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INTRODUCTION

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I grew up in a house on the campus of a boarding school in Connecticut. My parents were teachers there—my mother taught art and my father taught history and coached the wrestling team. My life was immersed in this insular world—the school was my home and my playground. On the weekends my sisters and I would play in the classrooms, the rooms were often hot and musty and smelled of teenagers and chalk.

Many years later, I find myself back in dusty, chalk-filled classrooms, photographing the chalkboards of eminent mathematicians. How did I get here? The deeper answer, most likely, is that it still feels like home to me. The more immediate reason, however, comes out of a conversation I had in another part of New England.

Every summer, I take a break from academia and leave my home in New York City to join my family in a small beach town on the outer tip of Cape Cod, Massachusetts. Over the years, I've become very close to my neighbors, who live in a house once occupied by my paternal grandfather. Amie Wilkinson and Benson Farb are a married couple and parents of two. They are also theoretical mathematicians who teach at the University of Chicago. Amie's research focuses on smooth dynamical systems and ergodic theory; Benson's work explores low-dimensional topology and geometric group theory.

Amie and Benson are theoretical or "pure" mathematicians, which means they do "math for math's sake." They are interested in ideas, abstraction, exploring the boundaries of pure reason without explicit or immediate application in the physical world—akin to art, philosophy, poetry, and music. Practitioners of applied math, on the other hand, use theories and techniques to solve

practical problems in the physical world. Although their objectives are different, these two branches of mathematics are inextricably linked: there are numerous examples of discoveries in pure math that would be revealed to have revolutionary applications years later. In fact, one could say that all modern technology is derived from pure mathematics.

One day on the Cape, I watch Benson work at his dining room table. He is listening to Indian ragas with the volume so loud I can hear the thumping beats of the tabla through his headphones. For several hours, he sits, thinking, creating, jotting down the occasional note. It looks like he has a secret. I feel like he is creating something so expansive and beautiful it is beyond words, something that exists only in his head.

When I ask him to explain what he is working on, he pauses, appears to struggle for the right words, and replies, simply, "No. I can't."

Later, after giving it some thought, he tells me he is working on "configuration spaces," thinking about all of the possible configurations of, say, 10,000 distinct points moving around in a 4-dimensional space. Benson notes that this is a 40,000-dimensional problem. He's trying to discover patterns within this extremely complex system. Or in his more technical terms: "I'm trying to decompose the cohomology of the space of configurations of n points on a manifold as a direct sum of irreducible representations of the symmetric group S_n , where n can be any number (like 10,000)."

Mathematics at Benson's level is not easily translatable to the layperson. Some things are just complicated and cannot be reduced in a meaningful way to a concise,

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easy-to-understand statement. It is like a special foreign language, unknowable to all but a few.

I have always used my camera as a way to understand and explore the world. At sixteen, I left my home in the Northeast to spend a year at a boarding school on a hill station in southern India. (My mother had attended the same school.)

Before I left, my father gave me a gift: a twin-lens reflex camera, medium format, heavy, clunky, with a beautifully worn brown leather strap and case. I loved it. I brought it with me to India and soon discovered that the camera is a powerful tool—it made me feel fearless and allowed me to connect to people and places that were seemingly inaccessible. This was exciting to me.

Thirty years later, I return to India, this time leading a group of my photography students on a study abroad trip. We are in Jaipur, Rajasthan. I arrange a visit to an elementary school so my students can take portraits of the children, and the children can practice their English with us. The principal takes us through a maze of back doors and staircases to the roof of the schoolhouse. We emerge to discover that there are chalkboards surrounding the perimeter. Some are on stands; some are on the ground leaning against a low wall. All are filled with lessons in Hindi, a language that is foreign to me. The boards are beautiful but inaccessible. I walk around the roof and photograph each one.

Back home, as I sort through my photographs from the trip, I keep coming back to the images of the chalkboards. They symbolize so much for me: the interplay of aesthetic beauty and practical use; the foreign and the familiar; understanding and mystery. The writing on the boards in

Jaipur reminds me of the mathematical symbols in Benson's notebook. I see this connection and the project begins to take shape.

The chalkboard is one of the oldest and most important analog tools for learning. At first, students used small, individual pieces of slate and would sit at their desks working on their boards. But their teachers had no way to share and demonstrate their work with an entire class. Although there is some dispute about the date, many believe that the first large chalkboard was installed in a classroom in 1801. Eventually it evolved into what we know now: a large object, typically with a black or green surface, hanging on a wall. No longer made of slate, most boards are made out of porcelain-enameled steel.

Despite technological advances (such as the creation of computers), chalk on a board is still how most mathematicians choose to work. As musicians fall in love with their instruments, mathematicians fall in love with their boards—the shape, the texture, the quality of the special Japanese Hagoromo chalk. The boards are their homes, their labs, their private thinking spaces.

When mathematicians start writing on an empty board, they often have no idea where their calculations will go. The board leads them there, like a blank page for a writer, or an empty frame for a photographer.

Working on a chalkboard is a physical, time-based act. It lets the narrative of solving a problem unfold in real time, slowing thought down and allowing the information to be more easily absorbed. The speed of thought, observation, and quiet contemplation doesn't always match the accelerated velocity of digital technology—faster isn't necessarily better when you're creating and discovering.

The tactile experience of holding chalk and drawing on a board influences the way you think. All of your senses are engaged, and your body is moving through space. Your brain lights up. Thoughts are exploding—erupting—at the moment of discovery.

I begin in New York City, my home. The mathematicians at Columbia University, CUNY Graduate Center, and the Courant Institute of Mathematical Sciences at New York University are gracious and welcoming. They are intimately familiar with the beauty of math, and they want to share it with me.

As I start shooting, I begin to set a few parameters. I will only photograph actual chalkboards—no glass or white boards—mainly for their inherent beauty and timeless quality. I ask the mathematicians to write or draw whatever they want on their boards. (Often I end up shooting whatever is already on the board—usually something they are currently working on.) I photograph in a literal, objective, straightforward way—showing the chalkboards as real objects—capturing their texture, erasure marks, layers of work, and all forms of light reflecting off their surfaces.

The formulas and mathematical symbols are inaccessible to me. And I don't mind not knowing. I actually like the tension of being seduced by the formal abstract beauty—the patterns, symmetry, and structure—while simultaneously feeling totally disconnected, not being able to fully access the meaning of their work. This friction of being drawn in and pushed away is exciting to me. I may not know the specific meaning of the theorems, but I do know that beyond the surface they are ultimately revealing (or attempting to reveal) a universal truth.

I am in Rio de Janeiro, Brazil. I travel here with my neighbor, Amie, who comes to judge a math competition. I visit the Instituto Nacional de Matemática Pura e Aplicada (IMPA). This is my first time in Rio, and I am amazed by the landscape—the city surrounded by dramatic mountainous terrain, ocean, and natural wilderness. At the institute, I can see the lush green jungle through the windows of the classrooms. I photograph these details to show the specificity of the place. Mathematics is the same no matter where you are in the world—you can be on 116th Street in New York City or in a small village in France. It is a universal language.

Back home, I crisscross the United States, traveling to the Institute for Pure and Applied Mathematics (IPAM) at the University of California, Los Angeles; Rice University; Princeton University; Yale University; Harvard University; Massachusetts Institute of Technology; Northwestern University; and University of Chicago. Without Amie as my tour guide and translator, I am on my own, and I dedicate myself to getting to know the mathematicians. They are gracious beyond my expectations.

I drive down to the Institute for Advanced Study (IAS) in Princeton. Academic institutes are places where the only job of the researcher is to think—ideally without interruption. Researchers are invited to work in their fields free of teaching responsibilities. Here they are in essence protected from the outside world; their physical environment gives them psychological space.

I meet Helmut Hofer, a German-American mathematician who is one of the pioneers in the field of symplectic topology. He also works in the areas of dynamical systems and partial differential equations. I observe as he draws

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an intricate formula on his board for me. It is an explosion of thought—with abstract, swirling, hypnotic lines. The picture shows the mathematical concept of “finite energy foliations” in “symplectic dynamics.” He tells me that this theorem represents many years of work and it describes a complex mathematical structure in a very condensed way. He says that this drawing is equivalent to several hundred pages of technical mathematical language. Midway through the shoot, he puts down his chalk and says, “Follow me.” It is teatime. I follow as instructed, and soon find myself having afternoon tea with fifteen brilliant mathematicians. Helmut introduces me to many of them, helping recruit more participants for my project.

I enjoy watching the mathematicians work on their boards. It is an ephemeral performance, only occurring once. (And the photographs are the only evidence that this performance has taken place.) They have a heightened aesthetic awareness, distinct styles and ways of using the chalk, just like a visual artist. Some of the formulas are intensely chaotic, with explosive energy, while others feel neat, quiet, serene, and carefully considered. There is also the sheer size and physicalness of the board. It is a large object hanging on the wall—you can live with the work, and multiple people can work on it simultaneously.

I am in Paris, once more with Amie, who is in town to meet with two of her collaborators, Artur Avila and Sylvain Crovisier. It is here, at the Henri Poincaré Institute, thousands of miles away from her home at the University of Chicago, that I photograph Amie’s board. Her chalkboard is made of slate and is more than one hundred years old. I think of its history, wondering who else has worked on this board, and what discoveries have been made on its surface.

Amie draws a formula that encapsulates the central argument of a theorem that she and another collaborator, Keith Burns, had worked on for ten years. She tells me that even after finally solving the problem, it still took some time to organize the argument and find the cleanest way to express the proof. There can be many variations of a proof. And the way the proof is expressed is important—it should be elegant and concise. It is like taking the scaffolding away to reveal a beautiful building that you’ve been constructing for years.

I feel a mounting excitement as I watch Amie fill up her board with colored chalk and three-dimensional shapes—it is extraordinary—and the lines seem to vibrate with energy and depth. I lean in and look into my viewfinder to watch her work. She sees only what is within the frame of the board, as I see only what is within the frame of the lens. She depicts a sequence of shapes that are equivalent to one another, starting with a spherical ball and ending with something called a julienne (since it looks like a thinly sliced vegetable). She tells me that this theorem identifies the criteria that define when a certain dynamical system is “chaotic.” Chaos theory explains that even a slight variation in an initial system can produce erratic, complex, and inconsistent effects—often referred to as the “butterfly effect”: if a butterfly flutters its wings in one part of the world, it might lead to a tornado in another part of the world.

While in Paris, I also visit the Institut des hautes études scientifiques (IHES), a mathematics and theoretical physics research institute located in Bures-sur-Yvette, just south of Paris. I arrive on foot, after walking several miles from the neighboring town of Orsay (where I

was visiting mathematicians at Université Paris-Saclay). As I wander up the steep hill toward the main building, I am surprised to see chalkboards outside. The boards are standing on thin metal poles, scattered in the grass and woods surrounding this institute. It is disorienting and surreal. I imagine the mathematicians working in solitude and silence among the trees. Patterns and beauty in mathematics are often reflections of the natural world, and the boards filled with formulas seem to be at home in nature.

I am interested in learning about the moment of discovery, the epiphany of solving a problem. Some mathematicians spend a lifetime on one problem and are never able to solve it, while others break through, find the truth, and reveal an elegant mathematical proof. When this moment of realization happens, many describe it as being transformative. It can happen while one is meditating in an isolated cabin in remote rural Ireland (David Gabai, Princeton University) or sitting on a beach in Maui watching a friend kiteboarding (Dimitri Y. Shlyakhtenko, IPAM/UCLA). There is a euphoric feeling when one has that rare flash of insight. Hélène Esnault, a French-German algebraic geometer, tells me, “After the fog, light comes. It is a short and silent moment of joy.” Jim Simons, mathematician-businessman-philanthropist, says, “One is perfused with a warm feeling of satisfaction.” Others describe this sensation in more visceral terms. My neighbor Benson Farb writes, “I often tell people that I feel like an addict, relentlessly in search of the high that comes from discovering something new, but that’s always been there (a mathematical truth was true even before the universe existed; it was there waiting for me all along).”

These breakthroughs come rarely, and typically come only after repeated failure.

Now, several years into my journey into the rarified world of higher mathematics, I think a lot about patterns. Patterns of images, to be sure, but also of processes. Of how mathematicians and how artists see the world. There are similar patterns not only in how we see but also in how we think, how we work, and how we create. Put differently, even if I still don’t understand all of what mathematicians do, I do very much relate to who they are. They live inside their imaginations, follow their intuition, get lost in thought, create with physicality, and explore the unknown.

Their work, like that of all great artists, should be preserved, honored, and recognized: they are expanding knowledge—seeking the truth—and building on what has come before. I am fortunate because I was able to see, and now share, something that most of the world never gets to enjoy—the world of higher mathematics. It is a remote, austere world, and is, to some extent, a secret society. I am deeply thankful to the mathematical community for inviting me in and allowing me to record their work and bring back the evidence that there is discovery, truth, mystery, and beauty all around us, hidden away in dusty, chalk-filled rooms around the world.

—Jessica Wynne

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CHALKBOARDS & REFLECTIONS

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PHILIPPE MICHEL

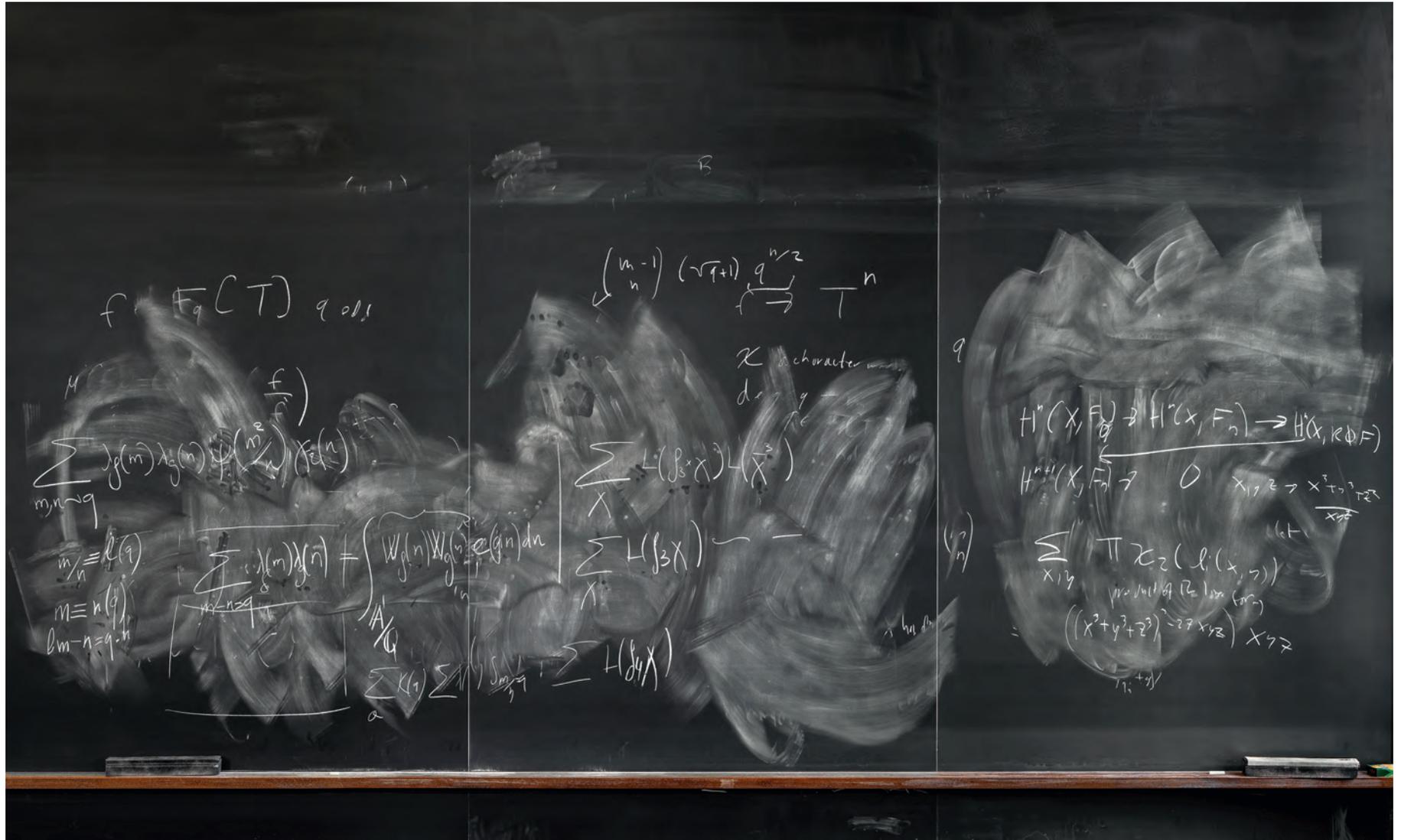
Philippe Michel is head of the chair of Analytic Number Theory at the Swiss Institute of Technology, Lausanne, where he has been on the faculty since 2008. He obtained his PhD in 1995 from Université Paris XI and subsequently held several positions in France.

Blackboards are a fundamental component of a mathematical life. The very first thing I did when I arrived in my office in Lausanne ten years ago was to ask that the ugly whiteboard, with its smelly red pen, be replaced by a true chalkboard. I got a very good one. A blackboard has to be as tall as possible so that its boundaries do not interrupt the flow of thinking and writing. In this regard, the immense blackboard in this photograph (which covers an entire wall of the common room in the mathematics department at Columbia) far exceeds expectations.

For a smooth and seamless writing experience, high-quality chalk is also important. I was particularly moved when a postdoc, returning from Christmas break one year, offered me two boxes of the legendary Japanese “Hagamoro Full Touch” chalk. (No need to break into my office; the stock is unfortunately exhausted.)

The last—but not least—piece of the trinity is cleaning. After years of training, I became adept at the Swiss (or is it German?) way. Here is the recipe: clean the board by applying water with a large cloth wiper, and dry it with a large rubber wiper (a colleague of mine in Zurich is even capable of doing both at once). Use that time to clear your mind. With a clean board and a clear mind, you’ll be ready to start anew.

This photograph was taken during a discussion with Will Sawin (who is assistant professor of mathematics at Columbia) concerning a joint project in number theory. Although number theory aims to describe the properties and structures of *whole* numbers, it borrows methods and techniques from many parts of mathematics; this blackboard is a good illustration of this. On this board, you can find ideas related to analysis (modular forms) as well as algebra and geometry (cohomology and sheaves).



AMIE WILKINSON

Amie Wilkinson is a professor of mathematics at the University of Chicago working in ergodic theory and smooth dynamics. She received her undergraduate degree at Harvard in 1989 and her PhD at Berkeley in 1995. She held post-doctoral positions at Harvard and Northwestern and rose to the level of professor at Northwestern before moving to Chicago in 2011. Wilkinson's research is concerned with the interplay between dynamics and other structures in pure mathematics—geometric, statistical, topological, and algebraic.

I work in an area called dynamics, which is the study of motion. The origins of the subject lie in physics; one of the earliest problems in dynamics was the stability of solar systems. My work focuses more on abstract mathematical spaces, but visualization and drawing pictures are key components of both understanding and discovery.

On this blackboard is a central argument in a paper I wrote with Keith Burns (professor of mathematics, Northwestern University) about a mechanism for chaotic dynamics. It depicts a sequence of shapes that, in a precise sense, are equivalent to each other, starting with a spherical ball and ending with something called a julienne, named for its resemblance to a thinly sliced vegetable. We are proud of this paper; as Keith likes to say, a good paper has one truly new idea, and this one has two.

Doing math on a blackboard is a tactile experience. You are constantly going back to change and modify things you have already written. Sometimes you write something and immediately realize you need to change it, and with a rub of your fist, you can erase the offending material and fix it. Or, when you are explaining something, you might draw an initial picture, write some formulas and explanations, and then go back to the original picture to add more—to explain in the picture what you just explained in words.

Chalk comes in many rich colors. You can shade things in with chalk, indicate depth and form. I love chalk, even though I hate the mess it makes and how it dries out my skin and hair. It is smooth and crisp on a clean blackboard and can be seen from a distance. It is one of my main tools of expression.

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