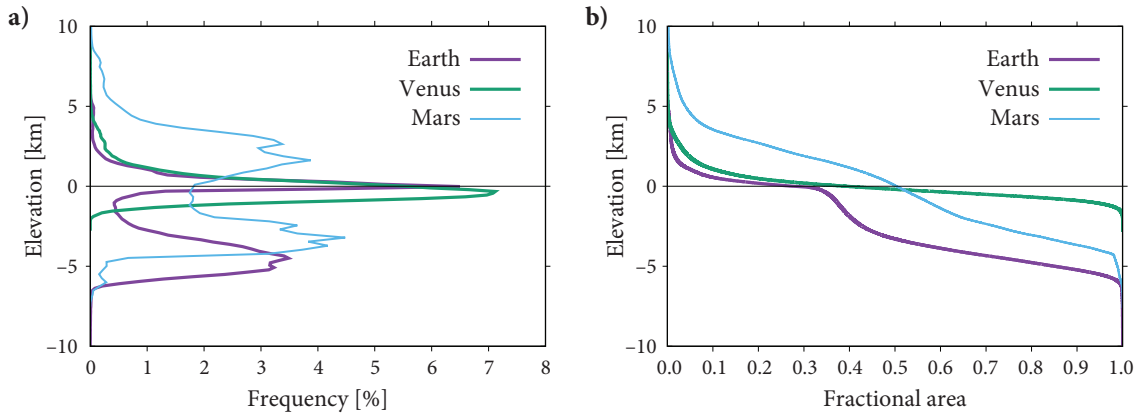


Figure 1.2: a) Fractional histogram of elevation for planetary topography from Fig. 1.1. b) Cumulative distribution function of elevation in area (i.e., hypsothetic curve) (y axes clipped) for objects as in a).

respectively. This dichotomy is profound, and is the result of thermochemical mantle convection. Oceanic plates are continuously generated and recycled with a lifetime of ~ 100 Ma. Continents are created mainly through fractionation at subduction zone arc volcanism, and subduction-driven accumulation of protocontinental fragments, e.g., created by plumes.

1.1.1 Isostasy

The bulk difference in oceanic and continental elevation of Fig. 1.2 can be explained by the floating equilibrium of different crustal columns. *Airy* (1855) first proposed that

Earth's crust lying upon the lava can be compared with perfect correctness to the state of a raft timber floating upon water.

This idea was based on 1730s survey results by Pierre Bouguer from the Andes, and in the Nineteenth century in India, including by George Everest close to the south Himalayas. Both studies showed that the plumb line deflection at the base of the mountain was less than expected from the extra mass above sea level.

One way to explain this is there was a negative mass anomaly at the base of the mountain due to the “substitution of a certain volume of light crust for heavy lava” (*Airy*, 1855), as for a floating iceberg. This is one way to state the principle of *isostasy*, which posits that a crustal density column should be in an (Archimedes) floating equilibrium over long time and length scales (Fig. 1.3). If this isostatic equilibrium holds, the *pressure* due to the overburden of a crustal column, p , has to add up to the same everywhere at some *compensation depth*, Z , i.e., $p(Z) = \text{const.}$ everywhere. Pressure arises from the forces pushing inward on an object from all directions, normalized by the surface area of the object.

How can we link this assumption about force balance to topographic or structural observables? When considering forces, it is often useful to start with Newton. His second law says that force, expressed as a *vector* (§C.3) pointing in some direction, F (units N, Newton), is mass m (units kg) times *acceleration*, g ,

$$F = mg, \quad (1.1)$$

where acceleration, g , is a temporal change in velocity (units of m/s^2). Eq. (1.1) means that it takes a force to change the rate of motion of an object due to inertia.

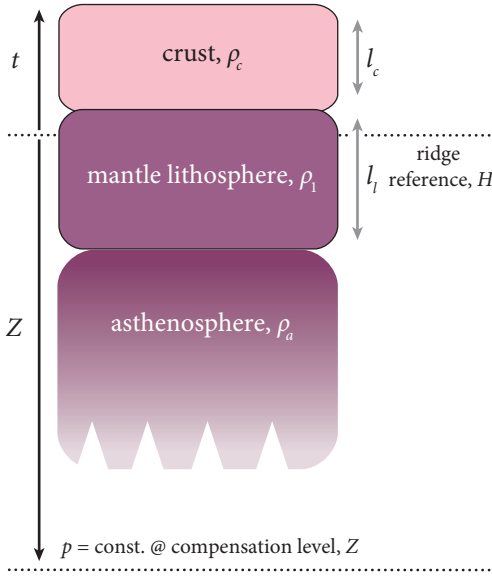


Figure 1.3: Isostatic balance column for load components from a crustal layer with thickness l_c and density ρ_c , and a mantle lithosphere with thickness l_l and density ρ_l (total lithospheric thickness $L = l_c + l_l$), floating over an asthenosphere of density ρ_a . The pressure p at the compensation depth Z is constant for a floating equilibrium, where the overburden pressure p_L , eq. (1.3), is computed for a column of height $h = Z + t$. Isostatic topography, t , is relative to a hypothetical spreading-center reference height H , where $l_c = l_l = 0$.

If g is the *gravitational acceleration*, i.e., the pull due to gravity, e.g., of our planet, then eq. (1.1) allows us to infer the weight of an object, and g points more or less vertically down toward the center of the Earth. If that m object is our mantle column considered for the floating equilibrium, with area A and volume V on top of the compensation depth is $m = \rho V = \rho h A$, with *density*

$$\rho = \frac{m}{V}, \quad (1.2)$$

and topography z and $h = t + Z$ (Fig. 1.3). The weight of that crustal column is thus $F = mg = \rho g h A$, with the amplitude, or length/norm, of g , $g = |g|$ (eq. C.22). The pressure (force per area, units of N/m² or Pa) is therefore (ignoring atmospheric pressure)

$$p_L = \rho g h = \int dz \rho(z) g(z), \quad (1.3)$$

where the integral over depth, $\int dz$, means sum up the area under a function (§C.2.4), and replaces the product if ρ and/or $g(z)$ are functions of depth. This lithostatic, or overburden, pressure is identical to the hydrostatic pressure in a fluid at rest, and the depth derivative of eq. (1.3),

$$\frac{dp}{dz} = -\rho g, \quad (1.4)$$

is called the *hydrostatic equilibrium equation*.

Let us assume that the crust is part of the thicker *lithosphere* (Greek *lithos* (λίθος) = stone, here: strong), which is a mechanical term that encompasses the relatively slowly deforming, cold surface regions of the Earth. It sits on top of the *asthenosphere* (Greek *asthenos* (ἀσθενής) = without strength, weak) which can flow so that things equilibrate. The timescales for this equilibration after load removal, e.g., due to erosion or deglaciation at the surface, or removal of dense layers by tectonic processes, depends on the creep behavior of the underlying mantle and the wavelength of perturbations (§6.7.1).

Assuming that density is constant with each layer (following Airy), this yields a balance for a column floating at topography z_{iso} under a cover of air (Fig. 1.3, and see exercise 2). Comparing the lithospheric and reference column in Fig. 1.3, we can write the pressure at compensation depth Z and asthenospheric density ρ_a as

$$\begin{aligned} g(\rho_c l_c + \rho_l l_l + (Z + t - l_c - l_l)\rho_a) &= gZ\rho_a \\ (\rho_c - \rho_a)l_c + (\rho_l - \rho_a)l_l + t\rho_a + Z\rho_a &= Z\rho_a \\ t &= \frac{\rho_a - \rho_c}{\rho_a}l_c + \frac{\rho_a - \rho_l}{\rho_a}l_l. \end{aligned} \quad (1.5)$$

Any asthenospheric layer below l_l thus cancels out irrespective of the actual choice for Z . However, this is only the case under the assumption that there are no asthenospheric density variations. Those complicate analysis of the nonisostatic residuals and considerations of the origin of topography (§9.2.1).

Allowing for water coverage, we can write the isostatic elevation of the continental lithosphere relative, e.g., to a hypothetical spreading center reference level, H , with no oceanic crust, $l_c = 0$, and zero plate thickness, $l_l = 0$, by definition, as

$$\begin{aligned} z_{\text{iso}}^{\text{land}} &= f_1 l_c + f_2 l_l - f_3 H \quad \text{if } z \geq 0 \text{ and} \\ z_{\text{iso}}^{\text{water}} &= \frac{z_{\text{iso}}^{\text{land}}}{f_3} \quad \text{if } z < 0, \end{aligned} \quad (1.6)$$

with

$$f_1 = \left(1 - \frac{\rho_c}{\rho_a}\right), \quad f_2 = \left(1 - \frac{\rho_l}{\rho_a}\right), \quad \text{and} \quad f_3 = \left(1 - \frac{\rho_w}{\rho_a}\right) \approx \frac{1}{1.44},$$

where ρ_w is the density of water, with $H \approx 2.6 \dots 2.7$ km (§7.3.2, §10.3.5). With $l_l = 0$ or no lithospheric variations, eq. (1.6) shows that high topography at constant crustal density has to be balanced by thicker crust, such that the depth to the base of the crust is variable (Airy, 1855). The factors relating crustal and lithospheric layer thickness and topography are of order $f_1 \sim 0.1 \dots 0.2$ and $f_2 \sim -0.02 \dots -0.01$, respectively (Fig. 1.4). This means that variations in crustal thickness have ~ 10 times the effect of variations in lithospheric thickness, albeit with larger uncertainties for l_l (§9.2.1).

Alternatively, the depth from sea level to the base of the crust or lithosphere, P , can be chosen to be the same as the compensation depth, $P = Z$. In this case, floating equilibrium for variable surface topography can also be achieved if the crustal density, ρ_P , is variable, e.g., reduced underneath high topography. This sort of compensation is attributed to Pratt, who in Pratt (1855) attempted to explain the Himalayan plumb line measurements with variable crustal density, akin to this line of reasoning. If compensation were to happen at a constant crustal bottom depth Z , the same kind of balance shows that the required density is

$$\rho_P = \rho_0 \frac{Z}{Z + t}, \quad (1.7)$$

where ρ_0 is the reference density at the spreading center. If a lithospheric column is perfectly in Airy isostasy or Pratt isostasy, it is called *fully compensated* (§3.2.2.2).

Granitic continental crust is lighter, i.e., less dense, than the basalt (extrusive) and gabbroic (intrusive) oceanic crust, which is also underlain by a relatively denser, colder convective boundary layer, and thus mainly thermally controlled, lithosphere. These bulk differences explain much of

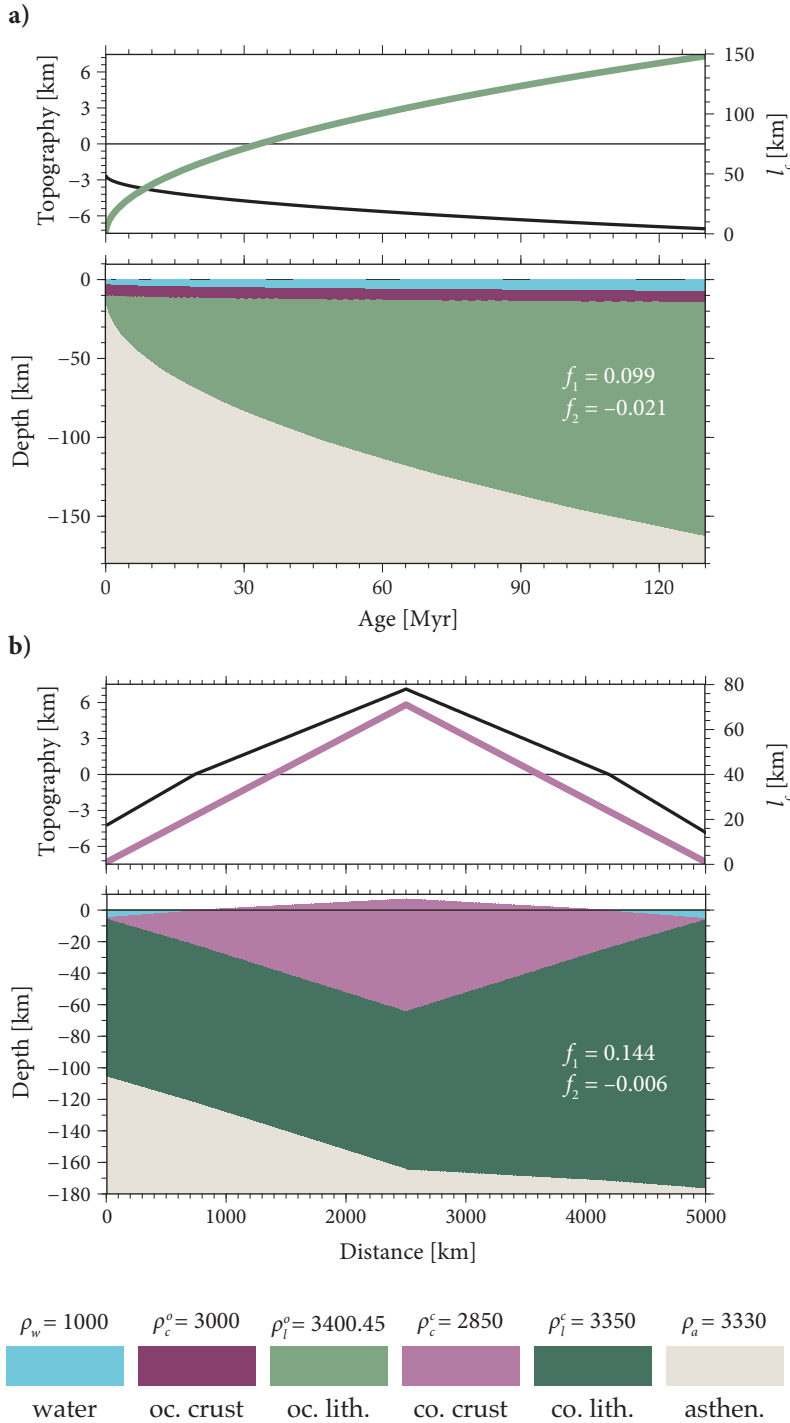


Figure 1.4: Nominal oceanic (a) and continental (b) lithospheric columns in isostatic balance, eq. (1.6), using the density values as provided in the color legend (units of kg/m^3). Top subplots show topography (black) and lithospheric, l_l^o , and crustal, l_c^c , thickness, respectively. The oceanic plate is represented by constant crustal thickness, $l_c^o = 7$ km, and a half-space cooling (§7.3.2) dependence of lithospheric thickness, l_l^o , following eq. (7.45), with constant lithospheric density, ρ_l^o , as per eq. (7.51), but can also be viewed as having constant l_l^o and variable ρ_l^o in a Pratt sense. In order to have the zero age, $l_l^o = 0$, plate rest at realistic $H \sim 2.6$ km water depth for nonzero l_c^o at the ridge, eq. (1.5) was corrected by adding $\Delta H l_c^o (\rho_c^o - \rho_a) / (\rho_w - \rho_a)$ to $\hat{H} = H + \Delta H \approx 3.6$ km. Continental crustal thickness is for a nominal orogen where the left and right half have constant l_l^c and constant $l_c^c + l_l^c$, respectively. Inset f_1 and f_2 values are the thickness multipliers of eq. (1.6).

the topography of Earth (Fig. 1.2) as a mix of mostly Airy isostasy for the continents and Airy and Pratt isostasy for the oceanic lithosphere (Fig. 1.4). There, we can apply the half-space cooling model to explain much of the deepening of the seafloor away from midoceanic spreading centers, one of the major achievements of geodynamics (§7.3.2).

Besides plate-scale features in the oceans and the continent-ocean difference, curious alien observers of our planet's topography might further identify linear chains of islands on the seafloor (Fig. 1.1a), and might make the connection to hotspot volcanism and moving plates (§10.1). They would also see that the suboceanic mountains show midoceanic ridges in the Atlantic but more subdued slopes within the Pacific. They might suggest different types of oceanic crust production, which we know to be due to slow and fast spreading, respectively (§10.3). Along the spreading centers, there are characteristic offsets by transform faults, another hallmark of Earth's style of plate tectonics and indicative of the generation of strike-slip-type motion by some means of strain-localizing rheologies (§8.5). Continents reflect a much longer geological record of more complex processes, with the high plateau of Tibet and the Andes being prominent topographic features, and other orogens, such as the North American Cordillera being more subdued, with evidence for water and wind-driven surface transport processes and erosion.

Even from such global-scale comparative, qualitative assessments of the outer terrestrial planets, it is thus clear that not only the surface morphology of planets but also their mass and energy transport processes, including climate and perhaps life, depend on interior dynamics through mantle convection. The latter is expressed, within our solar system only for Earth, as plate tectonics at the surface. However, the internal feedbacks are not limited to those expressed by the mantle at the surface. At its base, mantle convection controls core cooling, and hence magnetic field generation through the convective geodynamo operating in the liquid outer core (§1.4.1). The heat flow imposed on the core by the mantle may matter directly for the types and stability of the magnetic field, and indirectly through regulating inner core freezing. The field in turn may play a role for sustaining an atmosphere by shielding planets from solar radiation, another example of systems-level feedbacks. Our sister planet Venus is lacking a magnetic field at present, which is another indication of different planetary evolutionary trajectories (§9.4.1).

1.2 Geopotentials: Shape, spin, and geoid

If our aliens are able to put satellites into orbit around Earth and track their orbits precisely, as we have done for Earth, Mars, and Venus (Fig. A.5), they would be able to determine models of the spatial variation of the gravitational attraction one experiences in different places. Those variations arise due to internal density anomalies (Fig. 1.5) and lead to subtle fluctuations in the distance between the satellite and the planet, as exploited by terrestrial spacecraft missions of the GRACE series by having two satellites follow each other through the ups and downs of their orbits.

1.2.1 Gravitational potential

Newton's first law states that the gravitational force, F , exerted by a mass M , e.g., a planet, on a mass m , e.g., a satellite, is given by

$$\mathbf{F} = -G \frac{mM}{r^2} \hat{\mathbf{e}}_r \quad \text{or} \quad F = |\mathbf{F}| = G \frac{mM}{r^2}, \quad (1.8)$$

where r is the distance between the objects (their centers of mass, assumed as points), $\hat{\mathbf{e}}_r$ is a unity vector, i.e., $|\hat{\mathbf{e}}_r| = 1$, pointing in from the M object to the m object's center of mass, and G is the *universal gravitational constant* (one of nature's less well constrained fundamental constants;

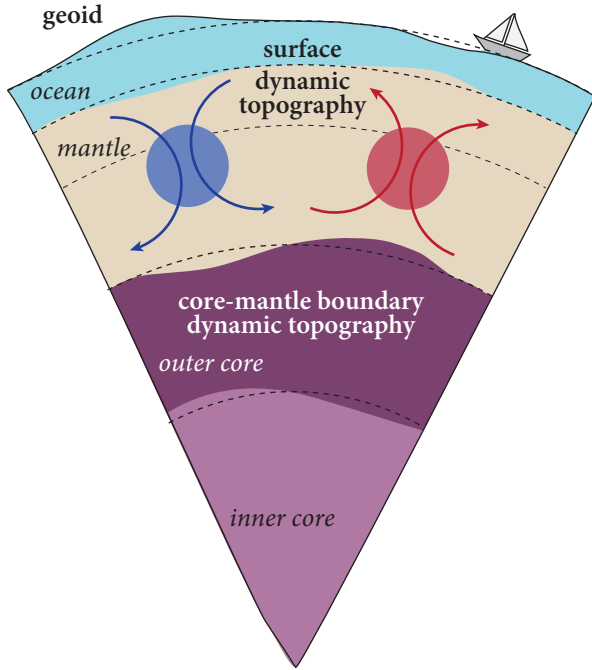


Figure 1.5: Drawing illustrating the definition of the geopotential surface of the actual geoid and its deflection from a reference geoid of ellipsoidal form shown with dashed lines (Fig. A.3). Internal density anomalies, such as due to a dense and light anomaly (blue and red, respectively), have a static and dynamic response on the geoid. Static, since the excess mass of a positive geoid anomaly means one has to move further away to experience the same gravitational pull, eq. (1.35). If the anomaly introduces flow, the resulting stresses also deflect the surface and core-mantle boundary, and those undulations lead to additional, dynamic effects on the geoid on timescales of $\sim$ Myr (§9.4). Here, we assume a higher viscosity lower mantle which leads to reduction of the surface relative to the bottom topographic deflection, and an effective positive geoid anomaly for a mid-mantle slab sinker (§9.4). Another time-dependent effect on the geoid is due to glacial isostatic adjustment, on timescales of melting, $\sim$ 26,000 yr for the last glacial maximum (§6.7).

Table A.1). If we consider the total work, W , involved in moving an object m from far away, at infinite distance, toward another with mass M at R , say, the surface of the Earth, we need to sum up, i.e., again integrate, the spatially dependent force that is required to do that. Using eq. (1.8) and integrating against that force (note sign),

$$W = \int_{\infty}^R F(r) dr = GMm \int_{\infty}^R r^{-2} dr = GMm [r^{-1}]_{\infty}^R = -GMm \left[\frac{1}{R} - \frac{1}{\infty} \right] = -\frac{GMm}{R}, \quad (1.9)$$

which defines the *gravitational energy* of the two masses, $E_p = W$. The energy is negative because work has to be done to separate them to infinite distances such that energy is conserved, and energy, or work, has units of force $\times$ length, i.e., Nm, or J (joules).

Since the gravitational force is also given by Newton's second law, eq. (1.1),

$$\mathbf{g} = -\frac{GM}{r^2} \hat{\mathbf{e}}_r, \quad \text{or} \quad |\mathbf{g}| = \frac{GM}{r^2} \quad (1.10)$$

holds, which defines the gravitational acceleration, $\mathbf{g}$, for the satellite, or for any observer at the surface of a homogeneous sphere with radius R such that $g = \frac{GM}{R^2}$, and the negative sign indicates that the acceleration is downward from the normal on the surface of the M object. Eq. (1.10) means that the gravitational attraction of the planet is fully described locally by $\mathbf{g}$; it is only the actual force (eqs. 1.1 and 1.8) that depends on the test mass m , e.g., of the satellite, that we are considering.

After things have settled down following planetary formation, any planet's mass distribution will mainly be layered, since denser material sinks to the bottom in the gravitational field, e.g., to form a predominantly iron core and silicic-type mantle. However, things get more complicated because there might be lateral variations of density, e.g., due to internal convective dynamics (Fig. 1.5). To capture these complexities, along with the possibility of having a rotating planet, we can express the gravitational force through a *potential*, U , which at a distance r from the center of the planet is defined as

$$U(r) = U_g + U_{\text{rot}} = -G \int_V \frac{\rho(\mathbf{r}')}{|\mathbf{r}|} dV' - \frac{1}{2} \Omega^2 r_0^2 \cos^2 \lambda \quad (1.11)$$

(see Expanded details 1), where $\int_V$ indicates integration over mass density at $\mathbf{r}$ from the mass center of the planet. In contrast to the vector $\mathbf{g}$, U is a scalar; the value of U depends on location, but there is only a single number defining it at that location, which is part of the motivation for introducing U . U is hence a *scalar field*, like topography expressed on a topo map.

For a spherical planet,

$$U_g(r) = -\frac{GM}{r} \quad (1.12)$$

is the *point mass potential* equivalent to a mass M at the center, and recovers eq. (1.9) for a unit test mass $m = 1$. The second term on the right-hand side (RHS) of eq. (1.11) is the rotational component of the potential, U_{rot} , and accounts for the centrifugal force due to a spin. The angular velocity of the Earth at the present day is $\Omega_E = 7.292, 211, 5 \cdot 10^{-5}$ rad/s as of the *World Geodetic System* (WGS-84; NIMA, 1984).

U_{rot} increases from zero at the pole of the rotation axes (latitude $\lambda = 90^\circ$) to maximum at the equator ($\lambda = 0$). Also in contrast to eq. (1.1), eq. (1.12) has a $\frac{1}{r}$ dependence, and bringing things back from U to g shows why. It takes work to move an object from $U(r_0)$ to $U(r_0 + dr)$ radially within the potential; that change is $dU = -g_r dr$, and we can recover g from the radial derivative of U (§C.2.2):

$$g_r = -\frac{\partial U}{\partial r} \quad \text{or, more generally,} \quad \mathbf{g} = -\left(\begin{array}{c} \frac{\partial U}{\partial x} \\ \frac{\partial U}{\partial y} \\ \frac{\partial U}{\partial z} \end{array} \right) = -\nabla U, \quad (1.13)$$

i.e., the gravitational attraction is a vector that aligns with the gradient of the potential field, U . The $\mathbf{g}$ vector points away from the steepest gradient, measured by the *gradient operator*, ∇ (which is a vector; §C.5). For a nonrotating, homogeneous sphere, eq. (1.12) holds, which also follows from integration of eq. (1.10) or the stationary part of eq. (1.11). One can show that eq. (1.13) is a sufficient condition for the gravitational force, eq. (1.1), to be conservative.

EXPANDED DETAILS 1: Moments of inertia and geopotentials of an ellipsoidal Earth

Consider the *moment of inertia* for rotation for a location at $\mathbf{r}$ given a spin vector $\boldsymbol{\omega}$,

$$I(\mathbf{r}) = \int (r' \sin \theta)^2 dM^{\text{point mass}} = mr^2$$

(units: kgm^2), where $r' \sin \theta$ is the distance between $\mathbf{r}$ and $\boldsymbol{\omega}$, and dM indicates integration over all mass elements, hence weighted by distance squared. The *angular momentum* is $\mathbf{L} = \mathbf{I}\boldsymbol{\omega}$ with a moment of inertia tensor, $\mathbf{I}$,

$$I_{xx} = \int (y^2 + z^2) dM, \quad I_{yy} = \int (x^2 + z^2) dM, \quad I_{xy} = - \int xy dM, \quad \text{and similar,}$$

and for the point mass

$$L = mr^2\omega, \quad (1.14)$$

which implies that since L is conserved for a closed system, r has to vary with any changes in ω , and vice versa, as for ice skaters extending their arms, for example.

In a reference frame with $\{\hat{x}, \hat{y}, \hat{z}\}$, I , the moment of inertia depends on direction cosines i_i , with, e.g., $i_3 = \cos \theta$, where θ is the angle between $I(\mathbf{r})$ and the $\hat{z}$ axis, the colatitude, and $I = i_1^2 A + i_2^2 B + i_3^2 C$ with $i_1^2 + i_2^2 + i_3^2 = 1$ and

$$A = \int (\hat{y}^2 + \hat{z}^2) dM, \quad B = \int (\hat{x}^2 + \hat{z}^2) dM, \quad \text{and} \quad C = \int (\hat{x}^2 + \hat{y}^2) dM. \quad (1.15)$$

If two moments of inertia are equal, $A = B$, then $I = A + i_1^2(C - A)$.

Let us move our mass integration for the potential into the center of mass, and further align the $\{\hat{x}, \hat{y}, \hat{z}\}$ so that they are the eigenvectors of I ; then I has no off-diagonal elements (§2.4.4). This makes A , B , and C the *principal moments of inertia*. In this moment of inertia frame, the general U_g term of eq. (1.11) is given by *MacCullagh's formula*,

$$U_g(r) = -\frac{GM}{r} - \frac{G}{2r^3} (A + B + C - 3I(\mathbf{r})). \quad (1.16)$$

For a spherically symmetrical body, $A = B = C = I$ holds, meaning that eq. (1.16) reduces to eq. (1.12).

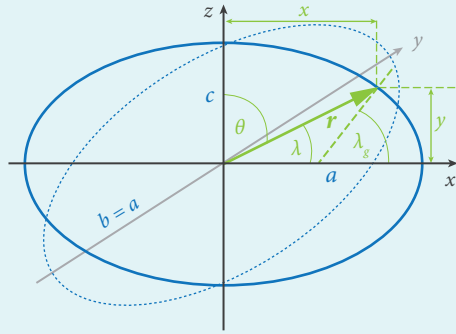


Figure 1.6: Coordinates and geometry for an oblate ellipsoid, or spheroid, with axes $a = b > c$. A location on the surface in geocentric coordinates $\mathbf{r} = x, y, z$ is described by longitude and *geocentric latitude* λ and colatitude $\theta = 90^\circ - \lambda$, or *geographic latitude* λ_g , defined with respect to the normal to the surface.

Now consider an Earthlike object, which is as an approximation an oblate, squashed *spheroid*. This is a special case of an even, long-axes, $a = b$ *ellipsoid*, with long axis a along the equator (distance from the center), and shorter axis c to the pole aligned with the rotational axis (Fig. 1.6). If we then define C to be the *polar moment* and A and B , to be *equatorial moments*, such that $\{\hat{x}, \hat{y}, \hat{z}\}$ are aligned with the a , b , and c axes of the ellipsoid, then

$$I(\mathbf{r}) = A + (C - A) \cos^2 \theta.$$

One can then show that

$$U_g(r) = -\frac{GM}{r} + \frac{G}{r^3}(C - A) \left(\frac{3}{2} \cos^2 \theta - \frac{1}{2} \right) = -\frac{GM}{r} + \frac{GMa^2}{r^3} J_2 \left(\frac{3}{2} \cos^2 \theta - \frac{1}{2} \right), \quad (1.17)$$

with a as the equatorial radius and J_2 as the *dynamic flattening parameter*,

$$J_2 = \frac{C - A}{Ma^2} \approx 1.081 \cdot 10^{-3}, \quad (1.18)$$

where J_2 can be determined in different ways. The relevance of this equation is that our aliens (or ourselves for other planets) can determine J_2 from remote observations of the gravitational potential, and eq. (1.18) links this to properties that depend on the internal density distribution of the object, e.g., the fractionation into a core and mantle once the geometrical property a is determined.

The potential on the reference ellipsoid best matching the spheroid form of our planet with axes a and c , U_{ref} , which we might use for navigation, defines the *reference geoid*, where a geoid is a surface of constant gravitational potential U . Thus U_{ref} is the closest approximation of the geoid (Fig. 1.5) that can be fit by a spheroid. Even though the ellipticity inherent in U_g from eq. (1.17) results from a planet's spin, that potential does not include the effects of rotation. That motion around an axis, $\hat{e}_z$, contributes

$$U_{\text{rot}}(\mathbf{r}) = -\frac{1}{2} \Omega^2 r^2 \sin^2 \theta = -\frac{1}{2} |\boldsymbol{\omega} \times \mathbf{r}|^2, \quad (1.19)$$

with $\boldsymbol{\omega} = \Omega \hat{e}_z$, where $\boldsymbol{\omega} \times \mathbf{r}$ denotes the cross product (§C.3.3.3). Eq. (1.19) follows from an integration like eq. (1.9) with unity mass, but instead of using the gravitational acceleration, $\mathbf{g}$, we use the *centrifugal acceleration*, $\mathbf{p} = \boldsymbol{\omega} \times (\boldsymbol{\omega} \times \mathbf{r})$.

The total potential outside a rotating, Earthlike reference spheroid is then

$$U(r) = -\frac{GM}{r} + \frac{GMa^2}{r^3} J_2 \left(\frac{3}{2} \cos^2 \theta - \frac{1}{2} \right) - \frac{1}{2} \Omega^2 r^2 \sin^2 \theta. \quad (1.20)$$

From eq. (1.20), at the poles where $r = c$ and at the equator where $r = a$,

$$U_{\text{ref}}^{\text{pole}} = -\frac{GM}{c} - \frac{GMa^2}{c^3} J_2 = U_{\text{ref}}^{\text{equator}} = -\frac{GM}{a} - \frac{GM}{2a} J_2 - \frac{1}{2} a^2 \Omega^2. \quad (1.21)$$

The *geometric flattening*, or ellipticity, of the reference geoid can then be derived as

$$f = \frac{a - c}{a} = J_2 \left(\frac{c}{2a} + \frac{a^2}{c^2} \right) + \frac{1}{2} \frac{a^2 c \Omega^2}{GM} \approx \frac{1}{2} (3J_2 + m), \quad f_{\text{WGS-84}} = \frac{1}{298,257,223,563}, \quad (1.22)$$

with what turns out to be the ratio between centrifugal to total gravity at the equator:

$$m = \frac{a^3 \Omega^2}{GM} = 3.461,391,899 \cdot 10^{-3},$$

with $a = 6,378,137.0$ m and $GM = 3.986,004,418 \cdot 10^{14}$ m³/s² of the World Geodetic System (WGS-84; NIMA, 1984). Eq. (1.22) yields another approximation for $J_2 = \frac{(2f-m)}{3} = 1.08141 \cdot$

10^{-3} . We get $U_{\text{ref}} = 62,636,860.8497 \text{ m}^2/\text{s}^2$, and then $c = 6356.7523 \text{ km}$; thus the difference between equatorial and polar radial axes is $a - c \approx 21 \text{ km}$.

The geodetically observed flattening as in, e.g., WGS-84 is actually slightly larger than that expected theoretically for a rotating, hydrostatic Earth, with expected values of $f_H = \frac{1}{299.829}$ (Nakiboglu, 1982) and $f_H = \frac{1}{299.981}$ (Chambat *et al.*, 2010) for different theoretical approaches. The difference between f and f_H becomes significant when interpreting geopotential field anomalies for glacial isostatic adjustment (Expanded details 18) or as a constraint for mantle convection (§9.4). The observed *hydrostatic bulge*, $a - c$, is 113 m larger than the $a - c$ that is expected from a hydrostatic Earth (Chambat *et al.*, 2010). Different corrections are shown in Fig. A.3, but it is clear that J_2 is dynamically supported by mantle convection where it evolves over timescales of $\sim 50 \text{ Myr}$ (Ricard *et al.*, 1984, 1993; Steinberger & O'Connell, 1997). On even longer, $\sim \text{Gyr}$ timescales, the slowdown of Earth's rotation through tidal dissipation will decrease J_2 (§1.2.2), and deglaciation since the last glacial maximum can be constrained by secular, present-day $\dot{J}_2$ (Yuen & Sabadini, 1985; Mitrovica & Peltier, 1993). The deglaciation trend is of order $\dot{J}_2 \sim -3 \cdot 10^{-11} \text{ yr}^{-1}$ and needs to be accounted for to understand present-day contributions due to melt water influx and climatic cycles (Expanded details 18; Cazenave & Nerem, 2004; Cheng *et al.*, 2013).

For our purposes here, $\dot{J}_2$ is super tiny ($\sim 0.02 \text{ m}$ of reduced bulge per 100 yrs) to require adjusting WGS-84 over decadal scales. Using the relationship between $\mathbf{g}$ and U from eq. (1.13) and eq. (1.20) and only considering the radial component of $\mathbf{g}$,

$$g(r) \approx \frac{GM}{r^2} - 3 \frac{GMa^2}{r^4} J_2 \left(\frac{3}{2} \cos^2 \theta - \frac{1}{2} \right) - \Omega^2 r \sin^2 \theta.$$

The distance from the center of the Earth to the spheroidal reference geoid, or the approximation of r for an ellipse, is

$$r_{\text{ref}}(\theta) \approx a \left(1 + \frac{f(2-f)}{(1-f)^2} \cos^2 \theta \right) \approx a(1 - f \cos^2 \theta). \quad (1.23)$$

Averaging of eq. (1.23) leads to the mean radius of the Earth, R_E (Table A.1). We can also derive a *reference gravitational acceleration* on the reference geoid at r_{ref} as

$$\begin{aligned} g_{\text{ref}}(\theta) &\approx \frac{GM}{a^2} \left(1 + 2f \cos^2 \theta - 3J_2 \left(\frac{3}{2} \cos^2 \theta - \frac{1}{2} \right) - m \sin^2 \theta \right) \\ &\approx \frac{GM}{a^2} \left(1 + \frac{3}{2} J_2 \sin^2 \theta - m + 2m \cos^2 \theta \right), \end{aligned} \quad (1.24)$$

which can be written as a function of latitude, $\lambda = 90^\circ - \theta$:

$$g_\lambda = 9.780,327(1 + 0.005,302,4 \sin^2 \lambda - 0.000,005,9 \sin^2 2\lambda) \text{ m/s}^2. \quad (1.25)$$

We can define a *gravity flattening* factor, analogous to eq. (1.22), as

$$f^* = \frac{g_{\text{pole}} - g_{\text{equator}}}{g_{\text{equator}}} = \frac{-\frac{3}{2} J_2 + 2m}{1 + \frac{3}{2} J_2 - m}, \quad (1.26)$$

and eq. (1.24) can then be rewritten like eq. (1.23) as another ellipse:

$$g_{\text{ref}}(\theta) = g_{\text{equator}} (1 + f^* \cos^2 \theta).$$

Within these approximations, *Clairaut's theorem* (from 1743) describes this interesting relationship between geometrical and dynamic properties,

$$f^* = \frac{5m}{2} - f,$$

i.e., f can be inferred from gravity observations alone, for example. The links of J_2 with the satellite geoid are further explored in Expanded details 2.

1.2.2 Reference geoid, spin, and the Earth-Moon system

On global scales, we can then use the geoid to describe variations in gravitational pull. The *geoid* is a constant gravitational potential surface, defined to coincide with the average sea surface height in the absence of $\lesssim$ yr timescale oceanographic fluctuations, i.e., *sea surface equipotential*, or *sea surface height* (Fig. 1.5). In continents, the geoid would be the level of water in an imaginary set of interconnected canals, ignoring any extra pull from any surrounding mountains.

We call the best-fitting spheroidal approximation to the geoid the *reference geoid*, based on evaluating the gravitational potential on the *reference ellipsoid*, which describes the oblate approximation to the shape of the Earth (Expanded details 1). Since the ellipsoidal geometry dominates the actual geoid, one typically makes a correction before analysis of *geoid anomalies*. The best-fitting spheroidal shape that is observed for Earth is that of an ellipsoid where the equator is ≈ 21 km further from the center of the Earth than the poles. While this shape is mainly due to the spin, the degree of ellipticity is slightly larger than that expected for a spinning planet with a 1-D Earth structure, the purely *hydrostatic geoid* (§A.4). This means that there is a dynamic, mantle-convection-related contribution to the equatorial bulge itself (§9.4). It has long been recognized that the Moon's ellipticity is likewise too large (*Laplace*, 1823), in fact much too large, at ~ 20 times the hydrostatic value (*Keane & Matsuyama*, 2014). Unlike for the Earth where dynamic processes sustain a slight extra ellipticity, the Moon has a *fossil bulge* (*Jeffreys*, 1915), frozen in during the early planetary evolution, perhaps at ~ 4 Ga (*Lambeck & Pullan*, 1980; *Qin et al.*, 2018). While that bulge is now static, its formation is one way in which the Earth and Moon are coupled through their orbits, internal dissipation, and angular momentum.

In particular, sedimentary deposits on Earth record tidal cycles, and such constraints indicate that our planet experienced ≈ 22 h days at the end of the Neoproterozoic (~ 620 Ma; *Williams*, 1989), and ≈ 19 h days at 1.4 Ga (*Meyers & Malinverno*, 2018), i.e., a $\approx 25\%$ faster spin rate. The effects of the Coriolis forces due to planetary rotation are important for the geodynamo as well as for organizing atmospheric and ocean circulation (Expanded details 15), and hence possibly paleoclimate and the biosphere (*Walker*, 1982; *Olson et al.*, 2020). Perhaps less exciting, faster spin also implies a $\sim 60\%$ larger ellipticity at that time, since the hydrostatic bulge $\propto \Omega^2$ (eq. 1.11). The geological timescale slowdown of Earth's spin toward the present day is caused by tidal dissipation, which depends on the internal structure of the planet for solid tides, and on ocean circulation dynamics as affected, e.g., by plate-tectonics-dependent bathymetry and continental distribution. Oceanic processes make up the bulk of Earth's tidal dissipation at present, but both contributions have evolved over planetary history (*Ross & Schubert*, 1989; *Bills & Ray*, 1999; *Tyler*, 2021). Since the total angular momentum of any closed system has to be conserved, the Moon has moved away from Earth at ≈ 2.2 cm/yr on average since 620 Ma (*Williams*, 2000) (eq. 1.14), and this separation rate has sped up to ≈ 3.8 cm/yr at present (*Dickey et al.*, 1994), prolonging our aliens' journey.

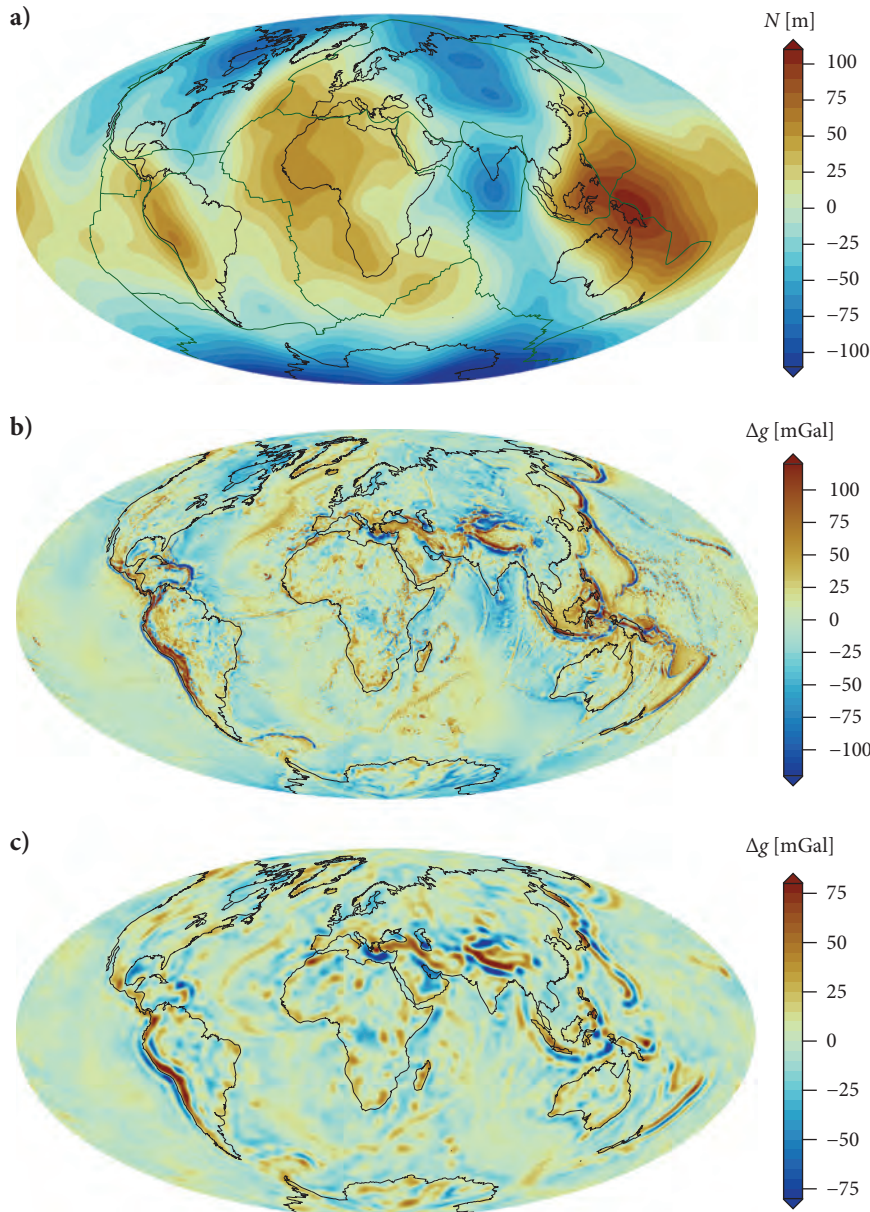


Figure 1.7: EGM 2008 (Pavlis *et al.*, 2012) geopotential model up to spherical harmonic degree $L = 512$ (§C.6.2). **a)** Nonhydrostatic geoid height anomalies, N of eq. (1.31), corrected following Chambat *et al.* (2010) (cf. Fig. A.3). **b)** Equivalent free-air gravity anomalies (cf. Fig. A.3d). **c)** Medium wavelength ($\ell = 10 \dots 80$, $\cos^2$ -smoothed), band-pass-filtered free-air gravity anomalies as often used to infer admittance-based dynamic topography, δz (e.g., $\delta z \sim \delta g / 50$ km/mGal; Craig *et al.*, 2011). See Fig. A.5 for a comparison of topography-geoid spectra for terrestrial planets.

1.2.3 Geoid and gravity anomalies

Figure 1.7a shows geoid anomalies with respect to the expected hydrostatic shape of the Earth (Expanded details 2). This satellite-based geoid anomaly map reveals that the geoid is in reality quite a bumpy landscape for Earth. Latitudinally, most positive anomalies globally are found around the equator, and the poles show negative anomalies. This pattern might be expected if the planet were to continuously reorient its excess density in the mantle and crust with respect to the spin axis such that the major moment of inertia, eq. (1.15), is oriented with excess mass moved to the equator. This way of shifting the entire mass of the Earth, including its surface, with respect to the rotation axis is called *true polar wander* (TPW). Mantle convection as well as glacial isostatic adjustment further change the geoid on a range of timescales (§6.7 and §9.4).

Without constraints about the internal structure of the planet, our aliens could not make a hydrostatic correction, but they could nonetheless explore, e.g., the correlation between topography and the geoid (§A.5), and muse as to the mechanisms of generation of the anomalies at different wavelengths (§3.2.2). When viewed in light of the interpretation of the oceanic topography, the aliens may note that there is no clear relationship between spreading centers and geoid highs, for example, which we now interpret as implying that spreading is predominantly passive, i.e., not driven by active, hot upwellings. There are large, positive geoid anomalies in places that we now associate with the main drivers of plate motions, subducted slabs, and deep mantle anomalies, but their proper interpretation relies on more sophisticated models of internal dynamics (§9.4).

Given that the geoid is predominantly ellipsoidal, the gravitational pull also mainly varies with latitude, as we are experiencing different degrees of the centrifugal force, eq. (1.25). We therefore consider anomalies from that reference gravity (Expanded details 2). Given that geoid anomalies are typically expanded as spherical harmonics, eq. (1.30), it is straightforward to manipulate the field, and, e.g., *free-air gravity anomalies*, eq. (1.34), at the reference level can be obtained from the geoid deflection's (N of eq. 1.31), spherical harmonic coefficients by multiplying them, at a given spherical harmonic degree ℓ , by an $(\ell - 1)$ -dependent factor (Expanded details 2). The resulting free-air anomalies (Fig. 1.7b) therefore look much rougher, or more detailed, than the geoid (Fig. 1.7a), since the derivative operation of eq. (1.13) leads to an $\ell - 1$ enhancement of higher spatial frequency power, eq. (C.67) (Fig. A.4). Tectonic analysis often uses gradients of the free-air anomalies to further enhance local variations, but note that the true information content in all those gravity “products,” as in Fig. 1.7, is identical.

Expressing gravity in a harmonic basis also allows filtering by wavelength. At the shortest wavelengths, $\ell \gtrsim 100$, a strong lithosphere is able to support surface topography by flexure rather than isostasy, leading to large free-air gravity anomalies that are well correlated with topography (§3.2.2). On longer than flexural wavelengths, isostatic adjustment over geologic timescales leads to small gravity anomalies and decorrelation with topography, and for $\ell \lesssim 20$, we expect mantle dynamics to matter (§9.4).

While any direct interpretation of gravity anomalies is highly dependent on a range of assumptions, mid-wavelength free-air gravity (Fig. 1.7c) is sometimes used to identify “anomalous,” that is, uncompensated, or “dynamic” topography. There are a number of features in this map that we return to (§9.2), such as the W-E tilting of Arabia, indication of mantle support of the Atlas Mountains of NE Africa, a marked depression in central Africa, and a characteristic pattern of free-air highs and lows along subduction arcs.

EXPANDED DETAILS 2: Geoid, spherical harmonics, and gravity anomalies

If we apply the divergence theorem, eq. (C.55), on $\mathbf{g}$, and use eqs. (1.10) and (1.13) considering a small volume V of homogeneous mass with surface area $4\pi r^2$, we can heuristically derive *Poisson's equation* for the gravitational potential,

$$\nabla^2 U(\mathbf{r}) = 4\pi G \rho(\mathbf{r}), \quad (1.27)$$

where ρ is density and ∇^2 is the *Laplacian* operator, in Cartesian $\left\{ \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right\}$ (§C.5). Eq. (1.27) is called the *Laplace equation* if the RHS is zero, which is the case for the potential at and above the Earth's surface. Written in spherical coordinates with longitude ϕ and colatitude θ (eq. C.50),

$$\nabla^2 U = \frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial U}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left(\sin \theta \frac{\partial U}{\partial \theta} \right) + \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 U}{\partial \phi^2} = 0. \quad (1.28)$$

The type of second-order partial differential equation (PDE) the Laplace equation, eq. (1.28), belongs to is called *elliptic*, and indeed it has an ellipsoidal, J_2 -type lowest-order solution, eq. (1.21).

The general solution to associated Legendre functions is one of *solid spherical harmonics*; for the potential outside the mass distribution, U is given by the superposition of harmonic functions

$$U(r) = \sum_{\ell=0}^{\infty} \sum_{m=0}^{\ell} \left(\frac{1}{r} \right)^{\ell+1} (c'_{\ell m} \cos m\phi + d'_{\ell m} \sin m\phi) P_{\ell m}(\cos \theta),$$

where ℓ is degree, m is order, the $P_{\ell m}$ are associated Legendre functions, eq. (C.64) (Fig. 1.8), and c' and d' are spherical harmonic coefficients, where care needs to be taken as to different normalizations being in use (§C.6.2). Spherical harmonics are in general highly useful to characterize the spectral content of global features (e.g., §1.5, §C.6.2). The equivalent feature length, D , or half wavelength, λ , on the surface of the Earth (§C.6.2) is

$$D = \frac{\lambda}{2} = \frac{\pi R_E}{\sqrt{\ell(\ell+1)}} \approx \frac{\pi R_E}{\ell} \approx \frac{20,000 \text{ km}}{\ell}. \quad (1.29)$$

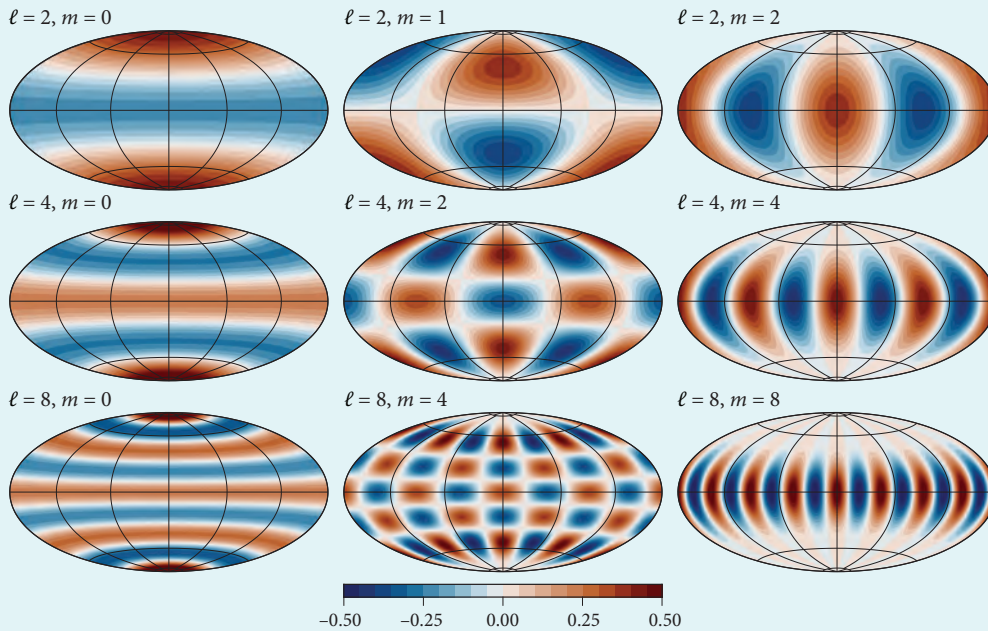


Figure 1.8: Examples of spherical harmonic basis functions (§C.6.2), with degree, ℓ , and order, m , showing the $\cos m\phi$ -type of contributions, or the $\mathcal{Y}_{\ell-m}$ spherical harmonics, eq. (C.63), in a projection centered on longitude $\phi = \pi$. The $m=0$ -type functions are called *zonal*; there is no longitude dependence. Wavelengths of features $\lambda \sim 40,000 \text{ km}/\ell$, eq. (1.29). Hammer equal area projection.

The actual geoid, e.g., as measured from satellite orbits for the Earth, is represented by a *band-limited* expansion with $\ell \leq L$ (cf. §C.6):

$$U(r) = -\frac{GM}{a} \sum_{\ell=0}^L \sum_{m=0}^{\ell} \left(\frac{a}{r} \right)^{\ell+1} (c_{\ell m} \cos m\phi + d_{\ell m} \sin m\phi) P_{\ell m}(\cos \theta). \quad (1.30)$$

The ellipsoidal U_g without rotation, eq. (1.17), is recovered by only considering $\ell = 0$ and $\ell = 2$ and $m = 0$ terms, for which $c_{20} = \frac{-J_2}{\sqrt{5}}$ for “fully normalized” harmonics (§C.6.2). At a certain location on the actual geoid, the difference between the value of the actual geoid when evaluated at the reference height, $U(r_{\text{ref}})$, and the reference geoid value, U_{ref} , is called a *potential anomaly*, and considering only verticals, is approximately

$$\Delta U = U(r_{\text{ref}}) - U_{\text{ref}}.$$

To first order, the difference in height away from the reference ellipsoid to where the actual U has the U_{ref} value is the *geoid undulation*, N , and *Bruns’ formula* relates the two:

$$\Delta U = -g_{\text{ref}}N. \quad (1.31)$$

This allows inferring N from the reference gravity (eq. 1.25), and potential measurements, as well as computing the geoid anomalies of Fig. 1.7a from an expansion of the form eq. (1.30).

The gravity varies on the geoid, equipotential surface, and we can define *free-air* gravity anomalies from the gravity g at some point at height r on the actual geoid relative to the gravity at the projection of this point onto the reference geoid, $g_{\text{ref}}(r')$:

$$\Delta g(r) = g(r) - g_{\text{ref}}(r').$$

From eq. (1.10), gravity changes with radial distance to first order as

$$\frac{\partial g}{\partial r} = \frac{\partial}{\partial r} \frac{GM}{r^2} = -\frac{2GM}{r^3} \approx -\frac{2g}{r_{\text{ref}}}, \quad (1.32)$$

which means that we can correct $g_{\text{ref}}(r')$ to $g_{\text{ref}}(r)$ by adding $\approx -\frac{2g_{\text{ref}}(\theta)}{r_{\text{ref}}N(r)}$. Using eq. (1.31) and $g(r) - g_{\text{ref}}(r) \approx \frac{\partial \Delta U(r)}{\partial r}$, the *fundamental equation of geodesy* results in

$$\Delta g(r) = \frac{\partial \Delta U(r)}{\partial r} + \frac{2}{r_{\text{ref}}} \Delta U(r), \quad (1.33)$$

where Δg are the measurements and the disturbing potential ΔU is the unknown, which has to be determined from eq. (1.28) since we are limited to measurements on the surface.

The most straightforward way of converting observed gravity values, g_{obs} , such as from a survey, collected at some height $h = r - r_{\text{ref}}$ above the reference geoid to some standard is thus by means of using the *free-air gravity anomaly*, Δg_{FA} (Fig. 1.7b). Close to the reference surface, we can write

$$\Delta g_{\text{FA}} = g_{\text{obs}} - g_{\lambda} + \frac{2g}{r}h = g_{\text{obs}} - g_{\lambda} + \gamma_{\text{FA}}h, \quad (1.34)$$

with $\gamma_{\text{FA}} \approx 0.3086 \text{ mGal/m}$, where mGal is 10^{-5} m/s^2 in SI units. Therefore, Δg_{FA} corrects for the decrease of gravitational pull away from the reference height as if there were air in between r_{ref} and r .

Free-air gravity anomalies are also linked through eq. (1.33) if there is an expansion of ΔU given by some set of anomaly coefficients $\bar{c}$ and $\bar{d}$ akin to eq. (1.30), in which case the corresponding N expansion is given by using the same $\bar{c}$ and $\bar{d}$ but dividing by $-g_{\text{ref}}$ as per eq. (1.31), and the free-air anomalies by scaling $\bar{c}_{\ell m}$ and $\bar{d}_{\ell m}$ by $-\frac{(\ell-1)}{a}$ (Fig. A.4). The equivalent, local

gravity anomalies, e.g., due to buried ore bodies or other lateral density variations, are tiny fractions of g_λ , and modern gravimeters used for surveying have relative μGal sensitivity, i.e., of order 10^{-9} of g .

The *Bouguer* gravity anomaly assumes that there is a laterally infinite slab of material with density ρ_B instead of air in between; with $\rho_B = 2670 \text{ kg/m}^3$, an equivalent $\gamma_B = 0.1967 \text{ mGal/m}$. We can compute the *surface gravity anomaly* dg of a buried density anomaly by integrating the incremental effect of all mass anomalies dM via eq. (1.9), $dg = \frac{G}{r^2} dm$. For example, a buried sphere of radius a with $\Delta\rho$ density anomaly yields

$$\Delta g = \frac{4\pi G a^3 \Delta\rho}{3r^2} \text{ or } \Delta g_b = \frac{4\pi G a^3 \Delta\rho}{3} \frac{b}{(x^2 + b^2)^{\frac{3}{2}}},$$

where Δg_b is the vertical gravity anomaly, x is the horizontal distance from the center of the sphere, and b is the burial depth to the center. Integration for complex geometries has to proceed numerically, and different-shaped anomalies may yield similar Δg expressions at the surface—a classic example of nonuniqueness of inversions, as expected from the solution of eq. (1.28) as required for eq. (1.33).

If a region is under isostasy, eq. (1.3) ensures that density anomalies down to compensation depth Z (Fig. 1.3) balance such that

$$\int_0^Z \Delta\rho(z) dz = 0,$$

and this can be achieved in numerous ways. However, the geoid anomalies need not be zero since those depend on the *dipole moment of the density distribution*,

$$\Delta N = -\frac{2\pi G}{g} \int_0^Z z \Delta\rho(z) dz = -\frac{3}{2\bar{\rho}R_E} \int_0^Z z \Delta\rho(z) dz \quad (1.35)$$

(Parsons & Richter, 1980), where $\bar{\rho}$ is the mean density of Earth, and the integral arises in computations of gravitational potential energy (§6.6).

If a crustal column of density ρ_c , thickness l_c , and topography t is in Airy compensation floating in a mantle of ρ_a , then

$$\Delta N_{t>0}^A = \frac{\pi G}{g} \rho_c \left(2l_c t + \frac{\rho_a}{\rho_a - \rho_c} t^2 \right) \quad \text{or} \quad \Delta N_{t<0}^A = \frac{\pi G}{g} (\rho_c - \rho_w) \left(2l_w t + \frac{\rho_a - \rho_w}{\rho_a - \rho_c} t^2 \right) \quad (1.36)$$

(Turcotte & Schubert, 2014; sec. 5.13), where the second equation accounts for an ocean basin, with water, ρ_w , coverage of depth l_w . For Pratt isostasy at a constant crustal base, depth $Z = P$ and using eq. (1.7) is

$$\Delta N_{t>0}^P = \frac{\pi G}{g} \rho_0 Z l_c \quad \text{or} \quad \Delta N_{t<0}^P = \frac{\pi G}{g} (\rho_0 - \rho_w) Z l_c. \quad (1.37)$$

Local topography variations of $\mathcal{O}(1 \text{ km})$ of isostatic crust thus correspond to $\mathcal{O}(10 \text{ m})$ geoid variations.

Geoid anomalies relate to ridge push via the density moment, eq. (1.35), associated with half-space cooling (exercise 7). Fig. 1.7a shows that spreading centers have, in general, no strong, excess positive geoid anomaly, meaning that spreading is passive in general, and not due to an active, hot mantle push component (§8.1.1). However, there are long-wavelength, large ΔN variations of the geoid; those are dynamically supported by the deep mantle (§9.4).

1.3 Internal structure, temperature, and composition of Earth

Remarkably, the USSR landed a number of probes on Venus during the Venera program starting in 1970. Those short-lived missions gathered photos and determined rock composition, which indicates basaltic-type rock cover in the lowlands. Our geophysical analysis of Venus so far is based on topography, geopotential, and inertial dynamics. However, given the more temperate climate, our enterprising aliens might be tempted to link appearances with internal qualities and deploy sensors on Earth.

Besides gravity, deep sounding estimates typically depend on electromagnetic or acoustic wave propagation, with the deepest and most detailed imaging achieved by listening to seismic waves using seismometers. Outside our planet, astronauts have deployed four such instruments on the Moon (Toksöz *et al.*, 1974; Nakamura, 1983), and a single sensor has recently greatly advanced our knowledge of the interior of Mars (Lognonné *et al.*, 2020; Khan *et al.*, 2021; Beghein *et al.*, 2022). What can our aliens say based on seismic wave propagation?

1.3.1 1-D structure of Earth as seen from seismology

Seismic waves are vibrations of the Earth where mechanical and kinematic energy gets transported away from a source, such as an earthquake or explosion, through an outwardly propagating wave front where material gets locally and transiently deformed. While more complicated in practice, we can think of this seismic wave by considering a raypath that is orthogonal to the wave front. That raypath will find the fastest path between source and receiver. If velocity increases with depth in the planet, the ray will thus not go straight from one side of the object to a recorder on the other side, but instead dive downward first and then be bent upward. This *refraction* is governed by *Snell's law* if velocities are layered, and differences in arrival time from a known source can be used to infer velocity with depth. A wave may also be *reflected* at a sharp contrast, such as a compositional layer, where the energy of transmission through an interface depends on the *impedance*, the product of wave speed and density. This provides constraints on the depth of the interface and the nature of the contrast.

Acoustic waves through air are of the volumetric (bulk) compressional type, but in an elastic solid, compressional, P , and shear, S , waves exist (§B.1). If we think of them as propagating through a planet as rays, we consider them to be two types of *body waves*. Their respective wave speeds correspond to different types of solutions of the wave equation (§B.1) and depend on different mechanical (elastic) properties and density as

$$v_P = \sqrt{\frac{K + \frac{4}{3}\mu}{\rho}} \quad \text{and} \quad v_S = \sqrt{\frac{\mu}{\rho}}, \quad (1.38)$$

where μ , K , and ρ are the shear modulus, incompressibility (§3.2), and density, respectively. Because $K, \mu \geq 0$, $\rightarrow v_P > v_S$, which means that the P wave arrives first, hence the labels of primary and secondary. The different wave types provide complementary constraints about the subsurface, e.g., since K and μ have different dependence on properties such as temperature and melt (§4.4.3). For wave propagation, v_P is like light speed for remote detection, in that no information can arrive earlier than as carried by the P wave. Earthquake rupture propagation typically happens at $\lesssim v_S$, but the related coseismic deformation can be transmitted remotely in near-instantaneous fashion by gravity signals traveling at actual light speed (Vallée *et al.*, 2017).

Based on their understanding of wave propagation, the analysis of refraction and reflection from a large amount of body wave records would then allow our aliens to infer a 1-D model of v_P and v_S for the Earth (e.g., AK135 in Fig. 1.9). For Earth, the interpretation of wave speeds in terms of solid and fluid structure has an interesting history (Brush, 1980). Wiechert (1897) proposed the existence of an iron core in a rocky shell, but the discovery of the core can be attributed to

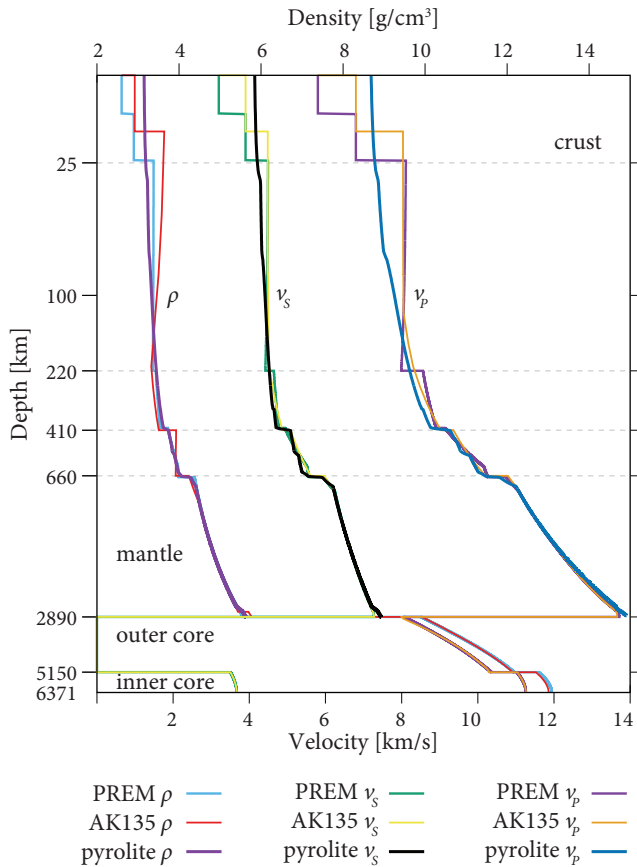


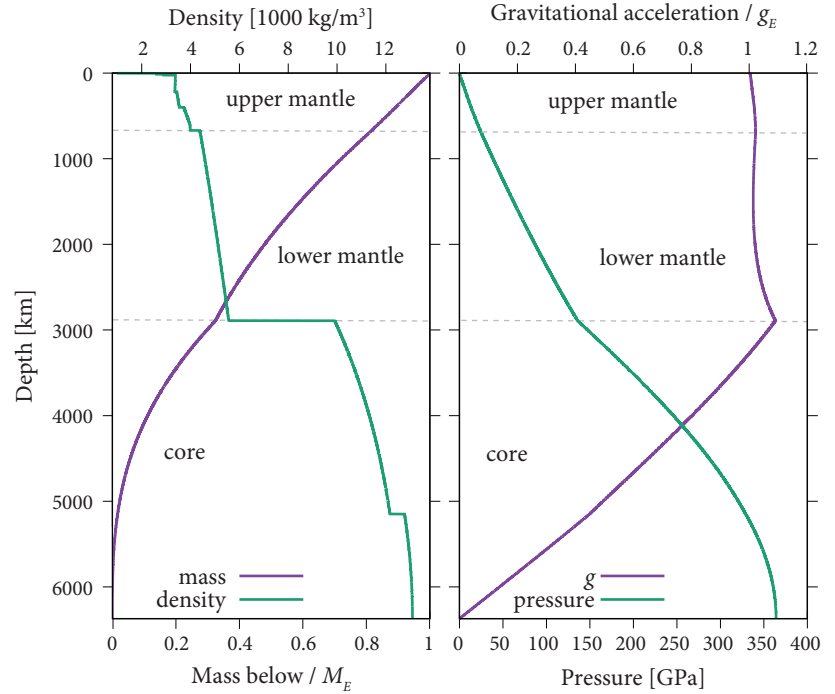
Figure 1.9: Layer averaged, 1-D Earth models showing density, ρ , as well as compressional, v_p , and shear wave, v_s , velocities from the long-period PREM (Dziewoński & Anderson, 1981) and shorter-period AK135 (Kennett et al., 1995) seismic models, and for the mantle only from a mineral physics prediction for pyrolite (HeFESTo computation; Stixrude & Lithgow-Bertelloni, 2011, 2022) and the geotherm of Fig. 1.13. PREM layer depths for 25 km (nominal crustal thickness) and 220 km (not globally observed), as well as actual major phase transition depths of 410 km and 660 km (670 km in PREM) (Fig. 1.14) are indicated; note log-scale for depth.

Oldham (1906). The depth of the core-mantle boundary (CMB) was placed at 2900 km by Gutenberg (1913). The CMB is one example where the differences in K and μ dependence of wave speed, eq. (1.38), jump out: a fluid cannot sustain shear, and the absence of shear wave arrivals once bent raypaths reach a certain depth in the deep Earth lead to the discovery of the fluid outer core, where $\mu = 0 \rightarrow v_s = 0$ (Oldham, 1906; Jeffreys, 1926).

The inner core was discovered by Lehmann (1936; paper title: “P”), and the major, global layers as seen in Fig. 1.9 were established in the Earth model by Bullen (1947). This is not to say that we fully understand the internal structure and dynamics of the Earth even on large scales. For example, there are indications of an anisotropically distinct innermost inner core (Ishii & Dziewoński, 2002; Pham & Tkalčić, 2023), and such inner core heterogeneity remains to be fully linked to planetary evolution. The existence of the inner core is also not a static feature, but arose some time over the last ~ 2.5 Ga (Buffett, 2002; Landeau et al., 2022), with revision of thermal conductivity estimates indicating a young, $\lesssim 700$ Myr origin (Labrosse, 2015), perhaps consistent with anomalously low magnetic field strength at ~ 500 Ma (Bono et al., 2019).

Properties in Fig. 1.9 are shown on a log scale for depth since the most interesting changes in velocities within the mantle happen in the upper ~ 700 km, which we focus on here. Figure 1.9 also shows the *preliminary reference Earth model* (PREM; Dziewoński & Anderson, 1981), which is mainly based on *normal modes*. Modes are global planetary oscillations that are excited by large earthquakes and can be described with spherical harmonics, like quantum mechanical orbitals, as the vibrations at various natural frequencies, *eigenfunctions*, combining to a range of possible oscillations. The surface displacements of spheroidal oscillations look like the examples of Fig. 1.8, but there are also toroidal types of oscillations, associated with twisting of the Earth.

Figure 1.10: Left: Density ρ (mean density ρ_E) and mass, in terms of total, M_E , below a given depth, as a function of depth. Density jumps at phase transitions are important for mantle convection, and likely overestimated in the case of the 670 km for PREM (cf. §12.12; Shearer & Flanagan, 1999). Right: Gravitational acceleration relative to the surface, $g_E = 9.807 \text{ m/s}^2$, and pressure. All based on integration of $\rho(z)$ from PREM (Dziewoński & Anderson, 1981) as in Fig. 1.9, cf. eqs. (1.8) and (1.39).



Higher-frequency (larger ℓ) normal modes localize deformation at the planet's surface, and their superpositions can also be identified as *surface waves*.

As the name implies, those waves require a free surface to vibrate. The displacement types of surface waves can be further divided into waves that oscillate in the horizontal, orthogonal to the propagation path (*Love waves*). Those waves require sphericity or a velocity increase with depth to exist, and consist of horizontally polarized shear, *SH*, waves and surface underside reflections of those (multiples). *Rayleigh waves* oscillate in the vertical and are a combination of vertically polarized shear, *SV*, and compressional, *P*, waves and can exist on any free surface, including a half-space. Love waves propagate faster than Rayleigh waves in the uppermost mantle, i.e., $v_{SH} > v_{SV}$, an example of the polarization dependence of wave speeds, or *seismic anisotropy* (§9.6).

1.3.2 Pressure and mass within the Earth

Since wave speeds depend on elastic moduli and density, eq. (1.38), the 1-D models of Fig. 1.9 provide further constraints for our aliens' assessments of Earth's internal mass distribution (Expanded details 3) and thus pressure conditions (Fig. 1.10). Whatever their density model, when ρ is integrated from the core up to radius r to obtain the mass underneath,

$$M(r) = 4\pi \int_0^r \rho(r') r'^2 dr' \quad \text{and} \quad M_E = M(R_E) = \frac{4\pi}{3} R_E^3 \rho_E, \quad (1.39)$$

the total mass of the Earth, M_E , needs to match the orbital constraints (§1.2). The mean density of Earth, $\rho_E = \langle \rho \rangle$, for an equivalent sphere is $\rho_E \approx 5513 \text{ kg/m}^3$. Plotting against depth, $M(r)$, yields the first curve in Fig. 1.10, which tells the aliens that only $\sim \frac{1}{3}$ of our planet's total mass, $M_E \approx 5.972 \cdot 10^{24} \text{ kg}$, is within the core, even though its iron-dominated composition has roughly three times the density of mantle rocks, which is $\langle \rho_m \rangle \approx 4454 \text{ kg/m}^3$. The mantle and crust contribute $M_m \approx 4 \cdot 10^{24} \text{ kg}$ (67%) and $M_c \approx 2.8 \cdot 10^{22} \text{ kg}$ (0.5%), respectively. These numbers matter for the assessment of composition and origin of the planet.

The gravitational acceleration due to the mass within an object is given by eq. (1.8). Figure 1.10 shows that $g(r)$ in the core roughly displays the linear increase that would be expected from $M \propto r^3$. However, the reduced densities of the mantle lead to a $\sim$ constant g within the mantle, with a volume-averaged mean value of $\langle g_m \rangle \approx 10.007 \text{ m/s}^2$, close to the surface value. The aliens can then also use eq. (1.3) to find the pressure conditions throughout the Earth (Fig. 1.10). For the upper mantle, p increases by $\sim 3.1 \text{ GPa}$ per 100 km; depths z in km can be converted to p in GPa and vice versa as

$$p \approx -0.04809 + 0.03077z + 7.30972 \cdot 10^{-6}z^2 \quad \text{or} \quad z \approx 2.98129 + 31.698p - 0.15771p^2,$$

which are coarse approximations (e.g., at the surface, p should be zero).

1.3.3 Complexities in the 1-D model

Considering shallow seismic wave speeds (Fig. 1.9), our aliens notice a sharp jump at $\sim 25 \text{ km}$. This is the assumed depth of a global crust-mantle interface for PREM, the *Moho*, first inferred by *Mohorovičić* (1910) based on P wave refraction from a single earthquake. The Moho marks a compositional change, from the basaltic- and granitic-type crust of oceanic and continental lithosphere, respectively, to the peridotitic, i.e., olivine- and pyroxene-dominated, mantle. PREM also divides the crust between an upper and lower part, whereas AK135 has only a single crustal layer of 18 km thickness. For the lithosphere, such global averages mean little, as discussed next.

1.3.3.1 The lithosphere and its topography

Using detailed regional seismological constraints, one finds that oceanic crust is of roughly constant thickness of $\sim 7 \text{ km}$ (Fig. 1.11b). This indicates that the fractionation process at midoceanic ridges is sampling an underlying upper mantle of fairly constant temperature overall (§10.3.4.2). There are, however, important local deviations, e.g., due to volcanic pulses leading to oceanic plateaus, as well as regional or temporal fluctuations of upper mantle temperature.

Within the continents, crustal thickness can be highly variable between $\sim 10 \dots 75 \text{ km}$ (Fig. 1.11b). Comparison of long-wavelength topography and crustal thickness estimates from seismology (Fig. 1.11a, b) leads our observers to expect that Airy isostasy does in fact broadly hold in continents. Particularly in continents, the depth of the lithosphere (or “plates”) is hard to define (Fig. 1.11c) since, to first order, the lithosphere is a mechanical layer. However, in oceanic lithosphere, the thermal, half-space cooling model can explain much of the surface topography through Pratt-type density variations (§7.3.2). A process difference between oceanic and continental lithosphere is that, once continental lithosphere is formed, it is then mainly floating at the surface because of compositional effects. This is partially because of its thick crust and partially because of melt depletion of its mantle-lithospheric part, in particular for old, cratonic regions (§10.4). Some continental material gets recycled into the mantle, but continental lithosphere is typically not subducted wholesale.

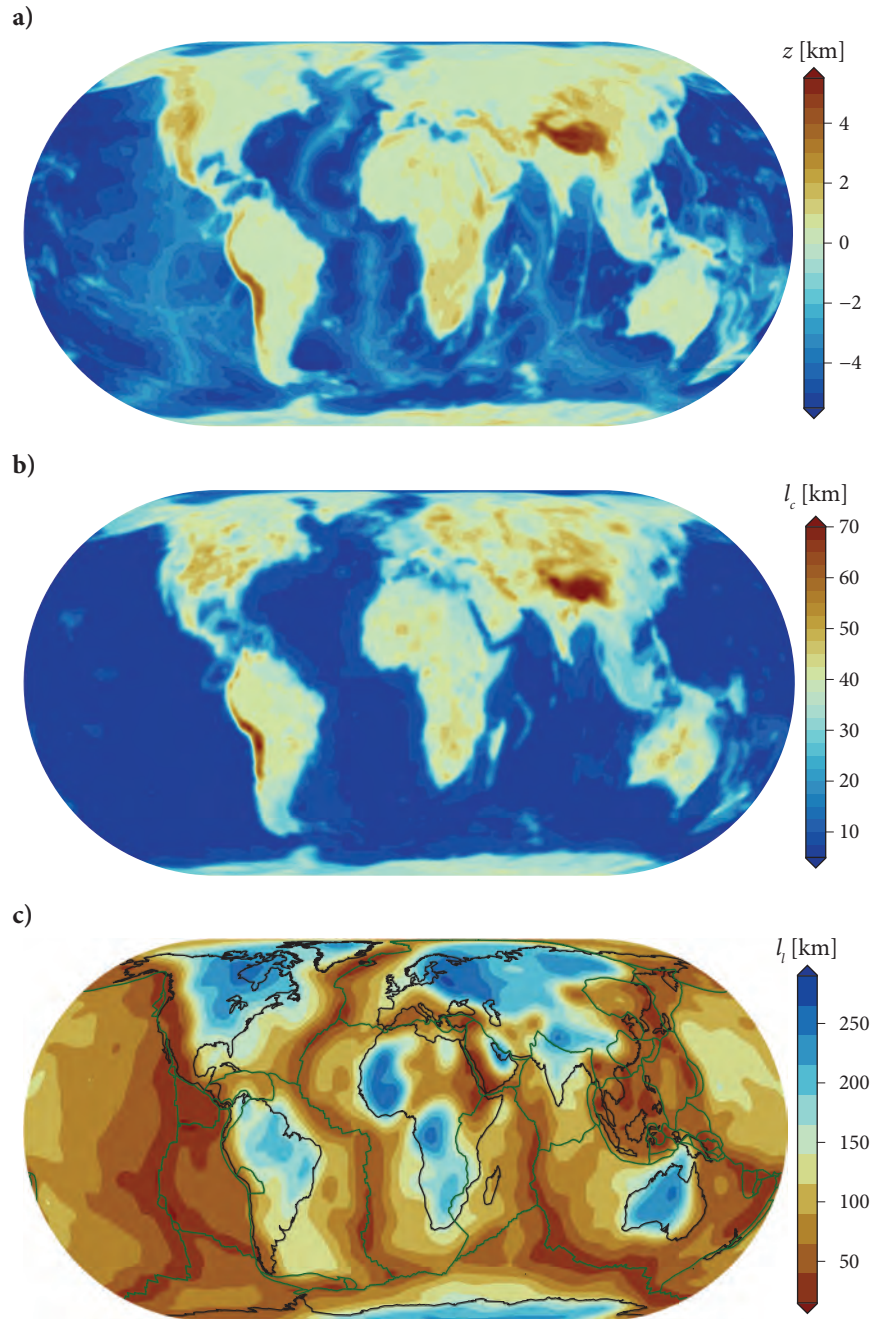
Partly as a consequence of such compositional heterogeneity, upon closer inspection, isostatic balance is often not exactly true. Figure 1.12 shows a global scatter plot for crustal thickness vs. *rock-equivalent topography*, z_{RET} . The latter removes the effect of water layers covering negative topography following eq. (1.6):

$$z_{\text{RET}} = z_{\text{bed}} + \frac{\rho_w}{\rho_c} \Delta W, \quad (1.40)$$

where z_{bed} is the bedrock elevation and ΔW the water layer thickness; it is the equivalent topography if water is replaced by an average crust.

There is an overall, broadly linear, isostatic trend (Fig. 1.12), but there is also large scatter and potentially different tectonic domains with different scalings (*Gvirtzman et al.*, 2016; *Ingalls et al.*, 2016). In particular, typical continental elevations of $\sim 500 \text{ m}$ correspond to a wide range of

Figure 1.11: **a)** Bedrock-equivalent topography (Hirt & Rexer, 2015), eq. (1.40), up to degree $\ell = 200$, and smoothed by a $\cos^2$ filter. **b)** Crustal thickness from an update of the CRUST1 model (Laske *et al.*, 2013) with regional studies (from Faccenna & Becker, 2020), filtered as for b). **c)** Lithospheric thickness model based on seismic tomography and the method of Steinberger & Becker (2018), their mean model with an attempt to remove subduction zone effects (see Steinberger & Becker, 2018). Note that lithospheric thickness estimates are highly uncertain, in particular close to orogens.



crustal thicknesses, which motivates the exploration of the sources of nonisostatic elevation. We can compute the deviation between actual topography and an isostatic model topography, z_{iso} ; this difference here defines *residual topography*. As per eq. (1.6), a change in crustal and lithospheric thickness, respectively, is expected to lead to isostatic elevation changes according to

$$\Delta z_c = f_1 \Delta l_c \quad \text{or} \quad \Delta z_l = f_2 \Delta l_l, \quad (1.41)$$

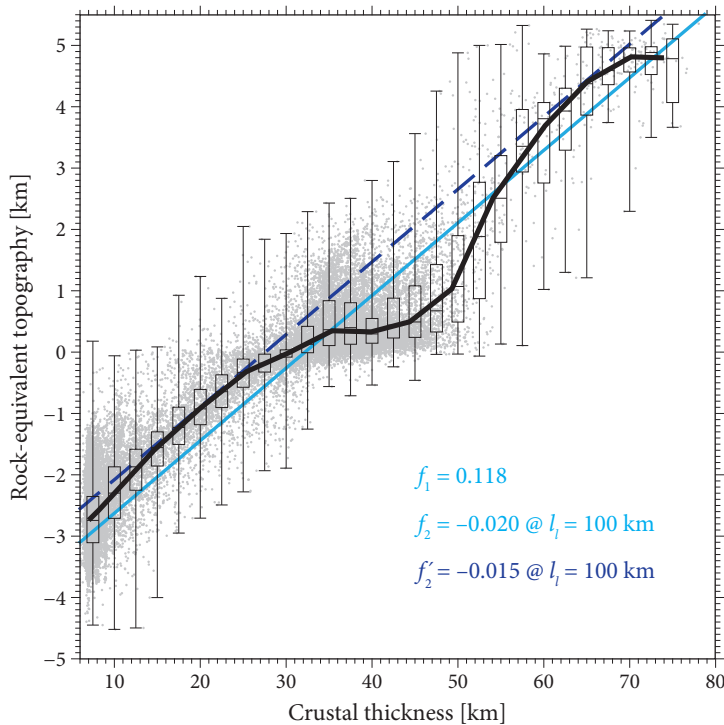


Figure 1.12: Relationship between crustal thickness (Fig. 1.11c) and rock-equivalent topography (eq. 1.40, Fig. 1.11a). Bar whisker plots (min and max as error bars, 25% and 75% quartiles as boxes) and black line are for the median at 2.5 and 5 km thickness bin width, respectively, and the blue line is a linear best fit. The $f_{1,2}$ values, eq. (1.6), are for an equivalent $H' = -1.8$ km and $l_l = 100$ km ($l_l = 97.5$ km is the mean of Fig. 1.11d). Dark blue dashed line corresponds to $f'_2 = -0.015$, which better matches the roughly linear regions at $l_c \sim 20$ and ~ 60 km. Compare to the synthetic columns and $f_{1,2}$ values of oceanic and continental domains in Fig. 1.4.

and the best-fit values of Fig. 1.12 are $f_1 \approx 0.12 \dots 0.17$ and $f_2 \approx -0.02$ and correspond to plausible, globally averaged density values (Fig. 1.4).

1.3.3.2 The asthenosphere

Below the Moho, the PREM model has another discontinuity at 220 km (Fig. 1.9): the *Lehmann discontinuity*. This depth and property jump is mainly a somewhat arbitrary choice of this particular model, and the velocity jump is not globally systematic. However, 1-D seismic velocities show a dip around $\sim 100 \dots 300$ km depth, particularly underneath oceanic plates, and regionally, there are sharp velocity contrasts at similar depths. Those may in some way (e.g., thermally, by means of partial melt variations, or mechanically) be associated with the transition between the lithosphere and the asthenosphere, sometimes referred to as the *lithosphere-asthenosphere boundary* (LAB; Fischer et al., 2010).

As mechanical terms, the L and the A are associated with relatively strong and weak parts of the mantle, deforming within mantle convection. Our aliens might thus study fluid dynamics and the deformation behavior of rocks in their laboratory by subjecting them to different deformation rates and varying pressures and temperatures. Depending on their patience, they might have to extrapolate their results over vast orders of magnitude since Earth's deformation is quite slow on experimental timescales. However, it is thought that the strength contrast between the asthenosphere and lithosphere is mainly achieved by temperature-dependent creep behavior of peridotitic mantle material (§5.1.6 and §7.3.2), with some contributions from composition and volatile variations. We can then associate the tectonic plates with the lithosphere, and the only compositionally distinct layer in Fig. 1.3 would be the crust. Figure 1.11c shows one seismology-based estimate of lithospheric thickness, based on the link between temperature and seismic velocity. However, there are many uncertainties, and the thickness estimate depends on which constraints are used to define the LAB (Rychert et al., 2020).

The mantle deforms by means of solid-state, slowly creeping flow, and a fluid's resistance to shear is called the *viscosity*. There is a moderate increase of viscosity with depth within the mantle, but most of the interior is low viscosity compared to the strong lithospheric plates on top, by several orders of magnitude (§9). We could thus associate the asthenosphere with all of the mantle below the lithosphere. However, the term more commonly refers to the depths of ~ 100 to ~ 300 km, where seismic velocity actually decreases slightly with depth (see v_S in Fig. 1.9 at ~ 200 km), the *low-velocity zone* of the upper mantle.

Most of seismic wave propagation happens elastically with tiny, reversible, elastic deformation of the rock through which the wave passes. However, dissipative processes lead to a reduction of wave amplitudes with distance besides the purely geometric spreading of energy. Such damping, or *attenuation*, of wave motion is frequency dependent (it increases with shorter periods) and also depends on depth and material properties (§4.4). Attenuation estimates thus provide additional constraints on the subsurface state (e.g., temperature, fluids, and melting), but this requires interpreting amplitudes rather than travel times, which is more complex as an inverse problem. That said, asthenospheric depths are found to be highly attenuating. The origin of this and the decrease of velocities with depth in the asthenosphere remains debated; it is probably not purely the effects of temperature and pressure causing this change, but small fractions of partial melting and anelastic processes close to the melting temperature likely contribute (§10.3.4; *Stixrude & Lithgow-Bertelloni*, 2005; *Takei*, 2017).

1.3.4 Thermal background state of the mantle

Our aliens thus cannot understand seismic structure without also figuring out internal temperatures. Below the low-velocity zone from ~ 200 km to the core-mantle boundary, we are seeing a monotonous increase in both v_P and v_S . Why is not obvious from eq. (1.38). Pressure increases with depth because of the overburden of overlying rock layers (Fig. 1.10); this squeezes mass into a tighter volume. Temperature also increases because of this pressure effect by means of *adiabatic compression* (Expanded details 3), but as Fig. 1.9 shows, the effect of pressure overwhelms that of temperature, and density does increase with depth in the mantle. Since velocities increase rather than decrease, this means that the increase of the elastic parameters with depth due to pressure is even greater.

An *adiabatic process*, or *isentropic process*, is one where there is no exchange of heat with the surrounding medium. The *mantle adiabat* describes the background increase of temperature within the mantle with depth only due to the effects of pressure. Using thermodynamics, one can show that this gradient can be linked to seismological properties; it works out to be $\sim 0.3^\circ\text{C}/\text{km}$ (Expanded details 3). This approach allows our aliens to add temperature to their Earth model (Fig. 1.13). The adiabatic gradient alone would predict an increase of temperature throughout the mantle of ~ 1000 K.

Dynamically, the 1-D average temperature within the mantle, the *geotherm*, is also expected to reflect the *thermal boundary layers* (TBLs) that arise due to mantle convection and show much stronger temperature gradients, $\approx 13.5^\circ\text{C}/\text{km}$ for the top ~ 100 km (§7.4.3). In fact, lithospheric plates and their motions are part of the top, cold thermal boundary dynamics of convection. Assuming an asthenospheric temperature of 1350°C below the plates (Table 1.1), using this as the TBL temperature difference, and assuming symmetry between the top and bottom layers adds $\sim 2700^\circ\text{C}$ to the adiabatic increase, so the aliens expect a core-mantle boundary temperature of $\sim 3700^\circ\text{C}$. Table 1.1 lists some additional constraints on the mantle geotherm, which overall are consistent with the high end of our convective plus adiabatic estimates. For the mantle, the symmetry between top and bottom TBL that purely bottom-heated, isoviscous convection would produce is an overestimate, and we expect the lower TBL to have a smaller temperature contrast because of significant internal heating (§8.1.1 and §10.1.4).

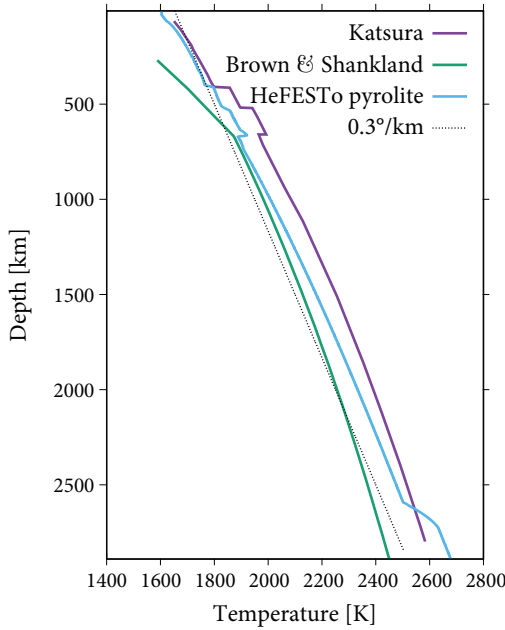


Figure 1.13: Adiabatic (i.e., constant entropy, no heat exchange) geotherm, accounting for latent heat due to phase transitions (Fig. 1.14), eq. (1.56), from Katsura (2022) for dry pyrolite, compared to seismology-based estimates from Brown & Shankland (1981), and pyrolite estimate as used for Fig. 1.9 (Stixrude & Lithgow-Bertelloni, 2011, 2022). For the latter, potential temperature, i.e., the intersection of the adiabat with the surface, is chosen as 1377°C. Also shown is a nominal adiabatic gradient, eq. (1.47), of 0.3°/km.

Boundary/region	Depth [km]	T [°C]	Reference
asthenospheric (potential) temperature			
half-space cooling (HSC)	~ 5	1365 ± 10	Carlson & Johnson (1994)
petrological	50	$1314 \pm 36(12)$	Brown Krein et al. (2021)
petrological, v_S , HSC	~ 100	1383 ± 40	Dalton et al. (2014)
phase transition anchor points (Fig. 1.14)			
olivine-wadsleyite	410	1490 ± 45	Katsura et al. (2004)
post-spinel	660	1600 ± 50	Katsura et al. (2004)
post-perovskite	2884	3730 ± 200	Hernlund et al. (2005)

Table 1.1: Temperature constraints (modified from Jaupart et al., 2007) with median $\pm$ one standard deviation recomputed for Dalton et al. and Brown Krein et al. for the same ridge segments (in parentheses: only considering the median cluster for Brown Krein et al.); cf. Fig. 1.13. For half-space cooling, see §7.3.2, eq. (7.53).

EXPANDED DETAILS 3: Thermodynamics of the adiabatic mantle temperature gradient

The *Maxwell equations* of thermodynamics can be written for the change of the *enthalpy*, H , type of energy, defined as internal energy, U , plus the product of pressure, p , and volume, V , $H = U + pV$. We can write the change in temperature, T (always in units of K for thermodynamics) due to change in pressure at constant *entropy*, S , as

$$\left(\frac{\partial T}{\partial p}\right)_S = \left(\frac{\partial V}{\partial S}\right)_p. \quad (1.42)$$

With mass $m = \rho V$, the total heat produced, ΔQ , relates to the change in temperature and entropy as

$$\Delta Q = mc_p \Delta T = V\rho c_p \Delta T = T \Delta S \quad (1.43)$$

for a reversible process, with *heat capacity* at constant pressure, c_p , which is defined through the limit case of the amount of heat ΔQ needed to raise temperature by ΔT at fixed pressure:

$$c_p = \lim_{\Delta T \rightarrow 0} \left(\frac{\Delta Q}{\Delta T} \right)_p, \quad (1.44)$$

where $c_p \sim 1000$ J/K/kg. The change in volume relates to change in temperature as

$$\Delta V = \alpha V \Delta T,$$

with *thermal expansivity* α , which links temperature and volume, or density, variations at constant pressure as

$$\alpha = \frac{1}{V} \left(\frac{\partial V}{\partial T} \right)_p = -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)_p; \quad d\rho = -\rho \alpha dT \quad \text{or} \quad d \ln \rho = -\alpha dT. \quad (1.45)$$

A typical value for the mantle is $\alpha_m \approx 2 \cdot 10^{-5} \text{ K}^{-1}$ (§4.4.3).

Plugging in those two relationships, we can rewrite eq. (1.42) for the increase of T with p as

$$\left(\frac{\partial T}{\partial p} \right)_{\text{ad}} = \frac{\alpha T}{\rho c_p}. \quad (1.46)$$

For a hydrostatic fluid, $dp = \rho g dz$, such that the *adiabatic gradient* of temperature with depth is

$$\left[\left(\frac{\partial T}{\partial z} \right)_{\text{ad}} = \frac{\alpha g T}{c_p} \right]. \quad (1.47)$$

This change in temperature is purely due to compressibility, and gradients are of order 0.3 K/km in much of the mantle, with values of ~ 0.6 K/km in the shallowest depths (Fig. 1.13).

Assuming homogeneity, we can integrate eq. (1.47) from the surface, *potential temperature*, $T_p(0)$, to a temperature at depth, $T(z)$, and simplify (§C.1.2, §C.2.4):

$$\int_{T_p}^T \frac{dT'}{T'} = \frac{\alpha g}{c_p} \int_0^z dz' \rightarrow \ln T - \ln T_p = \frac{\alpha g}{c_p} z \rightarrow \frac{T}{T_p} = \exp \left(\frac{\alpha g}{c_p} z \right). \quad (1.48)$$

Eq. (1.48) can be viewed as a way to correct some upper mantle temperature at depth, T , to its surface value, T_p , due to an adiabatic upwelling, without exchange of heat, i.e., constant entropy. Figure 1.13 shows that the actual adiabatic temperature profile does not follow this exponential shape because material parameters are depth dependent and phase transitions add complexity.

Eq. (1.47) is a combination of thermodynamic parameters and g , but we can further link the gradient to seismic parameters to build a planetary model. The *Grüneisen parameter*, γ ,

relates changes in volume to the properties of the crystal lattice and can be written as

$$\gamma = \frac{\alpha K_S}{\rho c_p} = \frac{K_S}{K_S^c} \quad \text{with} \quad K_S^c = \frac{\rho c_p}{\alpha}, \quad (1.49)$$

where K_S is the bulk modulus, or *incompressibility* (cf. eq. 3.6), denoted by subscript S for constant entropy:

$$K_S = \rho \left(\frac{\partial p}{\partial \rho} \right)_S = -V \left(\frac{\partial p}{\partial V} \right)_S. \quad (1.50)$$

The γ parameter is thus a nondimensional incompressibility with characteristic scale K_S^c , and $\gamma \approx 1 \dots 1.1$ throughout the mantle. One can also define an *Anderson-Grüneisen parameter*, γ_T , based on the change of expansivity as a function of volume change,

$$\frac{\alpha}{\alpha_0} = \left(\frac{V}{V_0} \right)^{\gamma_T}, \quad (1.51)$$

where γ_T has been suggested to be ~ 5 (e.g., discussion in *Katsura et al.*, 2004).

Based on γ of eq. (1.49), we can then rewrite eqs. (1.46) or (1.47) as

$$\left(\frac{\partial T}{\partial p} \right)_{\text{ad}} = \frac{\gamma T}{K_S} \quad \text{or} \quad \left(\frac{\partial T}{\partial z} \right)_{\text{ad}} = \frac{\gamma g T}{\phi}, \quad (1.52)$$

where ϕ is the *seismic parameter*:

$$\phi = v_B^2 = \frac{K_S}{\rho} = v_P^2 - \frac{4}{3} v_S^2 \quad (1.53)$$

with the *bulk sound velocity*, v_B , cf. eq. (1.38). These relationships provide the desired link between seismic constraints (Fig. 1.9) and thermal properties (§4.4.3), and can be used to integrate temperature down the adiabat. The Grüneisen parameter is often assumed to be $\sim$ constant within the mantle (Fig. 1.13), and also arises in *Birch's* (1952) *Adams-Williamson equation*, which, using eqs. (1.49) and (1.53), states that

$$\frac{d\rho}{dz} = -\frac{\rho g}{\phi} \quad \text{or} \quad \frac{1}{\rho} \frac{d\rho}{dz} = -\frac{\alpha g}{c_p \gamma}, \quad (1.54)$$

where the LHS can be integrated to yield an estimate of ρ as a function of depth, given ρ_0 and depth-dependent g and the seismically constrained ϕ :

$$\frac{1}{\rho} d\rho = -\frac{g(r)}{\phi(r)} dr \quad \rightarrow \quad \log \left(\frac{\rho}{\rho_0} \right) (r) = - \int_{r_0}^r \frac{g(r)}{\phi(r)} dr.$$

If we plug the definition of K_S into eq. (1.52), we can also relate temperature and density change as

$$\frac{dT}{T} = \gamma \frac{d\rho}{\rho}$$

or, after integration of both sides,

$$T = T_0 \left(\frac{\rho}{\rho_0} \right)^\gamma,$$

where the subscript zero refers to the values at zero pressure.

1.3.5 Composition and mantle phase transitions

Given their knowledge of thermodynamics and seismic constraints, our aliens have thus assembled a density, pressure, and temperature model of our planet. What is Earth made out of, though, i.e., which compositions fit the isostatic topography variations, as well as the bulk mass and the depth dependence as seen in their reference model?

They could drill, but that is tough (since T and p increase significantly with depth), and none of our deep drilling efforts has penetrated the continental Moho yet. They could also conduct experiments using different compositions, but what speeds up the search is the consideration of comparable objects. For Earth, we consider two types of meteorites as representative: in very broad strokes, iron-type meteorites for the core, which is made out of iron and some lighter element (*Li & Fei, 2005; Hirose et al., 2013*; sulfur, carbon, oxygen, and hydrogen have been suggested) and silicic-type meteorites (carbonaceous chondrites) to define the *bulk silicate Earth* (BSE; *McDonough & Sun, 1995; Palme & O'Neill, 2005*; §10.1.7).

Overall, mantle composition is like a peridotite rock. From the BSE, convection has extracted the oceanic and continental crust (§10.1.7), leaving a relatively depleted upper mantle component sampled by spreading centers, a midocean-ridge basalt (MORB) reservoir. From dredged samples, one can estimate the *depleted midocean-ridge mantle* (DMM; *Workman & Hart, 2005*), and estimate a hypothetical pyrolite material to match DMM, e.g., by mixing a fraction f_b of basalt produced with the $(1 - f_b)$ fraction of depleted harzburgite residuum, $f_b \approx 0.2$ (*Xu et al., 2008*). Such f_b values are broadly consistent with seafloor creation rates and operation of plate tectonics over $\gtrsim 3$ Gyr (*Phipps-Morgan & Morgan, 1999*).

Figure 1.9 compares thermodynamic estimates for pyrolite with the seismic models; there is a good fit overall, within uncertainties (*Cammarano et al., 2005*). However, turned around, seismic models do not provide sufficient constraints to resolve some of the more subtle questions, e.g., the degree of compositional variations from a mean bulk composition (*Mattern et al., 2005; Stixrude & Lithgow-Bertelloni, 2012*). There is also significant uncertainty about BSE composition itself. All of this means that we have a good overall understanding of the rocky mantle composition and what happens when fractionation modifies it. Yet, questions such as the degree of internal heating in the lower mantle and the degree to which compositional anomalies matter for midmantle convection remain open.

With these caveats in mind, we can return to the sharp increases in velocities and density seen in the transition zone (Fig. 1.9), typically defined to be below the asthenosphere, $\gtrsim 300$ km and $\lesssim 700$ km. In between, PREM identifies jumps at 410 km and 670 km, where the lower boundary is closer to 660 km according to more modern estimates. Those jumps are, to first order, not related to any compositional change, but rather a transition of the same pyrolitic material to closer packing due to *phase transitions* (Fig. 1.14). The pressure at which a phase transition occurs (and hence the depth) depends on the temperature via the *Clapeyron slope*,

$$\Gamma = \frac{dp}{dT_{\text{transition}}}, \quad (1.55)$$

(continued...)

Index

- Aa flow, 397
- A-type CPO, 386
- absolute plate motion, 43, 389
- absorption frequencies, 195
- acceleration, 6
- accretion, 325
- accretionary
 - margin, 500
 - orogeny, 568, 570
- acoustic emissions, 117
- activation enthalpy, 326
- active rifting, 416
- Adams-Williamson equation, 31
- adiabatic
 - compression, 28
 - gradient, 30
 - process, 28
- adjoint equations, 363
- admittance, 116, 366
- advected, 64
- advection, 269
- advective time derivative, 63
- aftershock, 137
- afterslip, 171, 204, 536
- aging law, 161
- Airy
 - functions, 667
 - isostasy, 8
- Amontons-Coulomb law, 120
- Ampère's law, 39
- amplitude, complex number, 679
- ancient reservoir, 409
- Anderson-Grüneisen parameter, 31
- andesite, 396
- andesitic, 519, 610
- anelastic
 - behavior, 196
 - creep, 189
 - liquid approximation, 323
- angle of repose, 120
- angular
 - frequency, 677
 - momentum, 12
- anharmonic behavior, 196
- anisotropy, 271
 - elastic, 105
- seismic, 106
- seismic, extrinsic, 385
- seismic, intrinsic, 386
- viscous, 130, 176
- antiderivatives, 683
- antisymmetric tensor, 68
- antithetic shears, 472
- aphroditoid, 657
- apparent
 - polar wander, 37
 - stress, 153
- Archean cratonic lithosphere, 464
- area under the curve $f(x)$, 683
- areoid, 657
- Argand number, 257
- arithmetic mean, 224, 707
- arrest zone, 150
- Arrhenius relationship, 208
- aspect ratio, 311
- asperities, 136, 538
- associated Legendre functions, 704
- asthenosphere, 7, 196, 313
- astrobiology, 644
- attenuation, 28
 - of seismic waves, 192
 - relationship, 489
- Australian-Antarctic discordance, 362
- avalanches, 336
- average elastic thickness, 116
- averaging, self-consistent, 226
- Avogadro number, 208
- axial
 - magma chamber, 443
 - summit caldera, 443
- axially symmetric, 71
- azimuthal anisotropy, 382
- b value, 135
- back-slip model, 145
- band-limited expansion, 19
- Barrovian pattern, 582
- barystatic term, 267
- basalt, 396
- basins and ranges, 436
- basis vectors, 694
- batch melting, 453
- beach ball pattern, 84
- Beaumont number, 598
- bending
 - moment, 111
 - stresses, 110
- Big Five, 413
- biharmonic stream function
 - equation, 252
- bimodality, 710
- Bingham fluid, 184
- black smokers, 450
- block
 - model, 146
 - rotations, 70
- Bode plot, 193
- body
 - forces, 74
 - waves, 22
- Boltzmann's constant, 269
- Bouguer gravity anomaly, 21
- boundary
 - condition, Dirichlet, 279
 - condition, flux, 279
 - condition, von Neumann, 279
 - conditions, 243
 - element methods, 149
- Boussinesq approximation, 304
- box model, 639
- breakdown stress drop, 158
- brittle
 - deformation, 98
 - ductile transition, 228
 - strength, 120
- Bruns' formula, 20
- bulge
 - flexural, 111
 - peripheral, 111
- bulging, 211
- bulk
 - modulus, 104
 - silicate Earth, 32, 274
 - sound velocity, 31, 197
- buoyancy number, 333
- Burgers
 - body, 191
 - vector, 209
- butterfly effect, 40
- Báth's law, 139
- Carreau fluid, 183
- Cartesian
 - decomposition, 693
 - reference frame, 695
- Cathles parameter, 263
- Cauchy
 - deformation tensor, 669
 - relation, 73
 - stress tensor, 72, 74
- cellular automaton, 136
- central
 - finite difference, 290
 - limit theorem, 710
- central Atlantic magmatic province, 414
- centrifugal acceleration, 14
- Chablis model, 457
- chain rule, 681
- chaos, 40
- chaotic mixing, 411
- characteristic
 - earthquakes, 137
 - equation, 693
 - temperature for internal heating, 308
- chondrites, 409
- chrons, 36
- circulation, 357
- Clairaut's theorem, 16
- Clapeyron slope, 32, 299
- clock advance, 123
- clustering, 137
- coaxial, 83
- Coble creep, 209
- coefficient
 - of friction, 120
 - of variation, 142, 707
- cohesion, 120
- cohesive
 - strength, 118
 - zone, 150
- colatitude, 695
- collisional-type orogeny, 572
- common component, 412

- compatibility equations, 666
- compensated linear vector dipole, 80
- compensated typography, 113
- compensation, 113
- compensation depth, 6
- complementary error function, 285
- completeness, 135
- complex conjugate, 678
- complex number, 678
- concavity parameter, 596
- concept of a continuum, 60
- conduction, 269
 - equation, 278
- conductive gradient, 280
- conductivity tensor, 271
- conservation
 - of energy, 666
 - general law, 663
 - of heat energy, 302
 - of mass, 241, 664
 - of momentum, 241, 664
 - of momentum for convection, 664
 - of momentum for the wave equation, 665
- constant strain-rate test, 185
- constitutive laws, 98
- continental
 - drift, 34
 - keels, 464
 - levering, 266
 - roots, 464
 - undertow, 609
- continental-type orogeny, 568
- continuity equation, 241, 664
- control variable, 169
- conveyor belt, 629
- coordinate system, 694
- Cordillera-type orogeny, 568, 570
- core complex
 - metamorphic, 423
 - oceanic, 445
- Coriolis force, 235
- corner flow, 253
- corner frequency, 159
- coronas, 4
- Couette flow, 244
- Coulomb stress, 123
- Courant-Friedrichs-Lewy condition, 291
- cracklike rupture, 158
- creep test, 189
- Cretaceous superchron, 36, 651
- critical
 - Rayleigh number, 306
 - slip distance, 159
- stiffness, 164
- taper, 504
- taper theory, 574, 587
- cross product, 689
- crustal-scale thrust wedges, 575
- crystallographically preferred orientations, 210
- Curie temperature, 36
- curl, 699
- curvature, 111
- damage, 117, 183
 - rheologies, 352
 - zone, 127
- damped wave equation, 288
- damping ratio, 192
- Darcy law, 272
- data assimilation, 357
- Deborah number, 187
- Debye peak, 194
- decadic log, 676
- decay constant, 276
- décollement, 126
- decompression melting, 452
- deformation
 - maps, 226
 - rate tensor, 69, 668
 - tensor, 668
- degree, 19, 703
 - of compensation, 113
- dehydration embrittlement, 541
- dehydration strengthening, 328
- delamination, 582
- density, 7
- depleted mid-ocean-ridge mantle, 274
- depleted midocean-ridge mantle, 32
- depocenters, 502
- depozones, 572
- derivative, 680
 - material, 64
 - operator, 680
 - partial, 680
 - total, 680
- detachment faults, 423, 445, 574
- determinant, 80, 687
- deterministic chaos, 40
- deviatoric
 - second invariant, 179
 - strain, 67
 - strain rate, 70
 - stress, 76
- dextral, 472
- diagenesis, 207
- differential stress, 76
- diffuse deformation, 493
- diffusion
 - equation, 282
- timescale, 283
- dilation, 67, 102
 - rate, 69
- dipole moment of density
 - distribution, 21, 256
- direct effect, 161
- directional data, 707
- Dirichlet boundary condition, 279
- discharge, 595
- discontinuities, 59
- discretization, 289
- dispersion, 195
 - relation, 677
- displacement, 61
 - gradient tensor, 68
- dissipation
 - number, 324
 - viscous, 179, 528
- dissolution creep, 207
- divergence, 699, 706
 - theorem, 701
- dot product, 688
- double couple, 84
- downscale, 52
- Drucker-Prager criterion, 182
- ductile deformation, 98
- dunite, 446
- DUPAL anomaly, 412
- duplexes, 502
- dyadic product, 68, 690
- dynamic
 - equilibrium, 73
 - flattening parameter, 14
 - friction, 140
 - pressure, 176, 182, 240
 - stress drop, 158
 - topography, 249, 362
 - viscosity, 176
- early enriched reservoir, 409
- earthquake, 130
 - early warning, 143
 - prediction, 143
 - simulators, 491
- eduction, 607
- effective
 - erosional efficiency, 596
 - medium, 119
 - normal stress, 121
 - principal stresses, 122
 - slab viscosity, 530
- effusive style, 396
- eigenfunctions, 23
- eigensystem, 75, 693
- eigenvalues, 693
- eigenvectors, 693
- Einstein summation convention, 67, 242, 688
- Ekman number, 236
- magnetic, 236
- elastic
 - modulus, 100
 - rebound theory, 140
- electrical conductivity, 236
- ellipsoid, 13
- elliptic, 19
- ellipticity parameter, 106
- elongation, 64, 669
- EM-1, 412
- EM-2, 412
- en echelon, 472
- endothermic, 33
- energy
 - flux, 269
 - rate, 269
- engineering shear strain, 65
- enthalpy, 29
- entropy, 29, 271
- epicenter, 133
- epidemic-type aftershock
 - sequence, 139
- episodic tremor and slip, 539
- equation of state, 31
- equatorial moments, 13
- erodability, 595
- erosion number, 596
- erosional efficiency, 596
- erosive margins, 500
- error function, 285
- Euclidean space, 695
- Euler
 - angles, 692
 - equations of motion, 664
 - formula, 678
 - pole, 62
 - vector, 62
- Eulerian reference frame, 63
- eustatic term, 267
- evolution
 - effect, 161
 - laws, 161
- exhumation, 601
- exoplanets, 644
- exothermic, 33
- expectation operator, 707
- explicit finite difference method, 290
- explosive style, 396
- exposure, 470
- extended rare earth diagram, 274
- extension, 64
- extremophiles, 451
- extrinsic seismic anisotropy, 385
- extroversion, 632
- factorial, 682
- failure envelope, 120
- fast azimuths, 385

- fatigue, 119
- fault
 - mirror, 127
 - plane ambiguity, 85, 93
 - readiness, 470
 - valving, 125
- felsic, 396
- fiber stresses, 110
- Fick
 - first law, 272
 - second law, 278
- field boundary
 - approach, 221
 - model, 223
- fingerprinting, 267
- finite
 - difference, 289
 - strain, 668
 - strain ellipsoid, 71, 669
 - strain tensor, 70
- first invariant, 79
- fission tracks, 276
- fixed point, 164
- fixist, 567
- flat slab, 612
- flat slab subduction, 525
- flattening, 71
 - dynamic, 14
 - geometric, 14
 - gravity, 15
- flexural
 - parameter, 111
 - rigidity, 110
- Flinn diagram, 670
- flip-flopping, 445
- flood basalt, 397
- flower structures, 472
- flux boundary condition, 279
- flysch to molasse, 572
- focal zone, 412
- fold-thrust belts, 572, 573
 - thick skinned, 574
 - thin skinned, 574
- foot wall, 76
- forbidden zone, 515
- foreland basin, 572
- foreshock, 139
- forward finite difference, 289
- fossil bulge, 16
- Fourier
 - coefficients, 702
 - law, 271
 - series, 702
- fractional melting, 453
- fracture mechanics, 118
- Frank-Kamenetskii
 - approximation, 327
- Fréchet kernel, 362
- free slip, 245
- free-air gravity anomalies, 18, 20
- frequency, 677
- frequency-magnitude
 - relationship, 134
- friction
 - dynamic, 140
 - kinetic, 140
- frictional sliding, 120
- fully compensated topography, 8, 113
- fundamental equation of geodesy, 20
- gas constant, 208
- Gauss
 - distribution, 709
 - integral, 683
 - theorem, 701
- geocentric
 - axial dipole hypothesis, 37
 - latitude, 13
- geodynamo, 36
- geographic latitude, 13
- geoid
 - anomalies, 16
 - undulation, 20
- geometric
 - flattening, 14
 - mean, 225, 708
- geoneutrinos, 276
- geosyncline, 569
- geotherm, 28, 279, 280
- geothermometers, 592
- giant impactor hypothesis, 325
- Gibbs phenomenon, 703
- glacial buzz saw, 586
- global
 - isostatic adjustment, 263
 - mantle circulation, 357
 - navigation satellite system, 59
 - positioning system, 59
- Goetze number, 424
- Goldilocks zone, 645
- Grüneisen parameter, 30
- gradient of a vector field, 68, 699
- gradient operator, 12, 698
- grain boundary
 - migration, 211
 - sliding, 211
- gravitational
 - acceleration, 7
 - constant, 11
 - energy, 11
 - potential energy, 256
 - sliding, 297
- gravity
 - anomaly, 20
 - flattening, 15
- Green's functions, 148, 374
- Griffith cracks, 125
- Griffith criterion, 118
- grounding line, 267
- growth rate, 304, 671
- Gutenberg discontinuity, 465
- habitability, 645
- Hagen-Poiseuille flow, 245
- Hale-Shaw flow, 254
- half-life, 276
- hanging wall, 76
- harmonic
 - functions, 304, 305
 - mean, 224, 708
- harmonics, 676
- harzburgite, 446, 460
- Haskell constraint, 260
- hazard, 469
- healing, 183
- heat
 - capacity, 30, 278, 666
 - flow paradox, 173
 - flux, 270
 - pipe, 638
 - pipe mode, 347
- heating rate, 273
- Helmholtz decomposition, 706
- high plateaus, 575
- hillslope diffusion, 596
- HIMU, 412
- hold time, 159
- homogenization, 61, 225
- homologous temperature, 208
- homopolar dynamo, 39
- Hooke's law, 100
- hotspot, 43, 324, 396
 - reference frame, 43
 - swell, 401
 - tracks, 396
- Howard's conjecture, 315
- hydraulic conductivity, 272
- hydrostatic
 - bulge, 15
 - equilibrium equation, 7
 - geoid, 16
 - pressure, 121
 - stress state, 76
- hydrothermal deficit, 450
- hyperarid, 613
- hyperbolic
 - equation, 665
 - functions, 678
- hyperextended margin, 419
- hypocenter, 133
- hypsometric curves, 4
- hysteresis, 346
 - heat transport state, 346
 - mechanical, 180
- parameter space, 321
- rheological, 350
- identity matrix, 691
- ill-determined problem, 694
- impedance, 22, 465
- in-sequence propagation, 573
- incompatible elements, 274
- incompressibility, 31, 104
- incompressible fluid, 70, 104, 176
- index fossils, 651
- infinite Prandtl number
 - approximation, 235
- infinitesimal deformation, 61
- initial condition, 283
- inner derivative, 681
- inner product, 179, 688, 691
- instability growth rate, 304, 671
- instantaneous solution, 241
- integrable singularity, 118, 685
- integral, 683
- intensity, 133
- interferometric synthetic aperture
 - radar, 152
- interferometry, 153
- internal
 - friction, 192
 - friction angle, 120
 - heat production, 273
 - heating Rayleigh number, 308
- intraplate orogeny, 572
- intrinsic seismic anisotropy, 386
- introversion, 632
- invariant, 79, 687
 - first, 79
 - second, 78
 - third, 80
- inverse theory, 707
- inviscid, 235
- irrotational, 70, 706
- isentropic process, 28
- isopycnic buoyancy, 463
- isostasy, 6, 109
- isotropic strain, 68
- J₂
 - gravity, 14
 - stress, 80
- Jacobian, 668
- jelly sandwich, 579
- joints, 125
- keel, 464
- Kelvin-Voigt element, 189
- kinematic viscosity, 234
- kinetic friction, 140
- kinetics, 336
- knickpoint, 597

- Kronecker δ , 68, 689
kurtosis, 710
- Lagrangian
 reference frame, 63
 strain tensor, 668
- Lamé constant, 101
- laminar mixing, 411
- Laplace
 equation, 18
 operator, 700
- large
 igneous provinces, 397
 ion lithophile, 517
 low shear wave velocity provinces, 52
- last glacial maximum, 259
- latent heat, 33, 514
- latitude, 695
 geocentric, 13
 geographic, 13
- least-squares solution, 694
- left-stretch tensor, 669
- Legendre functions, 704
- Lehmann discontinuity, 27, 465
- Leibniz notation, 681
- Levi-Civita
 permutation symbol, 689
 tensor, 689
- lherzolite, 509
- limit cycles, 164
- line of nodes, 692
- line of sight, 153
- linear, 71
 elasticity, 100
 fracture mechanics, 118
- liquidus, 451, 517
- lithosphere, 7, 293
 asthenosphere boundary, 27
 delamination, 582
- lithostatic stress state, 76
- local Rayleigh number, 315
- locking depth, 146
- log-normal distribution, 710
- logarithmic strain, 670
- logic tree, 489
- longitude, 695
- longitudinal strain, 101, 102
- Lorenz force, 39
- Love
 parameters, 106
 waves, 24
- low Urey number problem, 643
- low-angle normal faults, 416
- low-velocity zone, 28
- L-S tectonites, 71
- L-tectonites, 71
- Lyapunov exponent, 40
- MacCullagh's formula, 13
- mafic, 396
- magmons, 452
- magnetic
 diffusivity, 236
 Ekman number, 236
 permeability, 39
 Reynolds number, 236
- magnitude, 133
 of completeness, 135
 complex number, 679
- mantle
 adiabat, 28
 conveyor belt, 358
 plumes, 396
 wind, 359
- Marangoni-Bénard convection, 309
- marble-cake model, 411
- mass extinction, 413
- material derivative, 63, 666
- matrix, 687
 identity, 691
 inverse, 691
 orthogonal, 692
- Maxwell
 body, 185
 equations, 29
 time, 185
- mean
 arithmetic, 224, 707
 geometric, 225, 708
 harmonic, 224, 708
- mean-mantle fixed reference frame, 44
- mechanical
 hysteresis, 180
 lithosphere, 293
 twinning, 207
- median, 710
- megathrust, 131, 534
- melting, 453
- memory, 351
- metamorphic
 core complexes, 423
 facies, 510
- metastability, 337
- Mg-#, 463
- midlithospheric discontinuity, 465
- midocean-ridge basalts, 274
- mid-pleistocene transition, 259
- Milankovitch cycles, 259
- mixed-determined, 694
- mixing
 chaotic, 411
 laminar, 411
- mobile
 belts, 520
 lid, 330
- mobilitic, 567
- mode switches, 492
- modulo function, 698
- Moho, 25
- Mohr
 circle, 122
 Coulomb failure criterion, 120
- molasse, 572
- mole, 208
- moment
 of inertia, 12
 magnitude, 133
 release rate, 91
 tensor, 85, 86
- multiplication operator, 682
- multitaper, 703
- mylonite, 130
- Nabarro-Herring creep, 209
- nappes, 567
- natural
 frequency, 192
 logarithm, 676
 strain, 670
- Navier-Stokes equation, 233, 238, 664
- net rotation, 43, 389
- neutral line, 110
- neutrinos, 276
- Newtonian, 175
 fluid, generalized, 180
- no-net-rotation, 45
- no-slip boundary condition, 240
- non-coaxial, 83
- nonvolcanic tremors, 539
- norm, 686
- normal
 distribution, 709
 modes, 23
 strain, 64
 stress, 72, 73
- normalized vectors, 686
- nucleation length, 169
- number
 Argand, 257
 buoyancy, 333
 Deborah, 187
 dissipation, 324
 Ekman, 236
 erosion, 596
 Goetze, 424
 Nusselt, 311
 Peclet, 249
 Prandtl, 235
 Rayleigh, bottom-heated, 304
 Rayleigh, internal heating, 309
 Rayleigh, local, 315
 Reynolds, 234
- Rossby, 236
- Urey, 641
- numerical modeling, 289
- Nusselt number, 311
- obducted, 446
- obduction, 499
- ocean siphoning, 266
- oceanic
 core complex, 445
 spreading centers, 37
- octahedral stress, 80
- Omori-Utsu law, 137
- operational earthquake
 forecasting, 490
- operators, 698
- optimally oriented faults, 123
- order, 19, 703
- orientational data, 707
- orogenic belts, 567
- orogeny, 567
 accretionary-type, 568, 570
 collisional-type, 568, 572
 continental-type, 568
 Cordilleran type, 568, 570
 subduction-type, 568
- Orowan's equation, 209
- orthogonal, 686, 692, 695, 702
 basis, 693
 basis functions, 702
- orthonormal, 694
- orthoverion, 633
- oscillator
 damped, 192
 driven, 193
- outer
 derivative, 681
 product, 690
 rise, 112
- overdetermined, 694
- overshoot, 158
- overturn time, 317, 321, 559
- P* shears, 472
- p* values, 709
- paleobotany, 592
- paleomagnetic, 36
- paleoseismology, 142
- Pangea, 34
- parabolic, 278
- parameter space, 306
- partial derivatives, 680
- partial melt fractions, 451
- passive
 margin, 545
 rifting, 416, 430
 upwellings, 396
- Pearson correlation coefficient, 711

- Peclet number, 249
- pelitic rocks, 128
- peridotite, 446
- period, 677
 - doubling road to chaos, 40
- periodic
 - isotropic two-layered model, 385
 - orbits, 40
- peripheral bulge, 111
- permeability, 272
- perturbation growth rate, 304, 671
- phase
 - buoyancy parameter, 333
 - complex numbers, 679
 - delay, 677
 - space, 41, 164
 - transitions, 32
 - velocity, 677
- phonon, 270
- Piola-Kirchhoff stress tensor, 73
- plane
 - strain, 71, 81, 104
 - stress, 81, 104
- plastic dissipation, 118
- plate model, 455
- plate reorganization, 545
- plume
 - head, 397
 - tail, 397
- point mass potential, 12
- point source approximation, 153
- Poisson's
 - equation, 18
 - ratio, 101
- polar decomposition, 693
- polar moment, 13
- polflucht, 34
- poloidal, 45
 - potential field, 706
- pore pressure factor, 121
- postglacial rebound, 263
- postrift stage, 419
- potential, 12
 - anomaly, 20
 - temperature, 30
- power law, 179
- power per degree ℓ and unit area, 705
- Prandtl number, 235
- Pratt isostasy, 8
- preliminary reference Earth model, 23
- pressure, 6
 - dynamic, 176
- pressure solution, 207
- primary creep, 191
- primitive, 409
 - mantle, 274
- primordial reservoir, 409
- principal
 - axis system, 75
 - strain axes, 71, 669
- principal moments of inertia, 13
- pro wedge, 572
- probabilistic seismic hazard assessment, 489
- probability density function, 709
- process zone, 127, 150
- product
 - cross, 689
 - dot, 688
 - dyadic, 690
 - inner, 688
 - vector, 689
- product rules, 684
- propagating wave, 677
- pseudotachylites, 127
- pull-apart basins, 472
- pulselike ruptures, 158
- pure shear, 82
- purely deviatoric deformation states, 82
- Pythagoras, 676
- quadratic elongation, 669
- quality factor, 192
- quasi-static approach, 164
- R shears, 472
- radial
 - anisotropy, 106, 382
 - correlation functions, 54
 - strain, 101
- radian, 676
- radiation, 269
- radiation efficiency, 154
- radioactive decay, 276
- radiometric dating, 276
- radius of curvature, 111
- rake, 72
- rank n tensor, 701
- rank sorted, 710
- rare earth element, 274
- rate-and state-dependent friction, 160
- Rayleigh
 - number, 249
 - number, bottom-heated, 304
 - number, internal heating, 309
 - number, local, 315
 - number, power-law-based, 332
 - waves, 24
- Rayleigh-Bénard problem, 238, 303
- Rayleigh-Taylor instability, 670
- real surface spherical harmonics, 703
- receiver functions, 465
- recovery, 210
- recrystallization, 211
- reference
 - ellipsoid, 16
 - geoid, 14, 16
 - gravitational acceleration, 15
- reference frame
 - Eulerian, 63
 - Lagrangian, 63
- reflection, 22
- refraction, 22
- regularization, 61, 694
- relative sea level, 266
- relaxation
 - of stress, 187
 - time, 185
- relaxed modulus, 190
- reservoir
 - ancient, 409
 - early enriched, 409
 - primordial, 409
- residence time, 314
- residual topography, 26, 362
- resonance, 193
 - frequency, 193
 - peak, 193
- retardation time, 189
- retro wedge, 572
- Reuss average, 224
- reversal timescale, 651
- reverse faults, 76
- Reynolds number
 - magnetic, 236
 - thermal, 249
 - viscous, 234
- Reynolds transport theorem, 663
- rheology, 98
- Richter rolls, 459
- ridge
 - push, 257, 297
 - transform faults, 475
- Riedel structures, 472
- rift, 415
- rifting
 - active, 416
 - passive, 416
- right-hand rule, 689, 695
- right-handed system, 695
- right-stretch tensor, 669
- Ring of Fire, 546
- ringing, 703
- risk, 469
- rock
 - equivalent topography, 114
- rock uplift, 592
- rock-equivalent topography, 25
- Rodrigues's formula, 704
- root mean square (RMS), 707
- roots, 464
- Rossby number, 236
- rotation, 61
 - tensor, 669
 - vectors, 685
- rotation-rate tensor, 70
- rules of plate tectonics, 42
- rupture, 171
 - barriers, 538
- S-tectonites, 71
- Sarle's bimodality coefficient, 710
- scalar
 - field, 12, 685
- schlieren, 411
- scientific notation, 675
- screw dislocation, 126
- sea level equation, 266
- sea surface
 - equipotential, 16
 - height, 16
- seafloor spreading hypothesis, 38
- seaward dipping reflectors, 419
- second
 - deviatoric invariant, 179
 - invariant, 78
- secondary creep, 191
- sectoral spherical harmonics, 705
- secular cooling, 275, 641
- seismic
 - anisotropy, 24, 106, 385, 386
 - coupling, 89
 - gap, 489
 - moment, 85, 133
 - parameter, 31
 - period, 141
 - potency, 85
 - wave equation, 665
- seismicity, 133
- seismogram, 133
- seismotectonic, 89
- self-consistent average, 119
- self-consistent
 - averaging, 225
- self-organized critical state, 136
- sensitivity kernel, 362
- separation of variables, 186, 288, 305
- series approximations, 682
- serpentine, 446, 476
- shallow slip deficit, 493
- shape
 - factors, 71, 670
 - preferred orientation, 384
 - ratios, 78

- shear
 - delamination, 582
 - modulus, 101
 - strain, 65
 - strain, engineering, 65
 - stress, 72
 - viscosity, 176
 - zone, 211
- similarity variable, 283
- simple shear, 82, 102
- singular matrix, 692
- singular value decomposition, 694
- singularity, 685
- sinistral, 472
- site amplification, 133
- skew symmetric, 68
- skewness, 710
- skin depth, 288
- slab
 - flat, 612
 - ponding, 557
 - pull, 297, 527
 - stagnation, 557
- slabets, 559
- slip law, 161
- slip-predictable model, 141
- slip-weakening law, 157
- slope, 680
- slow slip event, 172, 536, 539
- small amplitude, 304
- small-scale convection, 458
- Snell's law, 22
- solenoidal, 70, 706
- solid spherical harmonics, 19
- solidus, 451, 517
- soliton, 452
- solution of the wave equation, 679
- Spearman rank-order correlation, 711
- spherical harmonics, 703
- spheroid, 13
- spidergram, 274
- spin tensor, 70
- spreading
 - centers, 439
 - ridges, 439
- spring slider system, 164
- square matrix, 687
- stagnant lid regime, 330
- stagnation
 - distance, 254
 - point, 254
- staircase trajectory, 126
- standard deviation, 707, 709
- standard linear solid, 189
- state variable, 160
- static
 - equilibrium state, 73
 - fatigue, 119
 - friction, 140
 - stress drop, 141, 157
- statistical moments, 710
- Stefan problem, 288
- stepovers, 472
- stick-slip cycle, 140
- sticky air, 251
- Stokes
 - condition, 178
 - equation, 241, 242
 - law, 248
 - Navier equation, 233
 - sinker velocity, 248
 - theorem, 701
- Stokeslet, 248
- storage capacities, 371
- strain, 64
 - intensity, 670
 - partitioning, 472, 481, 484
 - tensor, 66, 67
- strain rate, 62
- strain-dependent weakening, 183
- strain-rate tensor, 69, 70
- strange attractor, 41
- stream
 - function, 252
 - power, 595
- strength, 228
 - excess, 157
- stress, 72
 - compatibility equations, 667
 - concentration factor, 117
 - differential, 76
 - diffusivity, 200
 - intensity factor, 118
 - relaxation test, 187
 - shadow, 123
 - singularity, 118
 - tensor, 72
 - tensor, Cauchy, 72
 - tensor, Piola-Kirchhoff, 73
- stress drop, 140
 - dynamic, 158
 - static, 157
- stretch, 64
- stretching factors, 421
- strict isopycnic hypothesis, 463
- strike, 72
- strike-slip faults, 76
- strip-yield model, 150
- Student's
 - t -distribution, 711
 - t -test, 709
- stylolites, 207
- subadiabatic, 323
- subduction
 - channel, 500, 601
 - delamination, 582
 - initiation, 350, 544
 - orogeny, 568, 570
- subduction-transform edge
 - propagator, 487
- subgrain
 - boundaries, 210
 - rotation, 211
- summation operator, 682
- supercycles, 492
- superswells, 362
- surface
 - gravity anomaly, 21
 - uplift, 591
 - waves, 24
- symmetric tensor, 67
- synchronous
 - deformation model, 221
 - model for recrystallization, 223
- syn-rift deposits, 417
- synthetic motion, 472
- système international (SI), xxv
- tapering, 703
- Taylor expansion, 66, 682
- tectonic
 - accretion, 500
 - erosion, 500
 - melanges, 601
 - tremor, 539
- tectosphere, 463
- tensor, 70, 685, 701
 - antisymmetric, 68
 - deformation rate, 69
 - displacement gradient, 68
 - finite strain, 66
 - infinitesimal strain, 67
 - invariants, 79
 - left-stretch, 669
 - norm, 85, 687
 - right-stretch, 669
 - rotation rate, 70
 - strain, 66
 - strain-rate, 69, 70
 - stress, 72
 - symmetric, 68
- tesseral functions, 705
- test volume, 74
- Theia, 325
- thermal
 - blanketing, 350
 - boundary layer, 28, 286, 310
 - catastrophe, 642
 - conductivity, 270
 - diffusivity, 278
 - expansivity, 30, 249, 304
 - lithosphere, 293
 - parameter, 504
 - relaxation, 419
 - Reynolds number, 249
 - runaway, 543
- thermochemical boundary layer, 464
- thermochronology, 592
- thermodynamic pressure, 177
- thermostat, 641
- thin viscous sheet models, 255
- third invariant, 80
- Thixotropy, 177
- thrust faulting, 76
- time-predictable model, 141
- toroidal, 45
 - flow, 338
 - potential field, 706
- torque balance, 373
- total derivative, 680
- trace, 79, 687
- traction, 72
- transcurrent faults, 467
- transfer
 - faults, 467
 - function, 114
- transform faults, 467
- transformational faulting, 543
- transient time, 559
- translation, 61
- transpression, 472
- transension, 472
- transverse strains, 102
- transversely isotropic, 106
- traps, 398
- true polar wander, 17, 47, 266
- turbulent
 - mixing, 411
- twinning, 207
- two-phase flow, 452
- U-Pd dating, 276
- ultralow-velocity zone (ULVZ), 413
- ultracataclasites, 128
- unclamping, 123
- underdetermined, 694
- undershoot, 158
- uniaxial, 71
 - compression, 83
 - strain, 102, 104
 - stress, 101, 102
 - tension, 83
- unit
 - circle, 678
 - hyperbola, 678
 - vectors, 686
- universal flexural profile, 112
- universal gravitational constant, 10

- unrelaxed modulus, 190
- Urey ratio, 641
- vacancies, 209
- variance, 707
 - reduction, 694
- vector, 6, 685
 - calculus, 698
 - field, 61, 685
 - norm, 686
 - spherical harmonics, 706
- vectorial tomography, 382
- velocity, 62
 - gradient matrix, 69
 - strengthening, 162
 - weakening, 162
- viscoplastic, 184
- viscosity, 28
 - frequency-dependent, 195
- viscous, 528
 - anisotropy, 176
 - dissipation, 179
 - drag, 234
- Voigt
 - average, 107, 224
 - notation, 106
 - Reuss-Hill average, 225
- volumetric heating rate, 273
- von Mises
 - criterion, 181
 - plasticity, 182
 - stress, 80
- von Neumann boundary
 - conditions, 279
- vorticity, 706
 - tensor, 70
- vulnerability, 470
- Wadati-Benioff zone, 496
- water
 - concentration, 217
 - fugacity, 213, 217
- wattmeter model, 221, 223
- wave
 - number, 305, 677
 - solution, 679
 - vector, 679
- wave length, 677
- wavelet analysis, 703
- weakly isopycnic, 464
- weakly isopycnic state, 462
- WGS-84, 14
- white smokers, 451
- Wilson cycles, 47
- world
 - geodetic system, 12
 - stress Map, 93
 - uncertainty, 47
- xmas-tree diagram, 231
- yield criteria, 181
- Young's modulus, 101
- zero
 - curl, 45
 - divergence, 45
- zonal spherical harmonics, 19, 705