CONTENTS

CHAPTER 1

Optimization and the Visual Arts?

Optimization is the branch of mathematics and computer science concerned with optimal performance, with finding the best way to complete a task. As such, it is extremely applicable, as everyone from time to time attempts to perform some task at the highest level possible. A UPS driver, for instance, may sequence their stops to minimize total distance traveled, time spent on the road, fuel costs, pollutant emissions, or even the number of left turns. Finding an optimal tour, or at least one that is close to optimal, will benefit not only the driver and UPS, but also their customers (through lower prices) and the rest of society (through reduced pollution).

Some optimization problems are easy, while others are extremely difficult. Which is the case depends in large part on the *constraints*—the rules, the restrictions, the limitations—that specify the underlying task. If every stop on the UPS driver's list falls on the same thoroughfare, then finding the optimal route—and proving it to be optimal—is trivial. But if the city is filled with one-way streets, the stops are scattered throughout the city, and some stops must be made during specified time windows, then determining how to perform this task at a high level can require considerable algorithmic ingenuity and computing power.

Optimization has been put to good use in a great number of diverse disciplines: from advertising, agriculture, biology, business, economics, and engineering to manufacturing, medicine, telecommunications, and transportation (to name but a few). Numerous excellent books describe these important, practical applications, and if you turn to the bibliography, you will find my favorites.

The book you hold in your hands is quite different. It is a highly personal account of my more than sixteen-year-long obsession with using mathematical and computer-science-based optimization techniques to create visual artwork. As obsessions go, it is a harmless one, and not nearly as strange as it sounds! Within these pages, I will provide evidence that supports a bold claim: that the mathematical optimizer and the artist have more similarities than differences.

The mathematical optimizer studies problems that involve optimizing—that is, maximizing or minimizing—some quantity of interest (profit or total cost, for example, in business applications). The optimizer's goal is to come up with an optimal solution—perhaps a way of making the profit as large as possible or the total cost as low as possible. In some cases, the optimizer will be satisfied with a *local optimum*, a solution that is better than all neighboring solutions. If you find a local optimum, you can be confident that when you present it to the board, no one sitting there will be able to improve upon your solution by making minor tweaks to it. But in other cases, the optimizer will not rest until they find a *global optimum*, a solution that is provably better than every other solution. If you find a global optimum, you will be able to get a good night's sleep before the board meeting, for you will be certain that no one there—or anywhere—will be able to find a solution that is better than yours.

The artist is also a problem solver and a seeker of high-quality solutions. The creation of a piece of artwork can be considered a problem to be solved. And isn't it difficult to imagine an artist who, when creating a piece, does *not* try to do their best? For some small number of artists, the goal may be to maximize profit, but for most, the goal may be to make the piece as beautiful as possible, or to have as great an emotional impact on viewers as possible. Beauty and emotional impact are impossible to quantify, but haven't we all been in the presence of the critic, the museum-goer, or the gallery-opening shmoozer who in a burst of enthusiasm blurts out something like, "Don't you just *love* this piece? Don't you think that if the artist had added anything more to it, or had left anything out, it would have failed to have the same impact?" (an assertion, to the mathematical optimizer, about local optimality).

Mathematical optimizers are mindful of the roles that constraints play. They know that in some cases, if they impose additional constraints on an optimization problem, the problem will become much more difficult, but in other cases it will become considerably easier. Some constraints seem to be structured in such a way that in their presence, algorithms have trouble working their way to the best part of the *feasible region* (the set of all feasible solutions—the solutions that satisfy all the constraints), whereas other constraints provide the equivalent of handholds and toeholds that form an easily traversed path to optimality.

Artists are similarly mindful. Artists are well aware that they must deal with constraints. They must work within budgets. They must meet deadlines. If they enter competitions or juried shows, they must make sure that their pieces satisfy the rules of entry. If they take commissions, they must follow their clients' instructions. And no matter what media they choose to work with, they must deal with the particular constraints—imposed by the laws of physics—that govern how those media work. Painting with watercolors is different from painting with oils, and painting on rice paper is different from painting on canvas.

So, given that artists are creative, we might think that if it were up to them, they would do away with constraints. After all, constraints *constrain*. They restrict. They limit our choices. It would seem that constraints inhibit creativity.

But actually there is much evidence to the contrary. Many artists embrace constraints. Some need deadlines to be able to finish their work, and some believe that when their choices are limited, they are much more focused and creative. Joseph Heller (while paraphrasing T. S. Eliot) wrote,

> When forced to work within a strict framework the imagination is taxed to its utmost—and will produce its richest ideas.

And the psychologist Rollo May wrote,

> Creativity arises out of the tension between spontaneity and limitations, the latter (like the river banks) forcing the spontaneity into the various forms which are essential to the work of art or poem.

In fact, many artists go so far as to create their own constraints. Consider George-Pierre Seurat. While viewing his painting *A Sunday on La Grande Jatte–1884* from up close, one sees a mass of colorful dots. While backing away from it, one's eyes merge all of the dots into an image of a group of Parisians relaxing on an island on the Seine. To create this masterpiece, Seurat set himself the task of producing the best possible depiction of what he saw on the riverbank, subject to two highly restrictive, self-imposed constraints: he had to keep his colors separate, and he could only apply paint to the canvas with tiny, precise, dot-like brush strokes. Seurat's self-imposed constraints gave rise to a spectacular piece of artwork, the most widely reproduced example of what we now call Pointillism.

In the mosaicking arena, self-imposed constraints abound. Every time a mosaicist states, "I will build a mosaic out of ______," another self-imposed constraint is born (or at least conceived). In 400 BCE, the ancient Greeks were building mosaics out of differently colored pebbles, and around 200 BCE, they started building them out of specially manufactured tiles (tesserae) made out of ceramic, stone, or glass. Today's mosaicists still use these traditional materials, but they also use whatever else they have on hand: dice, dominos, LEGO bricks, Rubik's Cubes, toy cars, spools of thread, baseball cards, photographs, and even individual frames of films like *Star Wars* and *It's a Wonderful Life*.

Some mosaicists like to go beyond the inherent materials constraints. The domino mosaics of Ken Knowlton, Donald Knuth, and myself are not only made out of dominos, they are made out of *complete sets* of dominos. Knowlton's *Joseph Scala (Domino Player)* (from 1981) was made out of 24 complete sets of double-nine dominos, so it contains 24 dominos of each type: exactly 24 blank dominos, exactly 24 zero-one dominos, and so on. My domino portrait of President Obama, the 44th president of the United States, uses 44 complete sets. Knowlton's portrait of Helen Keller is composed of the 64 characters of the Braille writing system, and each of these characters appears 16 times. Chris Jordan's *Denali/Denial* mosaic arranges 24,000 (digitally altered) logos from the GMC Yukon Denali sports utility vehicle (six weeks of sales in 2004) into an image of Denali (also known as Mount McKinley). And a Robert Silvers photomosaic, commissioned by *Newsweek* for its 1997 pictures-of-the-year issue, portrays the late Princess Diana as a mosaic formed from thousands of photographs of flowers. All of these artists use computer software—usually computer programs that they have developed themselves—to design their mosaics.

In theory, you can design a photomosaic without software. You can take the senior portrait photos from your high-school yearbook, cut them out, and then arrange them in a rectangular grid so that from a distance, they will collectively resemble a photo of your favorite teacher. This is possible—some of the photos will be brighter and others will be darker. But you will need a good eye to assess the brightness of each photo, and even then, you will have a tough time determining the best position for each photo. Likewise, you can make a domino mosaic without software—by printing the target image on a large piece of paper and then placing dominos on top of the print, saving the brightest dominos (the nine-nines) for the brightest sections and the darkest dominos (the zero-zeros or blanks) for the darkest sections. Here, though it is clear which dominos are brighter than others, it still will be difficult to determine where to place each domino.

With mathematical optimization it is quite easy to design photomosaics, and it isn't all that difficult to design domino mosaics. With mathematical optimization, the artist/mathematician (or mathematician/artist) can explore all manner of constraints systems. This book is an account of my explorations of this world.

INDEX