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CHAPTER 1

Overview

Most of the fundamental ideas of science are essentially simple, and may, as a rule, be expressed in a language comprehensible to everyone.
—ALBERT EINSTEIN

1.1 The legacy of Maxwell

Historians tell us that the most important victories of the greatest Roman generals were celebrated with a triumph. Adoring crowds would line the streets, cheering wildly in support of their hero as he passed by in a grand procession. But the Romans astutely realized the need for a counterpoise, so an enslaved person would ride with the general, whispering in his ear, "All glory is fleeting."

All glory is fleeting—and never more so than in theoretical physics. No sooner is a triumph hailed but unforeseen puzzles emerge that could not possibly have been anticipated before the breakthrough. The mid-nineteenth-century reduction of all electromagnetic phenomena to four equations, the Maxwell equations, is very much a case in point.

The 1861 synthesis provided by Maxwell's equations united electricity, magnetism, and optics, showing them to be different manifestations of the same field. The theory accounted for the existence of electromagnetic waves, explained how they propagated, and showed that the propagation velocity is $1/\sqrt{\epsilon_0\mu_0}$ (ϵ_0 is the permittivity, and μ_0 the permeability, of free space). This combination is precisely equal to the measured speed of light, even though the permittivity and permeability constants are determined, respectively, from electrostatic and magnetostatic measurements. Light is therefore a form of electromagnetic radiation! The existence of electromagnetic radiation was later verified by brilliant experiments carried out by Heinrich Hertz in 1887, in which the radiation was both generated and detected.

But when one probed, Maxwell's theory, for all its success, had disquieting features. For one, there seemed to be no provision in the theory for allowing the velocity of light to change with the observer's velocity. The speed of light was aways $1/\sqrt{\epsilon_0\mu_0}$. A related point is that simple Galilean invariance was not obeyed; i.e., absolute velocities appeared to affect the physics, something that had not been seen before. Lorentz and Larmor in the late nineteenth century discovered that Maxwell's equations, in fact, had a simple mathematical velocity transformation that did leave them invariant, but it was *not* Galilean, and most bizarrely, it involved changing the *time.* The non-Galilean character of the transformation equation

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relative to the "aetherial medium" thought to host the waves was put down, a bit vaguely, to electromagnetic interactions between charged particles that truly changed the length of the object. In other words, the non-Galilean transformation was somehow electrodynamic in origin. As to the time change . . . well, one would just have to put up with it as an aetherial formality.

All was resolved in 1905 when Einstein showed how, by adopting as postulates (i) the speed of light is constant in all frames (as had already been indicated by a body of very compelling experiments, most notably the famous Michelson-Morley null result); and (ii) the truly essential Galilean notion that relative uniform velocity cannot be detected by any physical experiment, that the "Lorentz transformations" (as they had become known) must follow. In the process, electromagnetic radiation in vacuo took on a reality all its own: it no longer needed to be viewed as some sort of aetherial displacement. The increasingly problematic aether host medium could be abandoned: the waves were self-sustaining, "hosted" by the vacuum itself. Most stunningly, *all* equations of physics, not just electromagnetic phenomena, had to be invariant in form under the kinematic Lorentz transformations, even with the peculiar relative time variable. The transformations themselves really had nothing to do with Maxwellian electrodynamics; they were far more general. It just so happened that the Maxwell equations were the first such set discovered that adhered to the correct transformation laws of physics, because they pertained directly to light propagation. The other equations had to be fixed up!

These ideas, and the consequences that ensued collectively from them (e.g., the equivalence of energy and inertia) became known as *relativity theory*, in reference to the invariance of form with respect to relative velocities. (Einstein preferred the term "invariance theory," though it never caught on.) The relativity theory that emerged from the excavation of the Lorentz transformations from within the Maxwellian formulation of electrodynamics is rightly regarded as one of the crown jewels of twentieth-century physics: in other words, a triumph.

1.2 The legacy of Newton

Another triumph, another problem. If indeed all of physics had to be compatible with relativity, what of Newtonian gravity? By the early twentieth century, it had been verified with great precision, even predicting the existence of a new distant planet (Neptune) in the solar system. But Newtonian gravity is manifestly *not* compatible with relativity, because the governing Poisson equation

$$\nabla^2\Phi = 4\pi G\rho \tag{1.1}$$

implies instantaneous transmission of changes in the Φ field from source to observer. (Here Φ is the Newtonian potential function, G the Newtonian gravitational constant, and ρ the mass density.) Wiggle the density locally, and throughout all space there must *instantaneously* be a wiggle in Φ, as given by equation (1.1). Gravity was an outlier from relativity theory, a loner status it retains today in its obstinate refusal to accommodate quantum field theory.

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In Maxwell's theory, the *electrostatic* potential also satisfies its own Poisson equation, but the appropriate time-dependent potential obeys a wave equation:[1]

$$\nabla^2\Phi - \frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2} = -\frac{\rho}{\epsilon_0}, \tag{1.2}$$

and solutions of this equation do not propagate instantaneously throughout all space. They propagate at the speed of light c. At least in retrospect, this is rather simple. Might it not be the same for gravity?

Alas, no. The problem is that the source of the signals for the electric potential field, i.e., the charge density, behaves differently from the source for the gravity potential field, i.e., the mass density. The electrical charge of an individual bit of matter does not change when the matter is in motion, but the gravitating "mass," in the form of relativistic energy, does. This deceptively simple detail complicates everything. Relativistic energy, not just ordinary matter, must also be a source of a gravitational field, and this includes the distributed energy of the gravitational field itself. In other words, any relativistic theory of gravity would have to be nonlinear. In such a time-dependent theory of gravity, it is not even clear a priori what the appropriate mathematical objects should be on the right side or the left side of the wave equation. Come to think of it, should we be using a wave equation at all?

1.3 The need for a geometric framework

In 1908, the mathematician Hermann Minkowski presented a remarkable insight. He argued that one should not view the Lorentz transformations as a mere set of rules for how space and time coordinates change from one constant-velocity kinematic reference frame to another. Minkowski's idea was that these coordinates should be regarded as living in their own sort of pseudo-Euclidean geometry—a *spacetime,* if you will: Minkowski spacetime. There was even a ready-made mathematical structure at hand to be used: hyperbolic geometry.

To understand the motivation for this, start simply. We know that in ordinary Euclidean space we are free to choose any set of Cartesian coordinates we like, and it can make no difference to the description of the space itself, e.g., in measuring how far apart objects are. If (x, y) is a set of Cartesian coordinates for the plane, and (x', y') another coordinate set related to the first by a rotation (including a possible shift or origin), then

$$dx^2 + dy^2 = dx'^2 + dy'^2; \tag{1.3}$$

i.e., the distance between two closely spaced points is the same number, regardless of the coordinates used. The quantity $dx^2 + dy^2$ is said to be an *invariant* under rotational transformations.

Now, an abstraction. From a mathematical viewpoint, there is nothing particularly special about the use of $dx^2 + dy^2$ as our metric for closely spaced points. Imagine a space in which the metric invariant was $dy^2 - dx^2$. We needn't worry about the plus/minus sign;

[1] The knowledgeable reader will recognize that this form relies on the choice of the Lorenz gauge, in which causality in the potential is most transparent.

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mathematically speaking, an invariant is an invariant. Indeed, the hyperbolic geometry arising from this form of invariant had been studied by Bolyai and Lobachevsky (and was known to Gauss) long before. With $dy^2 - dx^2$ as our prescribed invariant, we are, in fact, describing a hyperbolic Minkowski space, with $dy = c\,dt$ and dx a space interval as before. Under the simplest form of the Lorentz transformations, $c^2dt^2 - dx^2$ emerges as an invariable quantity, which is precisely what is needed to guarantee that the speed of light is always constant . . . an invariant, if you will. With this formulation, the interval $c^2dt^2 - dx^2$ always vanishes for light propagation along the x direction, in any coordinates related by a Lorentz transformation, which we may think of as separate observers, if we so wish. The obvious metric generalization

$$c^2dt^2 - dx^2 - dy^2 - dz^2 = 0 \tag{1.4}$$

will guarantee the same is true in any of the three spatial directions. A better way to express this is that our Universe is a structure of four dimensions, but its hyperbolic geometry picks out one of these dimensions to appear with a different sign in formulating the invariant metric interval. This component of dimensional separation, therefore, has a different character from the other three and is perceptively experienced in its own distinct way. We feel the need to separate this one particular form of dimensional separation from the others; we cannot "spatially" point to it. Instead, we give it its own name: time. And it is not just our subjective consciousness that treats this dimension separately. When we actually solve a partial differential equation (PDE) like the Laplace equation, with all coordinates on an equal Euclidean footing, we specify the value of the function to be solved for (or its gradient) on closed surfaces everywhere and allow the PDE to fill in the rest of the space. But when we solve a PDE with the structure of the wave equation, in a mathematically hyperbolic space, we fix the special timelike coordinate and specify the function (and its time derivative) on all the spatial (nontime) surfaces. Then, we solve an initial value problem going forward in the timelike direction. The Laplace equation is an example of what is known as an *elliptical PDE;* the wave equation, in contrast, is known as a *hyperbolic PDE.* Our conscious reality, flowing from one moment to the next in some external, fourth dimension of separation, "feels" like the solution to a hyperbolic PDE.

To summarize: Minkowski took what seemed to be a purely kinematic requirement—that the speed of light needs to be a universal constant—and gave it a geometric interpretation in terms of an invariant quantity in a hyperbolic Minkowski space—rather, to be more explicit, Minkowski *spacetime.*

Pause. And so what? Call it whatever you like. Who needs obfuscating mathematical pretense? Eschew obfuscation! One could argue that lots of things add together quadratically, and it is not particularly helpful to regard them as geometric objects. The Lorentz transform stands on its own, thank you very much. That was very much Einstein's initial take on Minkowski's pesky mathematical meddling with his theory.

However, as the reader has no doubt sensed, it is the geometric viewpoint that is truly deeper and much more fundamental, a perspective that time (our fourth dimension!) has vindicated. In Minkowski's 1908 tour-de-force lecture, we find more than just a simple analogy or mathematical pretense.[2] We can understand *why* the kinematics is changing

[2] Minkowski, H. 1908, Space and Time. The 80th Assembly of German Natural Scientists and Physicians, Cologne, September 21, 1908, 133.

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from Galilean-Newtonian: it stems from the core underlying geometry of the spacetime itself. In Minkowski's work, we find the first presentation of relativistic 4-vectors and tensors, of the Maxwell equations in manifestly covariant form, and the remarkable realization that the magnetic vector and electrostatic potentials combine seamlessly to form their own 4-vector. Gone are the comforting kinematic tools and pedagogical aids of Einstein's 1905 relativity paper, the clocks, the rods, the quaint Swiss trams. This is no mere "überflüssige Gelehrsamkeit" (superfluous erudition), Einstein's dismissive term for the whole business.

It took until 1912 for Einstein to finally change his viewpoint. Five years earlier, in 1907, he had had the seed of a very important idea but had been unable to build upon it. It was in 1912 that Einstein discovered what it was that he had discovered. His great revelation was that *the effect of the presence of matter (or its equivalent energy) throughout the Universe is to distort Minkowski's spacetime and that this embedded geometric distortion manifests itself as the gravitational field.* Minkowski spacetime is not some mere mathematical formalism; it is physical stuff, you can mess with it, and it takes real, tangible energy to do that. The spacetime distortions of which we speak must become, when sufficiently weak, familiar Newtonian gravitational theory. You thought gravity was a dynamic force? No. Gravity is a purely geometric phenomenon.

Now that is one big idea, one whose implications physicists are still trying to fully take in. How did Einstein make this leap? What was the path that led him to change his mind? Where did the notion of a gravity-geometry connection come from?

It came from that "important idea" he had in 1907, a year before Minkowski published his geometric interpretation of relativity. In a freely falling elevator, or more safely in an expertly piloted aircraft executing a ballistic parabolic arc, one feels weightless. That is, the effect of gravity can be made to disappear locally in the appropriate reference frame—more precisely, the right coordinates. Let that sit. The fundamental physics of gravity must be intimately associated with coordinate transformations! This is because gravity has exactly the same effect on all types of mass, regardless of its bulk or elemental composition. But how does one go from "intimately associated with coordinate transformations" to a rigorous theory? In 1912, Einstein, with the help of his mathematician friend and colleague Marcel Grossman, grasped that this is precisely what would naturally emerge from a theory in which test objects were responding to background geometric distortions instead of to an applied force. In a state of free fall, gravity is in effect absent, and we locally return to the environment of an undistorted ("flat," in mathematical parlance) Minkowski spacetime, much as a flat Euclidean tangent plane is an excellent local approximation to the surface of a curved sphere. That is why it is easy to be fooled into thinking that the Earth is flat, if your view is too local. "Tangent plane coordinates" on small-scale road maps locally eliminate spherical geometry complications, but if we are flying from London to Hong Kong, Earth's curvature is important. Einstein's notion that the effect of gravity is to cause a geometric distortion of an otherwise flat Minkowski spacetime, but that it is always possible to find coordinates in which local distortions may be eliminated *to leading local order,* is the foundational insight of general relativity. It is known as the *equivalence principle.* We will have more to say on this topic.

Spacetime. Space*time.* Once again, bringing in time, you see, is everything. Who would have thought of doing that in a geometric theory? The theory of non-Euclidean geometry, as developed by the great mathematician Bernhard Riemann, begins with just the notion we've been discussing, that any distorted space looks *locally* flat. Riemannian geometry is thus the natural mathematical language of gravitational theory, a point impressed upon Einstein by

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Grossman. Indeed, half a century before Einstein, Riemann himself had the notion that gravity might arise from a non-Euclidean curvature in a space of three or more dimensions! He got nowhere, because time was not part of his geometry. Riemann was thinking only of space. It was the genius of Minkowski, who by showing how to incorporate time into a purely geometric theory, allowed Einstein to take the crucial next step, freeing himself to think of gravity in geometric terms, without having to agonize over whether it made any sense to have time as part of a geometric framework. In fact, the Newtonian limit is reached *not* from the leading-order curvature terms in the spatial part of the geometry, but from the leading-order "curvature" (if that is the word) of the dimension we associate with the most natural *time* coordinate. And that is why Riemann failed.

Indeed, as we have already noted, it is best to think of time not as something that has been *incorporated* into a four-dimensional geometry but as something that *emerges* formally from a natural four-dimensional hyperbolic space, which is our Universe in its pristine vacuum form, with metric $dw^2 - dx^2 - dy^2 - dz^2$. The dimension that comes in with the different sign, dw, is the separation we *experience* as time. We move through w much more rapidly than through x, y or z, which is why we think the "speed of light" is large and why w and x, y, z seem so decoupled from one another.[3]

In brief: Riemann created the mathematics of non-Euclidean geometry. Minkowski realized that the natural language of the Lorentz transformations was neither electrodynamic, nor even kinematic, it was ultimately geometric. But you need to identify time as a component of the geometric interpretation! Einstein took the great leap of realizing that the force we call gravity arises from the distortions of Minkowski's flat spacetime which are created by the presence of mass/energy.

Well done. You now understand the conceptual framework of general relativity, and that is itself a giant leap. From here on, it is just a matter of the technical details. But then, you and I also know how to draw like Leonardo da Vinci.

It is just a matter of the technical details.

[3] In the time it takes us to walk a meter or two, we have moved through the time dimension the equivalent of the distance from Earth to the Moon.

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