© Copyright, Princeton University Press. No part of this book may be distributed, posted, or reproduced in any form by digital or mechanical means without prior written permission of the publisher.

Contents

© Copyright, Princeton University Press. No part of this book may be distributed, posted, or reproduced in any form by digital or mechanical means without prior written permission of the publisher.

For general queries, contact webmaster@press.princeton.edu

© Copyright, Princeton University Press. No part of this book may be distributed, posted, or reproduced in any form by digital or mechanical means without prior written permission of the publisher.

For general queries, contact webmaster@press.princeton.edu

© Copyright, Princeton University Press. No part of this book may be distributed, posted, or reproduced in any form by digital or mechanical means without prior written permission of the publisher.

For general queries, contact webmaster@press.princeton.edu

© Copyright, Princeton University Press. No part of this book may be distributed, posted, or reproduced in any form by digital or mechanical means without prior written permission of the publisher.

Chapter One

Introduction

1.1 Time-Delay Systems

Time-delay systems, also called systems with aftereffect or dead-time, hereditary systems, equations with deviating argument, or differential-difference equations [42], are ubiquitous in practice. Some representative examples are found in

- chemical industry: rolling mills, milling processes, cooling systems, combustion systems, chemical processes,
- electrical and mechanical engineering: networked control systems, teleoperation, robotic manipulators, unmanned aerial vehicles,
- biomedical engineering: 3D printing/additive manufacturing, neuromuscular electrical stimulation,
- management and traffic science: traffic flow, supply chain, population dynamics.

The most common forms of time delay in dynamic phenomena that arise in engineering practice are actuator and sensor delays [133, 134]. Due to the time it takes to receive the information needed for decision-making, to compute control decisions, and to execute these decisions, feedback systems often operate in the presence of delays. As shown in figure 1.1, actuator and sensor delays are involved in feedback loops.

Another big family of time delays are transmission delays in networked control systems (NCS). As the name implies, NCSs are systems where the plant is controlled via a communication network. The typical feature of an NCS, as illustrated in figure 1.2, is that information (plant output, control input, etc.) is exchanged through a network among system components (sensor nodes, controller nodes, actuator nodes, etc.). In comparison with traditional feedback control systems, where the components are usually connected via point-to-point cables, the introduction of communication network media brings great advantages, such as low cost, reduced weight, simple installation/maintenance, and long-distance control. However, the performance of an NCS is heavily affected by the communication delays in both the controller-to-actuator and sensor-to-controller channels.

Although the presence of delays sometimes may have a stabilizing effect, for some special systems, time delay is a source of instability in general. In

For general queries, contact webmaster@press.princeton.edu

© Copyright, Princeton University Press. No part of this book may be distributed, posted, or reproduced in any form by digital or mechanical means without prior written permission of the publisher.

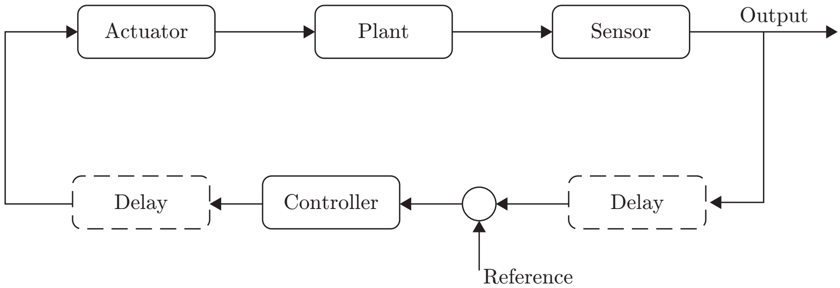

Figure 1.1. Feedback systems with actuator and sensor delay

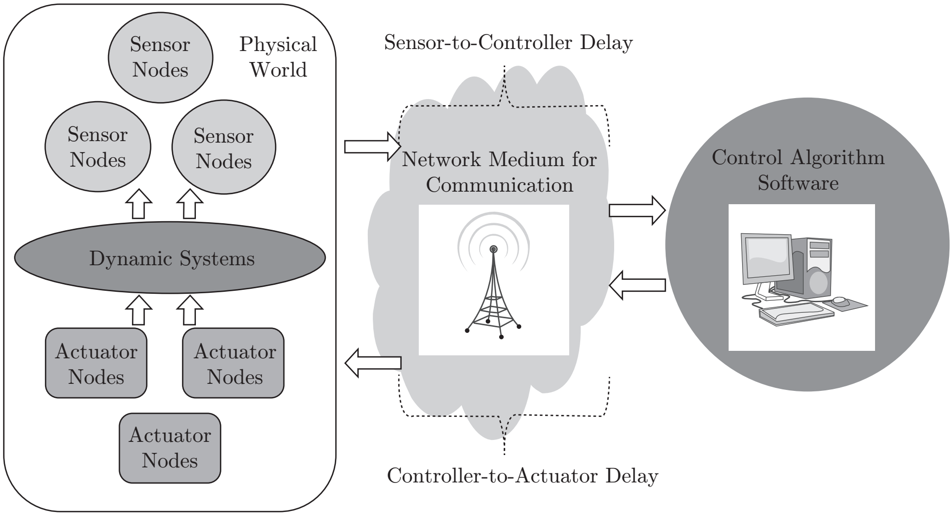

Figure 1.2. Networked control systems with transmission delays in controller-to-actuator and sensor-to-controller channels

accordance with Bode's performance limitation formulas, the existence of delays can severely hamper the performance of physical systems. As a result, the poor performance and catastrophic instability created by delays require the designer to take into account the delays in the control synthesis.

Besides their practical importance, time-delay systems also contribute many mathematical challenges to the analysis and design in control theory. As is well known, the time-delay systems belong to the class of functional differential equations that are infinite-dimensional, as opposed to the finite-dimensional ordinary differential equations (ODE). The state of the delay system is a function (or a vector of functions) rather than a vector. Its characteristic equation is not a polynomial, but involves exponentials. Thus the stability analysis requires Krasovskii

For general queries, contact webmaster@press.princeton.edu

© Copyright, Princeton University Press. No part of this book may be
distributed, posted, or reproduced in any form by digital or mechanical
means without prior written permission of the publisher.

functionals rather than Lyapunov functions. For the aforementioned reasons, time-delay systems are both of theoretical and practical importance, and they have drawn the attention of scholars including control engineers, scientists and mathematicians.

Over the past seventy years, delay systems have been an active area of research in control engineering, and major breakthroughs have been reported to deal with delays. The first systematic work of methodological significance may go back to 1946 in the paper by Tsypkin [129]. A couple of breakthroughs in stability analysis were reported in the late 1950s/early 1960s with the papers by Razumikhin in 1956 [115] and by Krasovskii in 1962–1963 [74, 75]. As a counterpart of Lyapunov methods for systems without delay, two main methods are named after their authors: the Krasovskii method of Lyapunov functionals and the Razumikhin method of Lyapunov functions. They are frequently used for stability analysis of time-delay systems. Almost during the same period, Smith introduced the celebrated Smith predictor in 1959 [119], which has seen success in industrial applications, and perhaps been second in popularity to proportional integral differential (PID) controllers, especially among process control practitioners [106].

The next well-known landmark in the control of time-delay systems occurred during the years 1978–1982, when the framework of "finite spectrum assignment" and the "reduction" approach were proposed by Manitius and Olbrot in 1979 [104], Kwon and Pearson in 1980 [94], and Artstein in 1982 [2]. Their basic idea was to make use of the future values of the state, i.e., the so-called "predictor state," in the feedback laws, to compensate for the delay. After the control signal reaches the plant, in a time interval equal to the delay, the state evolves as if there were no delay at all.

Another burst of research activity occurred in the 1990s after the introduction of linear matrix inequalities (LMI). Because of the unavailability of efficient numerical algorithms for the general form of LMI, most of the earlier works on the stability of linear systems via the Lyapunov method were formulated in terms of Lyapunov equations and algebraic Riccati equations. With the realization that LMI is a convex optimization problem, the direct Lyapunov method for linear ODEs leading to stability conditions in terms of LMIs became popular. The development of the efficient interior point method led to the formulation of many control problems and their solutions in the form of LMIs [21], which are capable of providing the desired stability/performance guarantees [41, 42].

In subsequent years, thousands of papers and dozens of books have appeared to deal with the control problem of time-delay systems by adopting various techniques coming from both finite-dimensional and infinite-dimensional systems. The most recent significant books and surveys summarizing the achievements are those by Gu and Niculescu [53], Niculescu [109], Gu, Kharitonov, and Chen [51], Zhong [136], Richard [116], Krstic [80] and Fridman [41, 42]. Still, many basic control problems for time-delay systems remain unsolved. Control of delay systems remains a very active area of research.

For general queries, contact webmaster@press.princeton.edu

© Copyright, Princeton University Press. No part of this book may be distributed, posted, or reproduced in any form by digital or mechanical means without prior written permission of the publisher.

1.2 Delay Compensation or Not

Among the basic control problems for time-delay systems, the most challenging ones may arguably be those in which the system has to be controlled through a delayed input, which are non-trivial even when the system is linear and even when the delay is constant. To illustrate the multitude of possible methods in control of time-delay systems, it is useful to consider control of the standard linear state-space model

$$\dot{X}(t) = AX(t) + BU(t-D), \tag{1.1}$$

where $X(t)$ is the state vector, $U(t)$ is the control input with the constant delay $D > 0$, and A and B are system and input matrices with appropriate dimensions. If the delay value is zero, the plant (1.1) is the standard linear time-invariant (LTI) system such that

$$\dot{X}(t) = AX(t) + BU(t) \tag{1.2}$$

and the standard control feedback is such that

$$U(t) = KX(t), \tag{1.3}$$

where K renders the closed-loop system matrix $A + BK$ Hurwitz.

1.2.1 No Compensation and State Delay

When the delay value is larger than zero and no compensation is employed for the input delay, the feedback law becomes

$$U(t-D) = KX(t-D), \tag{1.4}$$

which renders the closed-loop system such that

$$\dot{X}(t) = AX(t) + BKX(t-D), \tag{1.5}$$

$$= AX(t) + A_1X(t-D), \tag{1.6}$$

where $A_1 = BK$. The system (1.6) represents the simplest LTI system with a single discrete constant state delay. As a result, the control problem with input delay is transformed into a problem with state delay. Concentrating on the system (1.6), a large share of the literature investigates the control gain K and the maximum upper bound of delay that still guarantees the stability without compensation [49, 54, 109–111]. Furthermore, the control problem for the single discrete state delay can be nontrivially extended to the case of multiple discrete state delays such that $\dot{X}(t) = AX(t) + \sum_{k=1}^{N} A_kX(t-D_k)$, the case of

For general queries, contact webmaster@press.princeton.edu

© Copyright, Princeton University Press. No part of this book may be distributed, posted, or reproduced in any form by digital or mechanical means without prior written permission of the publisher.

distributed state delay such that $\dot{X}(t) = AX(t) + \int_{-D}^{0} A_d X(t+\theta)d\theta$, the case of time-varying state delay such that $\dot{X}(t) = AX(t) + A_1 X\big(t - D(t)\big)$, and so on. To deal with distinct types of state delay, a variety of LMI-based methods are proposed, such as the H_∞ control [45, 47, 125], the descriptor system approach [39], and so on. Different LMI techniques like the Schur complement lemma, Jensen's inequality, Wirtinger's inequality, and Halanay's inequality could result in delicate differences on the maximum upper bound of delay to ensure stability [38–47, 49–54, 118]. And the different Lyapunov-based functionals lead to different delay-dependent or delay-independent stability conditions, which have distinct conservatism. Furthermore, the sampled-data networked control systems, which are widely used in computer implementation nowadays, can also be transformed into time-delay systems and be analyzed using the methods for time-delay systems [95–100].

1.2.2 Delay Compensation by Predictor Feedback

As presented above, a large portion of the available results are finite-dimensional control laws without compensation for delay, whose applicability are limited to systems with "short" delays. In contrast, infinite-dimensional control laws are expected to compensate for the delay so that the stabilization of general systems with long delays can be achieved. Considering the LTI systems with input delay (1.1), instead of the control law (1.4) with no compensation, we aim to compensate for the delay by the control design such that $U(t-D) = KX(t)$ or $U(t) = KX(t+D)$. The effective method to compensate for the delay with arbitrarily finite length is the well-known "predictor feedback," which is also called the "finite spectrum assignment" or "reduction" method. Just as its name implies, the class of predictor feedback laws make use of the future (predicted) values of the state, so that they are able to compensate for the delay. In other words, after the control signal reaches the state of the plant, the delay is compensated for and the state evolves as if there were no delay at all. A challenge in developing predictor feedback laws is the determination of an implementable form for the future values of the state. This is overcome by a distributed integration of the actuator state over the past time interval. Having determined the predictor state, we then obtain the control law by replacing the state in a nominal state feedback law (which stabilizes the delay-free plant) with the state's predictor.

The first mathematically and practically significant development in control design for delay compensation was the introduction of the Smith predictor by Otto Smith in 1959 [119], in which the linear stable systems with constant input delays were addressed. Later on, the control design for the linear stable time-delay systems was extended to the linear unstable time-delay systems by Mayne in 1968 [105], Manitius and Olbrot in 1979 [104], and Kwon and Pearson in 1980 [94], in which the finite spectrum assignment technique for the compensation of input delays was introduced and the open-loop stability restriction was removed. In 1982 Artstein [2] systematized this methodology and named the method a "reduction" to emphasize that the stabilization problem for linear systems with

For general queries, contact webmaster@press.princeton.edu

© Copyright, Princeton University Press. No part of this book may be distributed, posted, or reproduced in any form by digital or mechanical means without prior written permission of the publisher.

input delays can be reduced to the stabilization problem of a delay-free (reduced) system.

Among the numerous difficulties in the design and analysis of predictor-based control laws, one key challenge is the construction of a Lyapunov-Krasovskii functional for stability analysis of the closed-loop system under predictor feedback. Such a construction is not easy because the overall state of the system consists of the finite-dimensional internal states of the plant and the infinite-dimensional actuator state. With the novel idea of treating the input delay as a first-order hyperbolic transport partial differential equation (PDE), in 2008 Krstic and Smyshlyaev [92, 93] introduced an infinite-dimensional backstepping transformation that enabled the construction of a Lyapunov-Krasovskii functional for linear systems with constant input delays under predictor feedback. Simultaneously, the first predictor feedback design for nonlinear systems was introduced by Krstic in 2008 [78]. After that, based on the PDE framework, the predictor feedback has seen a booming growth in compensating for various kinds of input delays in diverse system classes. For example, with the utilization of PDE and nonlinear control concepts [58, 60, 61, 62, 64, 65, 69–71, 86, 87, 130–131], for linear or nonlinear systems [4–20, 59, 63, 66, 67], delays are handled in forms of single-input delay [22–32], [153, 154, 159], multi-input delays [73, 127–128, 155–157], time-varying delays [33, 84], state-dependent delays [34, 35], distributed delays, and delays on the state, which are summarized in the books [15, 68, 80].

It is worth mentioning that the classic predictor-based controllers are feedback laws of an infinite-dimensional state and require care in their practical implementation [107]. Zhou developed a truncated predictor feedback that involves finite-dimensional state feedback in [137–148]. In [68] and the references therein, concentrating on both linear and nonlinear systems, Karafyllis and Krstic offered three different implementations of the predictor feedback law: the direct implementation, the dynamic implementation, and the hybrid implementation. The three different ways of implementing lead to the corresponding closed-loop systems with important differences.

1.2.3 Examples

For the purpose of more intuition and understanding of delay impact on systems, we offer several examples.

Example 1.1. **Networked Control Systems**

A general class of sampled-data networked control systems with time delay are shown in figure 1.3.

To concentrate on the input delay, we assume that the sensor is able to continuously measure the plant state and there is no transmission delay in the sensor-to-controller channel. Furthermore, we distinguish the actuator and the controller signals by denoting them as $V(t)$ and $U(t)$, respectively. As a result, the sampled-data NCSs in figure 1.3 are reduced to NCSs in figure 1.4.

For general queries, contact webmaster@press.princeton.edu

© Copyright, Princeton University Press. No part of this book may be distributed, posted, or reproduced in any form by digital or mechanical means without prior written permission of the publisher.

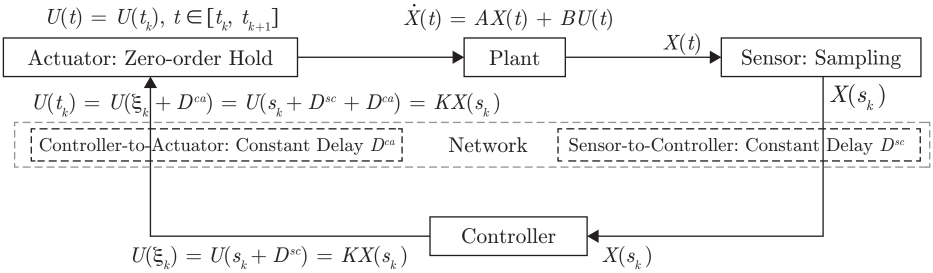

Figure 1.3. Sampled-data NCSs with controller-to-actuator and sensor-to-controller delays

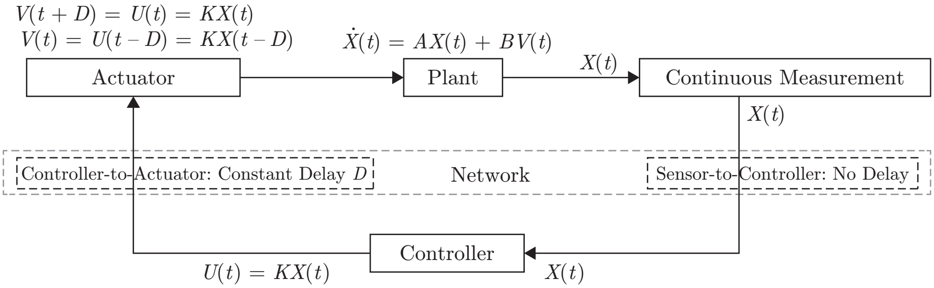

Figure 1.4. NCSs without compensation for input delay under continuous measurements

As shown in figure 1.4, there is no predictor feedback to compensate for the constant controller-to-actuator delay D. Thus we have

$$\begin{aligned}\dot{X}(t) &= AX(t) + BV(t)\\ &= AX(t) + BU(t-D)\\ &= AX(t) + BKX(t-D)\\ &= AX(t) + A_1X(t-D),\end{aligned} \tag{1.7}$$

where the input-delay problem is transformed into a state-delay problem. In this case, the delay D has to be short and the problem was discussed in section 1.2.1.

In contrast, for figure 1.5, the predictor feedback is brought in to compensate for the constant controller-to-actuator delay D. Thus we have, for $t \geq D$,

$$\begin{aligned}\dot{X}(t) &= AX(t) + BV(t)\\ &= AX(t) + BU(t-D)\\ &= AX(t) + BKX(t)\\ &= (A+BK)X(t).\end{aligned} \tag{1.8}$$

For general queries, contact webmaster@press.princeton.edu

© Copyright, Princeton University Press. No part of this book may be distributed, posted, or reproduced in any form by digital or mechanical means without prior written permission of the publisher.

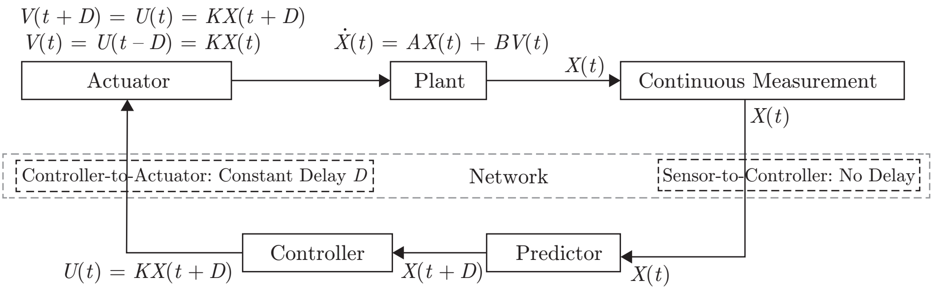

Figure 1.5. NCSs with compensation for input delay under continuous measurements

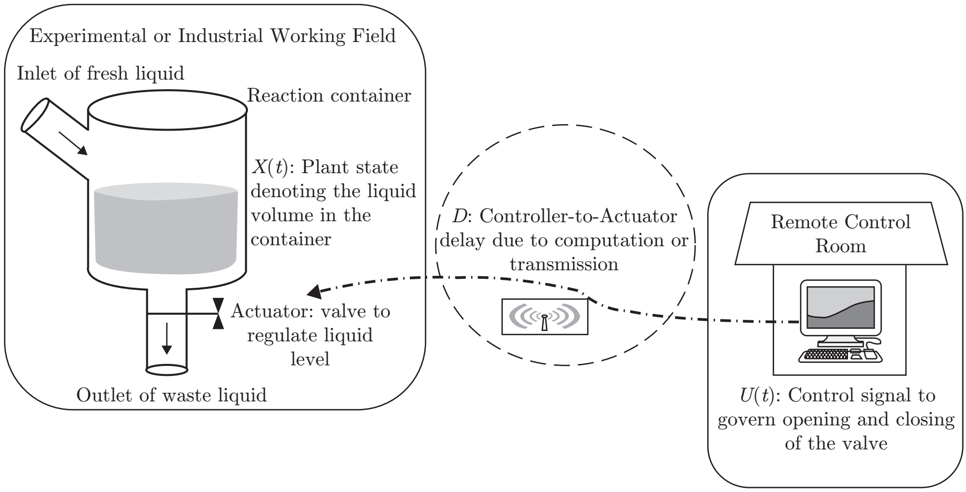

Figure 1.6. Liquid level systems in practice

As a result, the system evolves as if there is no delay at all after $t \geq D$, and the delay value D can be arbitrarily large, which was discussed in section 1.2.2.

Example 1.2. **Liquid Level Systems in Chemical Industry or Bioengineering**

As illustrated in figure 1.6, the liquid level systems exist widely in the chemical industry and bioengineering. For the liquid level system, the fresh liquid continuously enters the reaction container through the inlet, whereas the waste liquid is drained through the outlet, which is regulated by a valve. The plant state $X(t)$ represents the liquid volume in the container, and the control input is employed to govern the valve's opening and closing. The reaction container with the outlet valve is located in the working field, whereas the control command is produced from a remote control room, and they are connected by a

For general queries, contact webmaster@press.princeton.edu

© Copyright, Princeton University Press. No part of this book may be distributed, posted, or reproduced in any form by digital or mechanical means without prior written permission of the publisher.

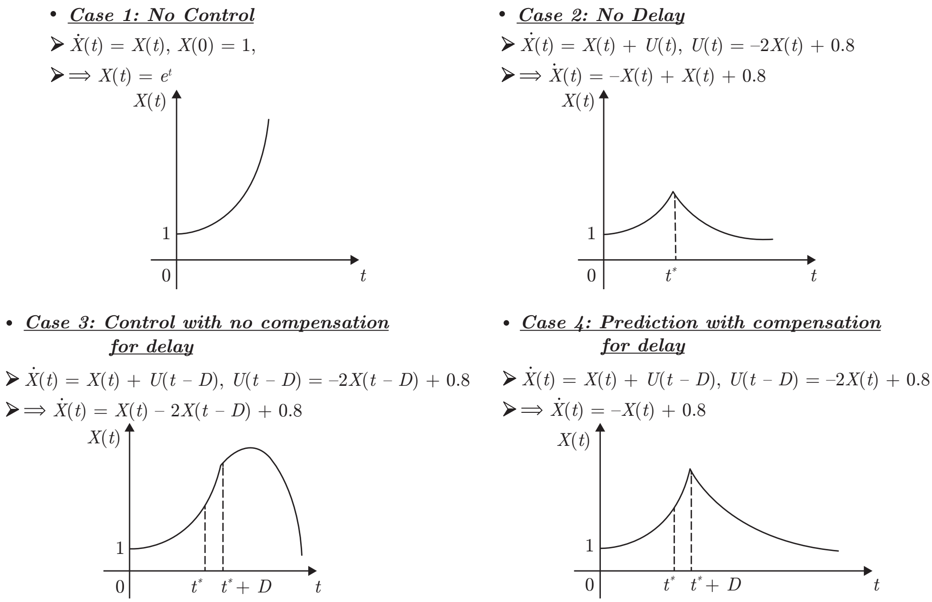

Figure 1.7. Four different cases of control of liquid level systems

long-distance communication network that is subject to a constant controller-to-actuator delay D. For simplicity, we assume that the liquid volume satisfies the following scalar dynamic equation with the initial condition $X(0)=1$:

$$\dot{X}(t)=X(t)+U(t-D). \tag{1.9}$$

As demonstrated in Case 1 of figure 1.7, the control input is zero such that $U(t-D)\equiv 0$. This means, if the outlet valve is kept closed, the liquid persistently enters the container through the inlet with an exponentially increasing volume.

For Case 2 of figure 1.7, there is no input delay such that $D\equiv 0$. This means, at a certain moment t^*, once the control command is activated, the valve is opened and the control input $U(t)=-2X(t)+X_r$ (where $X_r=0.8$ is a constant set-point representing the desired reference liquid level in the container) kicks in immediately. After the time instant t^*, the volume of liquid flowing into the container is less than the volume of liquid getting out of the container, and finally the liquid level will be reduced to a fixed position of the reference set-point and arrive at a steady state.

In Case 3 of figure 1.7, the input delay is not zero such that $D>0$ and there is no compensation for the delay. This means, at a certain moment t^*, we send a command to open the valve to slow down the rising liquid level and let $U(t)=-2X(t)+X_r$ by using the current state of that time. However, because

For general queries, contact webmaster@press.princeton.edu

© Copyright, Princeton University Press. No part of this book may be distributed, posted, or reproduced in any form by digital or mechanical means without prior written permission of the publisher.

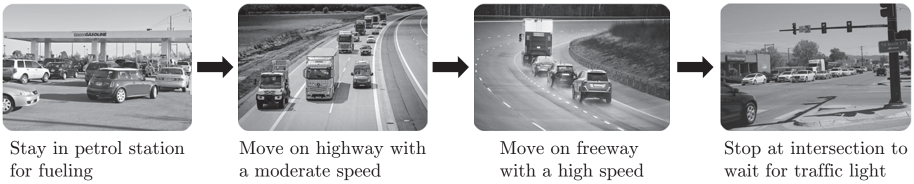

Figure 1.8. Car-following behavior in realistic traffic environment

of the delay, this valve is kept closed and the control $U(t) = -2X(t) + X_r$ does not act on the plant until D unit time later. Thus after that moment $t^* + D$, we have $\dot{X}(t+D) = X(t+D) + U(t) = X(t+D) - 2X(t) + X_r$, which is equivalent to $\dot{X}(t) = X(t) + U(t-D) = X(t) - 2X(t-D) + X_r$. From Case 3 of figure 1.7, when the delay value is large, it is apparent that the closed-loop time-delay system is not stable and the liquid level cannot be regulated to the desired set-point.

In Case 4 of figure 1.7, the input delay is not zero such that $D > 0$, but the predictor feedback is introduced to compensate for the delay. This means, at a certain moment t^*, we send a command to open the valve to slow down the rising liquid level. Due to the existence of a delay, we set $U(t) = -2X(t+D) + X_r$ by predicting the state of D unit time later. Thus at the moment $t^* + D$, this valve opens and the control $U(t) = -2X(t+D) + X_r$ acts on the plant and we have $\dot{X}(t+D) = X(t+D) + U(t) = X(t+D) - 2X(t+D) + X_r = -X(t+D) + X_r$, which is equivalent to $\dot{X}(t) = X(t) + U(t-D) = X(t) - 2X(t) + X_r = -X(t) + X_r$. That is to say, after $t^* + D$, the volume of liquid entering into the container is less than the volume of liquid exiting the container, and thus the liquid level will be reduced to a fixed position to achieve the reference set-point regulation.

Example 1.3. **Platoon Control of Car-Following Vehicles**

As demonstrated in figure 1.8, car-following behaviors for a platoon of vehicles (in which one vehicle follows another) are very common in realistic traffic environments. Generally speaking, the main task of platoon control for car-following models is to regulate a chain of vehicles to an ideal steady state, at which the following vehicles track the leading vehicle with a desirable velocity and simultaneously keep a safe and comfortable inter-vehicle spacing. The widely employed third-order linear vehicle dynamics [102, 113, 135] are taken into account as follows:

$$\dot{p}_i(t) = v_i(t), \tag{1.10}$$

$$\dot{v}_i(t) = a_i(t), \tag{1.11}$$

$$\dot{a}_i(t) = \frac{1}{\tau_i} u_i(t - D_i) - \frac{1}{\tau_i} a_i(t), \tag{1.12}$$

For general queries, contact webmaster@press.princeton.edu

© Copyright, Princeton University Press. No part of this book may be distributed, posted, or reproduced in any form by digital or mechanical means without prior written permission of the publisher.

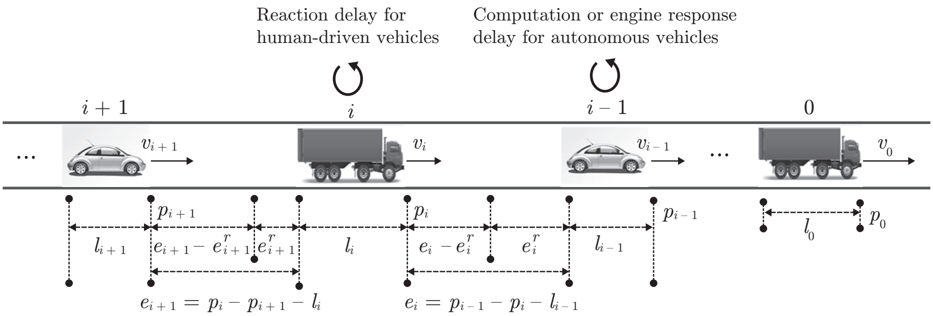

Figure 1.9. Platoon control of car-following vehicles with input delay

where $p_i(t)$, $v_i(t)$, and $a_i(t)$ denote the position, velocity, and acceleration of the ith vehicle; $u_i(t)$ is the ith vehicle's control input, which actually denotes the engine's driving/braking torque governed by the throttle/brake pedal command. τ_i is the engine time constant. D_i is the time delay, which might be either the reaction delay for human-driver vehicles or computation or engine response delay for autonomous vehicles.

As shown in figure 1.9, the inter-vehicle distance is introduced as

$$e_i(t) = p_{i-1}(t) - p_i(t) - l_{i-1}, \tag{1.13}$$

where l_i for $i = 0, 1, \cdots, n$ is the length of the vehicle i, and accordingly the desired/reference spacing is defined by $e^r_i(t)$. The control objective is to regulate the actual inter-vehicle spacing to achieve the desired value such that $\lim_{t\to\infty}\left(e_i(t) - e^r_i(t)\right) = 0$.

To achieve the above control target, the input delay D_i has to be handled carefully, otherwise a big time delay of the actuator could cause a catastrophic car accident.

1.3 Adaptive Control for Time-Delay Systems and PDEs

The stabilization of systems with actuator delay is not an easy task, especially when the system information is incomplete [36, 85, 114]. An effective method to deal with uncertainties in ODE systems is adaptive control [132, 152, 158, 160, 161], which has been extensively studied and summarized in the classic books [3, 48, 57, 108, 117, 126]. Early work on adaptive control of uncertain time-delay systems [37, 110, 112] focused on the uncertainty of parameters rather than the delay. Adaptive backstepping control of uncertain linear systems with input delay had been proposed by [150, 151]; however, by regarding the input delay as unmodelled dynamics, neither identification nor compensation for the delay is considered by the control design.

For general queries, contact webmaster@press.princeton.edu

© Copyright, Princeton University Press. No part of this book may be distributed, posted, or reproduced in any form by digital or mechanical means without prior written permission of the publisher.

The standard prediction-based compensation for input delay has the premise that the delay value of every actuator channel is known, as well as that the information is available on the plant and actuator state and plant parameters [2, 18, 20, 72, 73, 94, 104, 119, 127, 128, 142]. However, the fact is that such a prior knowledge may be hard to acquire in practice. Due to the infinite-dimensional nature of time-delay systems, conventional adaptive control methods for finite-dimensional ODE systems cannot be directly applied to addressing uncertain delays. Instead, adaptive control approaches for infinite-dimensional PDE systems provide potential solutions. In [101], a boundary control law and an adaptation law were proposed for Burgers' equation with unknown viscosity. In [90, 120, 121], adaptive boundary control schemes with different update laws (Lyapunov-based, passivity-based, and swapping-based) were developed for unstable parabolic PDEs with unknown parameters in the PDE or boundary condition. A systematic summary of adaptive control methodology for parabolic PDEs was given in [91, 122]. Focusing on another big family of PDEs—anti-stable wave PDEs—[30] and [79, 81], respectively, provided full-state and output feedback adaptive controls under the large uncertainty in anti-damping coefficient. The adaptive-based infinite-dimensional backstepping transformation initially motivated and developed for the boundary control and stabilization of uncertain PDE systems shed new light on addressing uncertain LTI-ODE systems with input delays.

An important step forward on compensation for input delay was made in [92]. With the novel idea of treating the actuator delay as a first-order hyperbolic PDE, LTI-ODE systems with input delays are converted into a class of ODE-PDE cascades. Under the assumption that all system information is available, a prediction-based boundary control and a backstepping transformation were developed. A tutorial introduction of such ODE-PDE cascade representation and PDE-backstepping-based predictor feedback with its extensions was offered in [82]. By regarding the input delay as a parameter in the transport PDE, this kind of PDE-based framework makes it possible for the application of adaptive control of PDEs to LTI-ODEs with input delays [88, 149].

1.4 Results in This Book: Adaptive Control for Uncertain LTI Systems with Input Delays

LTI systems with discrete or distributed input delays usually come with the following five types of basic uncertainties:

- unknown actuator delay,
- unknown delay kernel,
- unknown plant parameter,
- unmeasurable finite-dimensional plant state,
- unmeasurable infinite-dimensional actuator state.

For general queries, contact webmaster@press.princeton.edu

© Copyright, Princeton University Press. No part of this book may be distributed, posted, or reproduced in any form by digital or mechanical means without prior written permission of the publisher.

In this book, we design adaptive and robust predictor feedback laws for the compensation of the five above uncertainties for general LTI systems with input delays. When some of the five above variables are unknown or unmeasured, the basic idea of certainty-equivalence-based adaptive control is to use an estimator (a parameter estimator or a state observer) to replace the unknown variables in the nominal backstepping transformation and the predictor feedback of the uncertainty-free systems. The PDE boundary control law is designed to satisfy the stabilization boundary condition of the target closed-loop system, the observer is chosen to estimate the unmeasurable variables with a vanishing estimation error, and the adaptive update law of the unavailable signal is selected to cancel the estimation error term in Lyapunov-based analysis.

In addition, the main content of the book is mainly concerned with the delay-adaptive linear control of linear systems with input delays. Results for delay-adaptive control of nonlinear systems can be found in [29], and the predictor control of various nonlinear systems can be found in [20, 76, 83]. Results for delay-adaptive control of a class of linear feedforward systems with simultaneous state and input delays can be found in [8], whereas delay-robust control of nonlinear or linear systems with time-varying delays can be found in [16, 31, 67, 77].

1.5 Book Organization

The book has three parts.

Part I: In the first part of the book we consider adaptive control of uncertain single-input LTI systems with discrete input delay. The results in this part could be easily (or even trivially) extended to systems with multiple inputs when the delay is the same in all the input channels.

In chapter 2, by assuming all information of interest about the system is known to designers, we provide the introductory idea of predictor feedback and infinite-dimensional backstepping through the nominal single-input LTI systems with discrete input delay. To help readers in the field of control of delay systems who are not accustomed to the PDE notation, we first present an alternative view of the backstepping transformation based purely on standard ODE delay notation. Then the backstepping transformation is described in PDE and rescaled unity-interval transport PDE notation. It is convenient for readers to compare the relationship among the different notational frameworks and the subtle differences in distinct notations.

In chapter 3, a variety of adaptive predictor control techniques are provided to deal with different uncertainty collections from four basic uncertainties (delay, parameter, ODE state, and PDE state). In the presence of a discrete actuator delay that is long and unknown, but when the actuator state is available for measurement, a global adaptive stabilization result is obtainable. In contrast, the problem where the delay value is unknown, and where the actuator state is not measurable at the same time, is not solvable globally, since the problem is not

For general queries, contact webmaster@press.princeton.edu

© Copyright, Princeton University Press. No part of this book may be distributed, posted, or reproduced in any form by digital or mechanical means without prior written permission of the publisher.

linearly parameterized in the unknown delay. In this case, a local stabilization is feasible, with restrictions on the initial conditions such that not only do the initial values of the ODE and actuator state have to be small, but also the initial value of the delay estimation error has to be small (the delay value is allowed to be large but the initial value of its estimate has to be close to the true value of the delay). Please note that among different uncertainty combinations in chapter 3, we do not consider the case of coexistence of an unknown parameter vector and unmeasurable ODE state of the finite-dimensional part of the plant.

In chapters 4 and 5, output-feedback adaptive control problems involving uncertainties in both an unknown plant parameter and unmeasurable ODE plant state are taken into account. In this case, the relative degree plays a major role in determining the difficulty of a problem. As a result, chapter 4 considers the relatively easier case where the relative degree is equal to the system dimension, whereas chapter 5 considers the more difficult case where there is no limitation on the relative degree. Please note that trajectory tracking requires additional tools, as compared to problems of regulation to zero.

Part II: In the second part of the book we consider adaptive control of uncertain multi-input LTI systems with distinct discrete input delays.

In chapter 6, by assuming all the system information is known to the designer, we provide the elementary PDE-based framework of the exact predictor feedback and the infinite-dimensional backstepping transformation for the nominal multi-input LTI systems with distinct discrete input delays.

In parallel with the single-input systems, chapter 7 provides the adaptive global stabilization results for uncertain multi-input systems via the assumption that the actuator states in distinct control channels are measurable, whereas chapters 8 and 9 provide the adaptive local stabilization results for uncertain multi-input systems under the assumption that the measurements of actuator states are unavailable.

Part III: In the third part of the book we address adaptive and robust control problems of uncertain LTI systems with distributed input delays.

In chapter 10, under the premise of an uncertainty-free system, a fundamental predictor feedback framework to compensate for distributed input delays is developed for both single-input and multi-input systems. By modeling a linear system with distributed delay as an ODE-PDE cascade, we employ a couple of important conversions: the reduction-based change of variable converts a stabilization problem of a delayed system into a stabilization problem of a delay-free (reduced) system, and the forwarding-backstepping transformation contributes to a convenient stability analysis of the target closed-loop system.

When some information of the system (such as delay, delay kernel, parameter, ODE state, and PDE state) is unavailable, chapters 11 and 12 solve the adaptive and robust stabilization problems for single-input and multi-input systems, respectively. When both the finite-dimensional plant state and the infinite-dimensional actuator state are measurable, the global adaptive state feedback is used to handle the uncertainties in delay, delay kernel, and plant parameter. When the delay, delay kernel, and plant state are simultaneously uncertain, the

For general queries, contact webmaster@press.princeton.edu

© Copyright, Princeton University Press. No part of this book may be distributed, posted, or reproduced in any form by digital or mechanical means without prior written permission of the publisher.

Table 1.1. Uncertainty Collections of Linear Systems with Actuator Delays

Part I: Predictor Feedback of Single-Input LTI Systems with Discrete Input Delay

Organization of the Book	Input Delay	Parameter	ODE State	PDE State
Chapter 2	known	known	known	known
Section 3.2	known	known	*unknown*	known
Section 3.3	*unknown*	known	known	known
Section 3.4	*unknown*	known	known	*unknown*
Section 3.5	*unknown*	*unknown*	known	known
Section 3.6	*unknown*	known	*unknown*	*unknown*
Section 3.7	*unknown*	*unknown*	known	*unknown*
Chapter 4 and section 5.2	*unknown*	*unknown*	*unknown*	known
Section 5.3	*unknown*	*unknown*	*unknown*	*unknown*

Part II: Predictor Feedback of Multi-Input LTI Systems with Discrete Input Delays

Organization of the Book	Multi-Input Delays	Parameter	ODE State	PDE States
Chapter 6	known	known	known	known
Section 7.2	*unknown*	known	known	known
Section 7.3	*unknown*	known	*unknown*	known
Section 7.4	*unknown*	*unknown*	known	known
Chapter 8	*unknown*	known	known	*unknown*
Section 9.2	*unknown*	known	*unknown*	*unknown*
Section 9.3	*unknown*	*unknown*	known	*unknown*

Part III: Predictor Feedback of LTI Systems with Distributed Input Delays

Organization of the Book	Delay	Delay Kernel	Parameter	ODE State	PDE State
Chapter 10	known	known	known	known	known
Sections 11.1,12.1	*unknown*	known	known	known	known
Section 11.2	*unknown*	*unknown*	known	known	known
Sections 11.3,12.2	*unknown*	*unknown*	*unknown*	known	known
Sections 11.5,12.3	*unknown*	*unknown*	known	*unknown*	known
Sections 11.4,12.4	*unknown*	*unknown*	known	known	*unknown*

adaptive control problem may not be globally solvable in its most general form as the relative degree plays an important role in the output feedback. Thus the robust output feedback is employed. Similarly, when the delay and actuator state are unknown at the same time, the adaptive control can only achieve local stabilization both in the initial state and in the initial parameter error. For this case, instead of the adaptive method, the robust output feedback is considered.

To clearly describe the book is organization, different combinations of uncertainties considered in the book are summarized in table 1.1, from which the

For general queries, contact webmaster@press.princeton.edu

© Copyright, Princeton University Press. No part of this book may be distributed, posted, or reproduced in any form by digital or mechanical means without prior written permission of the publisher.

readers can make their own selections to address a vast class of relevant problems they face.

1.6 Notation

Throughout the book, the actuator dynamics (of the control input with delays) are modeled by a transport PDE whose state is $u(x,t)$, where t is time and x is a spatial variable that takes values in the interval $[0,1]$ or $[0,D]$. When the plant being controlled (or whose state is estimated) is an ODE, then the overall state of the system is the state of the ODE, $X(t)$, along with the state of the PDE, $u(x,t)$.

For a finite-dimensional ODE vector $X(t)$, its Euclid norm is denoted by $|X(t)|$.

For an infinite-dimensional PDE function $u(x,t)$, $u:[0,1]\times\mathbb{R}^+\to\mathbb{R}$,

$$\|u(x,t)\| = \left(\int_0^1 u(x,t)^2 dx\right)^{1/2}. \tag{1.14}$$

For general queries, contact webmaster@press.princeton.edu

© Copyright, Princeton University Press. No part of this book may be distributed, posted, or reproduced in any form by digital or mechanical means without prior written permission of the publisher.

Index

For general queries, contact webmaster@press.princeton.edu

© Copyright, Princeton University Press. No part of this book may be distributed, posted, or reproduced in any form by digital or mechanical means without prior written permission of the publisher.

For general queries, contact webmaster@press.princeton.edu