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INTRODUCTION

Statistics has become known in the twentieth century as the mathematical tool for analyzing experimental and observational data. Enshrined by public policy as the only reliable basis for judgments as to the efficacy of medical procedures or the safety of chemicals, and adopted by business for such uses as industrial quality control, it is evidently among the products of science whose influence on public and private life has been most pervasive. Statistical analysis has also come to be seen in many scientific disciplines as indispensable for drawing reliable conclusions from empirical results. For some modern fields, such as quantitative genetics, statistical mechanics, and the psychological field of intelligence testing, statistical mathematics is inseparable from actual theory. Not since the invention of the calculus, if ever, has a new field of mathematics found so extensive a domain of application.

The statistical tools used in the modern sciences have virtually all been worked out during the last century. The foundations of mathematical statistics were laid between 1890 and 1930, and the principal families of techniques for analyzing numerical data were established during the same period. The achievement of the founders of mathematical statistics—Pearson, Spearman, Yule, Gosset, Fisher, and others—was formidable, and is justly venerated by their scientific heirs. There is, however, another story to be told, of the background that made this burst of statistical innovation possible. The development of statistics was necessarily preceded by its invention. This was the contribution of the nineteenth century, the culmination of a tradition of statistical thinking that embraced writers from a variety of backgrounds working in areas which were otherwise unconnected.

The invention of statistics was the recognition of a distinct and widely applicable set of procedures based on mathematical probability for studying mass phenomena. Statistics was and continues to be seen as especially valuable for uncovering causal relationships where the individual events are either concealed from view or are highly variable and subject to a host of influences. The identification of statistics as a category of knowledge was first of all a scientific accomplishment, and not a purely mathematical one. To be sure, the central role of probability theory in the history as well as the logic of statistics is plain. But most of the probability mathematics needed by the founders of statistics had been available for almost a century, since the time of Laplace and Gauss. Indeed, practical techniques for the use of such mathematics in the analysis of numerical data were worked out with great sophistication during the early decades of the nineteenth century in the form of error theory, which was used widely in geodesy and observational astronomy. In retrospect, the history of error theory seems to abound in precursors to the principal accomplishments of mathematical statistics. If statistics were just mathematics, the "anticipations" of the error theorists would leave little basis for the claim that Quetelet, Lexis, and Galton were original thinkers in this field.

The identification of precursors, however, is almost always misleading, and it is no less so here. Identical mathematical formulations must yet be viewed as different if they are interpreted differently, especially when their purpose is scientific and not purely mathematical. As Stephen Stigler shows in his new book, the effective use of probabilistic techniques for estimating uncertainty in astronomy and geodesy was by no means sufficient to enable social scientists to apply similar analysis to the problems of their disciplines.[1] Decades after a sophisticated theory of errors had been developed, there remained the great problem of finding points of contact between the mathematical formulas of error theory and the scientific objects of the social and biological sciences, where variation was genuine and important. Once Galton and Pearson had in some measure solved that problem, the formulations of the error theorists could readily be seen to be applicable. In this new context, however, the analysis of error had become something quite different, a method for studying the causes of variation, and not just for measuring it. Only through their successful application to the refractory but rich problems of the social and biological sciences did the probabilistic techniques of error analysis grow into the powerful and flexible method of analysis that we know as mathematical statistics.

The study of variable mass phenomena had different origins from mathematical probability and error theory. It began instead with the development of numerical social science and the formation of what was regarded during the late nineteenth century as the characteristically statistical viewpoint. John Theodore Merz recognized this viewpoint in his 1904 *History of European Thought in the Nineteenth Century*, which

[1] See Stigler.

includes a chapter on "the statistical view of nature."[2] Merz's phrase referred not only to the mathematical techniques for analyzing data derived from probability theory, but also, and principally, to the strategy of investigation derived form the numerical science of society and of states. It was this science for which the term "statistics" had been adapted early in the nineteenth century. Its practitioners were initially called "statists," and only late in the nineteenth century did they assume the title of statisticians.

It is no accident that a quantitative, empirical social science of the nineteenth century should have given its name to a branch of applied mathematics, for the contribution of "statists" to the style of reasoning associated with the new mathematics was a fundamental one. To begin, statists familiarized the scientific world and the educated public with the use of aggregate numbers and mean values for studying an inherently variable object. Statistical writers persuaded their contemporaries that systems consisting of numerous autonomous individuals can be studied at a higher level than that of the diverse atomic constituents. They taught them that such systems could be presumed to generate large-scale order and regularity which would be virtually unaffected by the caprice that seemed to prevail in the actions of individuals. Since significant changes in the state of the system would appear only as a consequence of proportionately large causes, a science could be formulated using relative frequencies as its elemental data.

Practicing statists, of course, did not identify their enterprise with any doctrine so abstract as this. Theirs was a world of progress and discontent, of surveys and census figures, into which advanced mathematics and abstruse philosophy rarely intruded. They were, for the most part, reformers and bureaucrats. As nineteenth-century liberals, they were impressed by the power and dynamism of that complex entity, society, and were pleased to find evidence that it exhibited a stability which seemed not to be dependent on the intermittent wisdom of governing authorities. Hence they were delighted by the uniformity from year to year which was found to characterize not only natural events like births and deaths, but also voluntary acts such as marriages and even seemingly senseless and irrational phenomena like crime and suicide. From this was born Adolphe Quetelet's doctrine of "statistical law," which held that these regularities would continue into the future because they

[2] John Theodore Merz, *A History of European Thought in the Nineteenth Century* (1904-1912; 4 vols., New York, 1965), vol. 2, pp. 548-626.

arose necessarily from an underlying stability of the "state of society." Using statistics, it seemed to be possible to uncover general truths about mass phenomena even though the causes of each individual action were unknown and might be wholly inaccessible.

The doctrine that order is to be found in large numbers is the leitmotif of nineteenth-century statistical thinking. The regularity of crime, suicide, and marriage when considered in the mass was invoked repeatedly to justify the application of statistical methods to problems in biology, physics, and economics by writers like Francis Galton, James Clerk Maxwell, Ludwig Boltzmann, Wilhelm Lexis, and F. Y. Edgeworth. Indeed, the use of probability relationships to model real variation in natural phenomena was initially made possible by the recognition of analogies between the objects of these sciences and those of social statistics. We find them not only in veiled allusions; they are openly and explicitly developed in both popular and technical writings.

Thus this book is, in one sense, a history of the influence of ideas developed within social statistics. It is a study of the mathematical expression of what Ernst Mayr calls population thinking,[3] a phrase which points no less clearly than statistics to sources in the human sciences. As in all noteworthy cases of intellectual influence, however, the beneficiaries here were no mere passive recipients of social-scientific dogma. The leading characters in this story were "moral statisticians," economists, kinetic gas theorists, and biometricians. The objects of their work required them to find some way of studying mass phenomena profitably without first having to attain detailed knowledge of the constituent individuals. They were successful precisely because they were able to adapt existing methods and concepts to new objects. In doing so, they contributed as much to the statistical method as to their particular fields.

Just as statistical reasoning was closely associated with the idea of large-scale regularity during the nineteenth century, the history of statistical mathematics before 1890 is, for the most part, a history of the normal or Gaussian distribution. This is the familiar bell-shaped curve, known to nineteenth-century writers as the astronomical error law. Although it had earlier been used in connection with the classical "doctrine of chances," it became closely associated with astronomy as a consequence of its incorporation into the method of least squares for reducing astronomical observations. Since stellar objects have a real po-

[3] Ernst Mayr, *The Growth of Biological Thought: Diversity, Evolution, and Inheritance* (Cambridge, Mass., 1982), pp. 45-47.

sition at a given time, the existence of variation among observations was quite naturally interpreted as the product of error, and the error curve, accordingly, was conceived as descriptive of the imperfections of instruments and of the senses. The practical task of fitting curves of various sorts to astronomical observations, as well as the search for rigorous foundations for the method of least squares, led to much sophisticated mathematical work; but since the object of the exercise was always to manage or estimate error, there was little incentive to study variation for its own sake.

A major transition in thinking about the error law was initiated by the Belgian Adolphe Quetelet. Quetelet journeyed to Paris in 1823 to learn observational astronomy, but while there he was introduced to mathematical probability and was infected by the belief in its universal applicability, as championed by Laplace, Poisson, and Joseph Fourier. The new social science of statistics became for him a branch of the "social physics," patterned closely on celestial physics, for which he wished to lay the foundation. Every possible concept from physics was given a social analogue in this gradualist metaphysic of society, and the error law finally found its place in 1844 as the formula governing deviations from an idealized "average man." Quetelet interpreted the applicability of this law as confirmation that human variability was fundamentally error, but the effect of his discovery was to begin the process by which the error law became a distribution formula, governing variation which was itself seen to have far greater interest than any mere mean value.

The further development of statistical mathematics was, until the end of the nineteenth century, largely the result of work in other natural and social scientific disciplines. Quetelet's belief in the widespread applicability of the error law to variation of all sorts, although not his interpretation of it, won acceptance by the ablest workers on statistical mathematics of the late nineteenth century. James Clerk Maxwell, who learned of Quetelet's use of the error law from an essay review of one of Quetelet's books by John Herschel, proposed that the same formula governed the distribution of molecular velocities in a gas. He, along with Ludwig Boltzmann, made of statistical gas theory one of the great achievements of late nineteenth-century physics and formed an important part of the background to the new quantum theory as well as to Willard Gibbs's statistical mechanics. In social science, Wilhelm Lexis used similar formalism to provide a measure of the stability of statistical series, and Francis Edgeworth showed how error analysis and related techniques might be applied fruitfully to problems in economics such as index numbers. Francis Galton, who was introduced to Quetelet's ideas by the geographer William Spottiswoode, employed the error curve in his study of heredity and, in the end, found that his index of hereditary regression was in fact applicable as a general tool of statistical mathematics to the study of variation in data of all sorts.

At the same time as the mathematics of statistics was being advanced through its application to new objects, the statistical approach began to be seen as distinctive and even as challenging for conventional views of science and of natural law. Quetelet's contemporaries held no uniform view as to the nature of statistical science, but they were all in accord that their discipline consisted of the application of the tried and true method of natural science to a social object. The assertion of what may be called statistical determinism in relation to man and society, however, provoked a backlash of opposition to statistics and a critical analysis of the nature of statistical reasoning. On one level, this reassessment led to wide adoption of the view that statistics could provide little of scientific worth so long as attention was focused on mean values rather than on variation. More abstractly, critics of the idea of statistical law put forward by Quetelet and the historian Henry Thomas Buckle began to argue that the statistical method was inherently an imperfect one, applicable precisely because the remoteness or intrinsic variability of the constituent objects rendered exact deterministic knowledge inaccessible. The elaboration of this view, especially by kinetic theorist and social thinkers, inaugurated what Ian Hacking calls the "erosion of determinism," which has profoundly influenced the scientific world view of the twentieth century.

The modern field of mathematical statistics arose from the diverse applications to which statistical ideas and methods were put during the nineteenth century. It became, under Pearson, Fisher, and others, a mathematical resource for a variety of disciplines for which numerical data could be obtained through experiment or observation. Before 1890, however, and indeed for some decades thereafter, statistical methods and concepts were developed not by mathematicians but by astronomers, social scientists, biologists, and physicists. The development of statistical thinking was a truly interdisciplinary phenomenon for which mathematics had no priority of position; new ideas and approaches arose as a result of the application of techniques borrowed from one or more disciplines to the very different subject matter of another. The great pioneers of statistical thinking were widely read generalists, interested in historical, philosophical, or social issues as well as in their research areas. As Karl Pearson himself pointed out in some lectures he gave on the history of statistics:

> I ought, perhaps to apologise for carrying you so far afield in these lectures. But it is impossible to understand a man's work unless you understand something of his environment. And his environment means the state of affairs social and political of his own age. You might think it possible to write a history of science in the 19th century and not touch theology or politics. I gravely doubt whether you could come down to its actual foundations, without thinking of Clifford and Du Bois Reymond and Huxley from the standpoint of theology and politics. What more removed from those fields than the subject of differential equations? What more removed from morality than the theory of Singular Solutions? Yet you would not grasp the work of De Saint-Venant or Boussinesq unless you realised that they viewed Singular Solutions as the great solution of the problem of Freewill, and I hold a letter of Clerk-Maxwell in which he states that their work on Singular Solutions is epoch making on this very account![4]

Many contributed to a great range of scientific specialties, and all were alert to developments outside their own fields. The general development of a statistical method required effective communication between diverse studies, and this investigation reveals much about the links that have given science a certain measure of unity, making it more than a collection of isolated disciplines.

Statistics has been prominent not only on the margins between disciplines, but also in the nebulous and shifting border region that separates science from nonscience. Statistics has contributed essentially to a considerable expansion of the scientific domain, but it has been for two centuries or more a singularly problematical method of science. Probability was a suspect area of mathematics almost from the beginning, though from a purely technical standpoint its accomplishments were already impressive in the time of Montmort and De Moivre. Influential writers on error theory, such as Augustin Cauchy and James Ivory, ac-

[4] Karl Pearson, *The History of Statistics in the 17th and 18th Centuries Against the Changing Background of Intellectual, Scientific, and Religious Thought*, E. S. Pearson, ed. (London, 1978), p. 360.

cepted the method of least squares but refused to rest content with the probabilistic assumptions in its underpinnings.[5] Laplace himself was inspired by his ambitions for probability to redefine scientific certainty—to limit perfect knowledge to an omniscient but imaginary demon, and to insist that while events in the world are completely determined by preexisting causes, our knowledge of their outcome is necessarily subject to a certain domain of error. The statistical approach also presented serious problems for the kinetic gas theory, which reduced the deterministic laws of thermodynamics to mere regularities. While Maxwell and Boltzmann labored to incorporate the necessary refinements into the kinetic theory, others, such as Planck's student Zermelo, argued that this connection with statistics invalidated atomism altogether.[6]

That probability seemed to imply uncertainty clearly discouraged its use by physicists. From the standpoint of social science, on the other hand, statistical method was synonymous with quantification, and while some were skeptical of the appropriateness of mathematics as a tool of sociology, many more viewed it as the key to exactitude and scientific certainty. Most statistical enthusiasts simply ignored the dependence of statistical reasoning on probability, and those who acknowledged it generally stressed the ties between probability and that most ancient and dignified among the exact sciences, astronomy. The social science of statistics, in the hands of Quetelet and his admirers, constituted a self-conscious attempt to imitate the successful strategy of natural science. Statistical quantification in social science was more commonly seen as exemplary than as problematical, and the aspirations of statistics reveal much about what were taken to be the essential features of science during the nineteenth century.

Finally, the history of statistics sheds light on the relations between abstract science and what are often seen as its applications. In truth, practice was decidedly ahead of theory during the early history of statistics, and "pure" or abstract statistics was the offspring, not the parent, of its applications. The statistical techniques and approaches that were invented by Lexis, Edgeworth, Galton, Pearson, and their successors reflected at once the particular problems to which statistical methods had

[5] James Ivory, "On the Method of Least Squares," *Phil Mag*, 65 (1825), 1-10, 81-88, 161-168. On Cauchy, see Ivo Schneider, "Laplace and the Consequences: The Status of Probability Calculus in the 19th Century," in *Prob Rev*.

[6] Ernst Zermelo, "Über mechanische Erklärungen irreversibler Vorgänge," *Annalen der Physik*, 59 (1896), 793-801.

been applied and the context of ideologies and philosophical attitudes within which statistics had been pursued. Mathematics and physics may, as the Comtean hierarchy of the sciences suggests, have logical primacy over biological and social science, but historically the situation is much more complex and interesting.

There are, to be sure, special reasons why statistical theory should be more powerfully affected by considerations external to its specific subject matter than other areas of mathematics or science. Its main task, after all, has been to provide analytical methods by which practitioners of other disciplines can analyze their numerical data, and the statistician's quest is as much for useful techniques as for timeless truths. Nevertheless, the history of statistics should not be seen in this respect as utterly unique, but as an ideal type for one aspect of the historical process through which modern science has evolved.

Some changes of terminology almost always accompany the emergence of new areas of science, especially when, as in the present case, a significantly new style of thought is involved. A preliminary discussion of some key terms ought therefore to be helpful.

"Statistics" as a plural means to us simply numbers, or more particularly, numbers of things, and there is no acceptable synonym. That usage became standard during the 1830s and 1840s. It seems almost impossible now to talk about such numbers and numerical tables published before that time without using this anachronistic term. That all generations previous to the 1820s managed to get by without it reveals dimly how different was the world they lived in—a world without suicide rates, unemployment figures, and intelligence quotients. To be sure, this prenumerate age was not entirely deprived of statistical tables, but the great explosion of numbers that made the term statistics indispensable occurred during the 1820s and 1830s. The demands it placed on people to classify things so that they could be counted and placed in an appropriate box on some official table, and more generally its impact on the character of the information people need to possess before they feel they understand something, are of the greatest interest and importance.

In the nineteenth century, statistics designated an empirical, usually quantitative, social science. Before that, it was an ill-defined science of states and conditions. The term only came to be applied commonly to a field of applied mathematics in the twentieth century. "Statistics" gained a wide meaning, by which it could be used to refer to mass phenomena of any sort, mainly by analogy with its social object. After the mid-nineteenth century, it became common to investigate collective phenomena using what came to be called the statistical method, the method of reasoning about events in large numbers without being troubled by the intractability of individuals.

The forms of "probability" are less troublesome for the nineteenth century, though probability did not come into its own as a branch of pure mathematics until quite recently. Laplace's substitution of "calculus of probabilities" for the traditional "doctrine of chances" at the end of the eighteenth century was intended to make clear that rational belief or expectation rather than the outcome of games of chance was its proper object. Indeed, this view lingered on, but the mathematics of chance was almost unfailingly referred to as probability or calculus of probability after Laplace, and I see no reason to depart from the modern usage.

"Determinism" was until the mid-nineteenth century a theory of the will—a denial of human freedom—and some dictionaries still give this as its first meaning. Partly as a result of statistical discussion, it assumed during the 1850s and 1860s its modern and more general meaning, by which the future of the world is held to be wholly determined by its present configuration. It differs from fatalism in that it rests on natural laws of cause and effect rather than on some transcendant force. Its opposite, we may note, underwent a similar change. "Indeterminism" now refers to the view that some events in the world are not wholly determined by natural causes, but are, at least to some extent, irreducibly random or stochastic. Indeterminism may be contrasted with probabilism, which implies simply that our knowledge does not permit perfect prediction, though there may be no exceptions to complete causality in the world.

The phrase "law of large numbers" was coined by Poisson in 1835. To him, it referred to the proposition that the frequencies of events must, over the long run, conform to the mean of their probabilities when those probabilities fluctuate randomly around some fixed, underlying value. Virtually everyone else who recited the phrase in the nineteenth century made no distinction between Poisson's theorem and the one in Jakob Bernoulli's 1713 *Ars Conjectandi*, according to which the frequency of events must conform over the long run to the fixed probability governing each trial. It is most convenient here to adopt this undiscriminating usage, which really expresses simply the observed regularity of statistical aggregates. More recently, Poisson's phrase has been used to denote the rule that errors of mean values conform to the normal distribution. I know of nobody who used the phrase in this way before Emile Dormoy in 1874, whose work is mentioned in chapter 8.

There are, incidentally, a number of terms for the normal distribution, but, fortunately, there is little opportunity for confusion. The standard nineteenth-century phrases were "error curve" and "error law"; the eponymous "Gaussian" became common in the late nineteenth century, and "normal law" was used by Pearson in 1894, as was "standard deviation." The nineteenth-century measure of the width of a distribution was the "probable error," the magnitude of error which precisely one-half of the measurements would, over the long run, exceed. These few terms, fortunately, nearly exhaust the technical vocabulary of nineteenth-ceniury statistical thinking, apart from error theory and insurance mathematics. The technical and mathematical content in the following chapters is minimal, and, except for a few pages here and there, should be readily comprehensible even to readers with no mathematical training.

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