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1 Introduction

1.1 Assets and Markets

1.1.1 Markets

Financial markets comprise markets for stocks, bonds, currencies, and commodities. During the last decade, these markets have grown remarkably fast in number and volume of daily concluded transactions. Their expansion was paralleled by substantial qualitative improvements. The supply of financial products has increased in size, and several new and sophisticated products have been developed. As well, trading on major stock exchanges has become much faster due to computerized order matching systems that enhance market transparency and accelerate operations.

Financial markets satisfy various commercial and productive needs of firms and investors. For instance, the forward markets of futures on commodities ensure the purchases and future deliveries of goods at prices fixed in advance. Their activity reduces uncertainty in transactions and creates a safe environment for developing businesses. Stock markets satisfy essentially the demand of national and international companies for external funds. The possibility of issuing equity tradable on domestic markets and abroad offers easy access to many investors and allows diversification of shareholders. As for investors, the market value of stocks provides information on the performance of various companies and helps efficient investment decisions to be made.

Financial markets also serve some purely financial purposes: lending, risk coverage, and refinancing. Especially, bonds issued by the Treasury, various states, or companies represent the demand of these institutions for loans. The use of organized markets to collect external funds has several advantages: It allows for a direct match between borrowers and lenders; it extends the number of potential lenders by splitting the requested amount into the so-called bonds or notes; it facilitates the diversification of investments; and it allows financing of very risky plans with low probabilities of repayment (junk bonds and emerging markets). Moreover, the experience of past decades shows that the development of organized markets has contributed to significant growth of pension funds by providing sustained returns in the middle and long run.

Financial assets are also used by investors for coverage against various risks; in financial terminology, this is *risk hedging*. For example, a European firm that exports its production to the United States and receives its payments in US dollars within six months following a shipment, may wish to cover against the risk of a decrease in the exchange rates between the US dollar and the Euro. Similarly, an institution that provides consumption loans indexed on the short-term interest rate may need to seek insurance against a future decline of this rate. The demand for coverage of diverse types of risk has generated very specific products called *derivatives*, such as options written on exchange rates or interest rates.

Finally, we need to emphasize the role of secondary markets. A standard credit contract involves a borrower and a lender; the lender is entitled in the future to receive regular payments of interest and capital until the expiry date. Secondary financial markets provide the initial lender an opportunity to sell the rights to future repayments to a secondary lender. The trade of repayment rights is widely used by credit institutions as an instrument of refinancing. A related type of transaction involving mortgages is called *securitization*, which allows a bank or an institution that specializes in mortgages to create financial assets backed by a pool of individual mortgages and to trade them on the market. The assets created in the process of securitization are called *mortgage-backed securities* (MBS).

1.1.2 Financial Assets

Financial assets are defined as contracts that give the right to receive (or obligation to provide) monetary cash flows. Typically such a contract specifies the dates, conditions, and amounts of future monetary transfers. It has a market price and can be exchanged whenever there are sufficient potential buyers and sellers. The acquisition of a financial asset can be summarized in terms of a sequence of monetary cash flows, including the purchasing price. It is graphically represented by a bar chart with a horizontal axis that measures the times between consecutive payments and a vertical axis that measures the amounts of cash flows. The cash flows take positive values when they are received and are negative otherwise.

Figure 1.1 shows that, unlike standard real assets, the financial assets

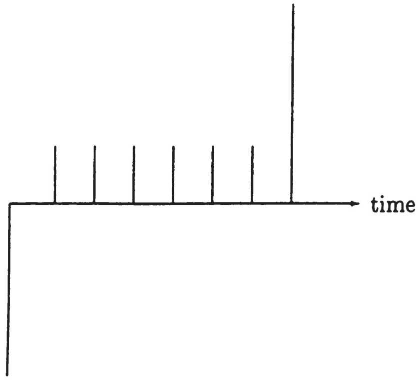

Figure 1.1 *Bar Chart*

need not exist physically. Instead, most financial assets are recorded and traded by computer systems.

Below are some examples of financial assets and the associated bar charts.

Zero-Coupon Bond (or Discount Bond)

A *zero-coupon bond* or *discount bond* is an elementary financial asset. A zero-coupon bond (Figure 1.2) with maturity date T provides a monetary unit (i.e., \$1) at date T. At date t with $t \leq T$, this zero-coupon bond has a residual maturity of $H = T - t$ and a price of

$$B(t,H) = B(t,T - t).$$

The zero-coupon bond allows for monetary transfers between the dates t and T.

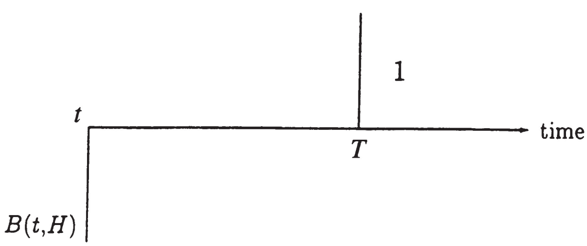

Figure 1.2 *Zero-Coupon Bond*

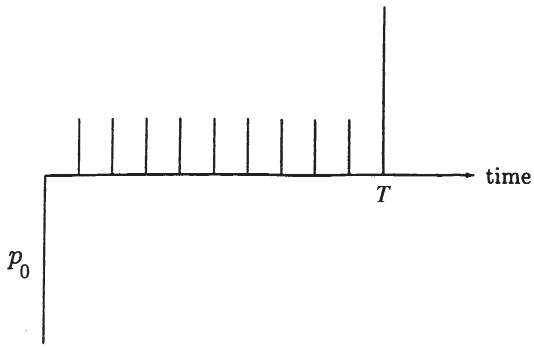

Figure 1.3 *Coupon Bond*

Coupon Bond

Coupon bonds are similar to loans with fixed interest rates and constant, regular repayment of interest. The contract specifies the lifetime of the loan (or *maturity*) and interest payments (or *coupons*) and states the method of capital repayment. The capital is usually repaid at the terminal date (or *in fine*). The coupon bond has a market price at any date after the issuing date 0. Figure 1.3 displays the bar chart at issuing date 0.

If the coupon bond is traded at any date t between 0 and the maturity date T, the bar chart needs to be redrawn. The reason is that the sequence of residual cash flows is altered since some payments prior to t have already been made. Therefore, intuitively, the price p_t differs from the issuing price p_0.

Stocks

Stocks are assets that represent equity shares issued by individual companies. They give to shareholders the power to inflict their opinion on the

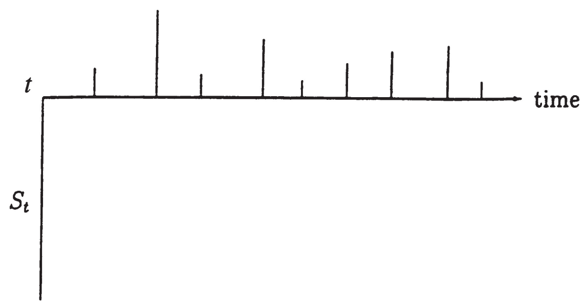

Figure 1.4 *Stock Indefinitely Held*

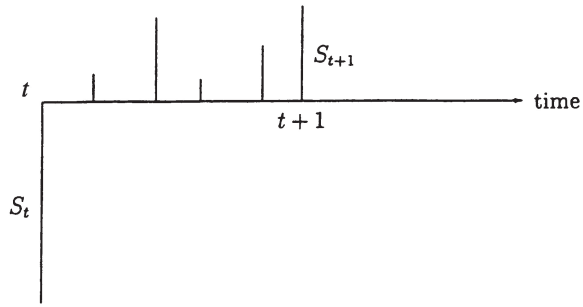

Figure 1.5 *Stock Sold at* $t + 1$

policy of the firm via their voting rights and to receive a part of the firm's profits (*dividends*). If we disregard the value of the right to vote, the current price S_t of a stock is equivalent to the sequence of future dividends, the amounts and payment dates of which are not known at t. Figure 1.4 provides the bar chart representing an indefinitely held stock, whereas Figure 1.5 provides the bar chart of a stock sold at $t + 1$.

Buying and Selling Foreign Currency

To demonstrate transactions that involve buying and selling foreign currency, let us denote by x_t the exchange rate between the US dollar and the Euro at date t. We can buy 1 Euro at t for x_t dollars and sell it at $t + 1$ for x_{t+1} dollars.

The bar chart of Figure 1.6 differs from the one that illustrates a zero-coupon bond because the future exchange rate (i.e., the amount of cash flow at $t + 1$) is not known at date t.

Forward Asset

Let us consider a simple asset, such as an IBM stock. A *forward buy contract* of this stock at date t and maturity H represents a commitment of a trader

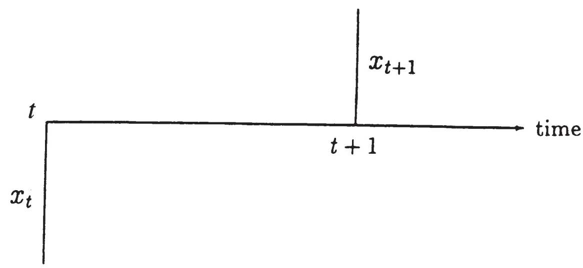

Figure 1.6 *Buying and Selling of Foreign Currency*

to buy the stock at $t+H$ at a predetermined price. Therefore, the buyer starts receiving the dividends after $t+H$. The existence of forward assets allows stripping the sequence of stock-generated cash flows before and after $t+H$ (Figure 1.7).

Options

Options are contingent assets that give the right to make a future financial transaction as described in the following example. A *European call* on IBM stock with maturity T and strike K gives the opportunity to buy an IBM stock at T at a predetermined price K. The cash flow received by the buyer at T is

$$F_T = \max(S_T - K, 0) = (S_T - K)^+,$$

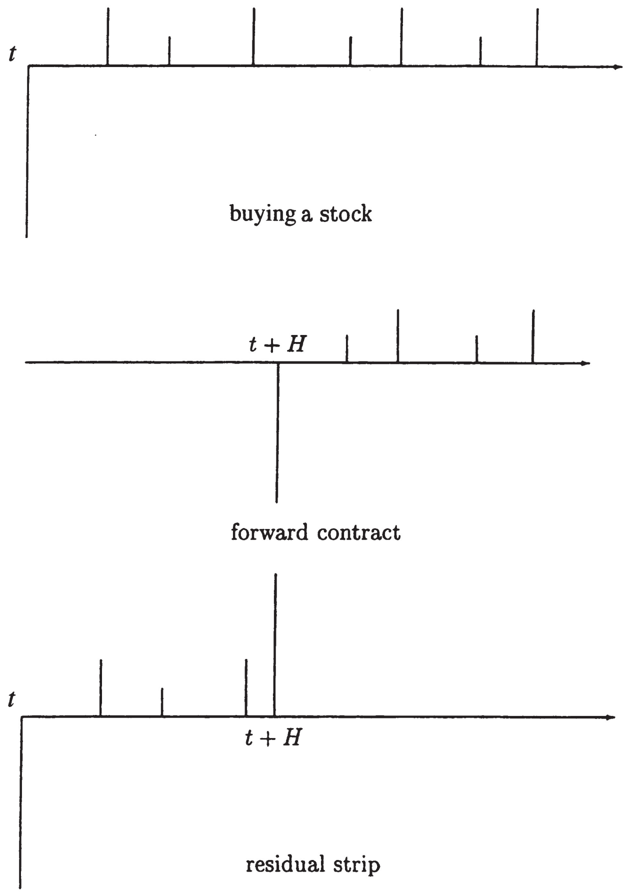

Figure 1.7 *Stripping of a Stock*

where S_T is the price at T of the IBM stock. Therefore, this cash flow is uncertain and depends on the future value S_T. It is equal to $S_T - K$ if $S_T > K$ and is zero otherwise. These two outcomes are illustrated in Figure 1.8. Here, $C_t(T,K)$ denotes the price at t of the European call.

1.2 Financial Theory

Financial theory describes the optimal strategies of portfolio management, risk hedging, and diffusion of newly tailored financial assets. Recently, significant progress has been made in the domain of market microstructures, which explore the mechanisms of price formation and market regulation. In this section, we focus attention on the theoretical aspects of dynamic modeling of asset prices. We review some basic theoretical concepts, not all of which are structural.

1.2.1 Actuarial Approach

The actuarial approach assumes a deterministic environment and emphasizes the concept of fair price of a financial asset. As an illustration, let

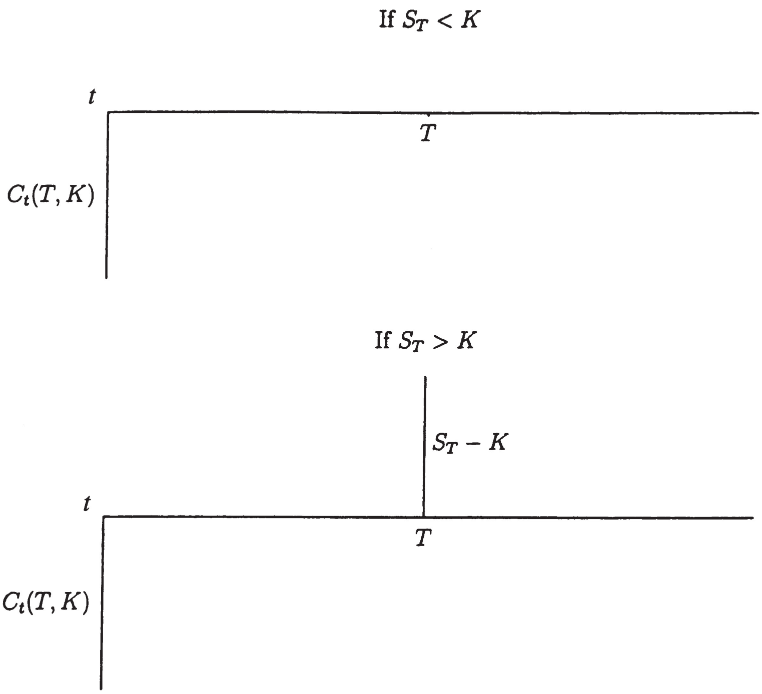

Figure 1.8 *European Call*

us consider at date 0 a stock that provides future dividends $d_1, d_2, \ldots, d_t$ at predetermined dates $1, 2, \ldots, t$. In a deterministic environment, the stock price has to coincide with the discounted sum of future cash flows:

$$S_0 = \sum_{t=1}^{\infty} d_t B(0,t), \tag{1.1}$$

where $B(0,t)$ is the price of the zero-coupon bond with maturity t. Moreover, if the short-term interest rate r_0 is assumed to be constant at all maturities, the above formula becomes

$$S_0 = \sum_{t=1}^{\infty} \frac{d_t}{(1+r_0)^t}. \tag{1.2}$$

Formulas (1.1) and (1.2) are the essential elements of the *actuarial calculus*. However, they are in general not confirmed by empirical evidence. The reason is that formulas (1.1) and (1.2) do not take into account the uncertainty about future dividends and the time variation of the short-term interest rate. Some ad hoc extensions of the actuarial formulas have been proposed in the literature to circumvent this difficulty in part. For instance, the literature on expectation models has come up with the formula

$$S_0 = \sum_{t=1}^{\infty} \frac{E_0(d_t)}{(1+r_0)^t}, \tag{1.3}$$

in which future dividends are replaced by their expectations evaluated at date 0. However, the pricing formula (1.3) disregards again the uncertainty about future dividends. Intuitively, the larger this uncertainty, the greater the risk on future cash flows is. Hence, the observed price will likely include a *risk premium* to compensate investors for bearing risk.

An alternative extension assumes the existence of a deterministic relationship between the derivative prices (e.g., an option written on a stock) and the price of an underlying asset (e.g., the stock). This approach is known as the *complete market hypothesis*, which underlies, for instance, the well-known Black-Scholes formula. The existence of deterministic relations between asset prices also is not confirmed by empirical research.

Essentially, the merit of concepts such as fair price or the deterministic relationship between prices lies rather in their theoretical appeal than in their empirical relevance.

1.2.2 *Absence of Arbitrage Opportunity*

Let us consider two financial assets; the first one provides systematically, at predetermined dates, the cash flows of amounts smaller than the second one. Naturally, we would expect the first asset to have a lower price.

For instance the price $C_t(T,K)$ of a European call with maturity T and strike K and written on an underlying asset with price S_t should be less than S_t. Its cash flow at the maturity date $(S_T - K)^+$ indeed is less than S_T.

The inequality between prices is a consequence of the *absence of arbitrage opportunity* (AAO), which assumes the impossibility of achieving a sure, strictly positive, gain with a zero initial endowment. Thus, the AAO principle suggests imposing deterministic inequality restrictions on asset prices.

1.2.3 Equilibrium Models

In the approach of equilibrium models, market prices arise as outcomes of aggregate asset demand and supply equilibrium. The equilibrium models are rather complicated due to the presence of assumptions on investor behavior and traded volumes involved in the analysis.

Various equilibrium models can be distinguished with respect to the assumptions on individual behavior. Basic differences among them can be briefly outlined as follows. The standard *Capital Asset Pricing Model* (CAPM) assumes the existence of a representative investor. The equilibrium condition concerns only a limited number of financial assets. The *Consumption-Based Capital Asset Pricing Model* (CCAPM) instead supposes joint equilibrium of the entire market of financial assets and of a market for a single consumption good. The *market microstructure theory* focuses on the heterogeneity of economic agents by distinguishing different categories of investors. This classification is based on access to information about the market and therefore makes a distinction between the informed and uninformed investors (the so-called liquidity traders), and the market makers. Microstructure theory also explains the transmission of information between these groups during the process of convergence toward equilibrium.

1.2.4 Predictions

The efficiency of portfolio management and risk control depends on the accuracy of several forecasted variables, such as asset prices, and their time-varying variance (called the *volatility*). A significant part of financial theory relies on the *random walk hypothesis*, which assumes that the history of prices contains no information useful for predicting future returns. In practice, however, future returns can often be inferred from past prices and volumes, especially when nonlinear effects are accounted for. There exist various methods to examine nonlinear temporal dependence, such as the technical analysis or the time series analysis of autoregressive conditionally heteroscedastic processes (ARCH).

1.3 Statistical Features

Statistical methods for estimation and forecasting of prices, returns, and traded volumes exploit the informational content of past observations. We give below a few insights on various types of variables used in statistical analysis and on the selection of sampling schemes and methodology.

1.3.1 Prices

Many financial time series represent prices of financial assets. It is important to understand the nature of available data before proceeding to the statistical analysis. The mechanisms of financial markets do not differ substantially from those of standard good markets (see Chapter 14 for more details). The trades are generated by buyers and sellers, whose demand and supply are matched directly by computer systems or by an intermediary. On some stock markets, called *order driven*, the prices offered by traders who wish to buy or sell (i.e., the *quotes*), are displayed on computer screens accessible to the public. The quotes are ranked starting with the best *bid* (proposed buy price) and the best *ask* (proposed sell price). This type of market includes the Toronto Stock Exchange (TSE) and the Paris Bourse (PB), for example. On other stock markets, such as the New York Stock Exchange (NYSE) and the National Association of Securities Dealers Automated Quotation (NASDAQ), asks and bids are determined by market makers and include their commissions.

The price at which assets are effectively exchanged can therefore be equal to the bid, the ask, or even a different amount, especially in the presence of market makers. Accordingly, the price records may contain the bids, asks, and/or traded prices. Also, prices per share depend not only on the exchanged assets and times of trade, but also on the traded quantities (*volume*) and individual characteristics of investors and may eventually include the commission of an intermediary. Moreover, in particular cases, the publicly displayed prices may differ from the true trading prices. Therefore, even on well-organized financial markets for which information is accurate and available on line in real time, it is important to know the genuine content of price records. In particular, we have to consider the following questions:

1. Do the available data contain the true trading prices, quotes, or proxies for trading prices computed as geometric averages of bids and asks?
2. Empirical analysis may occasionally concern separately the buyer-initiated (ask) or seller-initiated (bid) trades. In such cases, only sequences of ask and bid prices (*signed transactions*) need to be extracted from records.

3. Do the prices include transaction costs or commissions of intermediaries? Are they corrected for the tax transfers effectuated by either the buyer or seller?
4. Is the market sufficiently liquid to eliminate noncompetitive effects in price formation? This issue arises in the empirical analysis of infrequently traded assets.
5. Have the prices been adjusted for inflation to facilitate their comparison at different dates? This question is especially important for bonds with coupon payments that commonly are discounted.

1.3.2 Frequency of Observations

Recent expansion of financial markets has entailed increasing numbers and frequencies of trades due to the implementation of electronic order-matching systems. Until the early 1980s, data on prices were registered daily at either market openings or market closures. Accordingly, daily traded volumes were also recorded. Therefore, a sample spanning, for example, four years of asset trading would amount to about 1,000 daily observed prices (there are about 250 working days per year).

The electronic systems now allow instantaneously updated records to be kept of all transactions. They register on computer screens all movements that reflect all changes in the list of queued orders (called the *order book*) and have an accuracy of a fraction of one second. Therefore, the size of data files comprising the so-called tick-by-tick data or high-frequency data may be extremely large. A four-year sample may contain more than 1 million records on trades of a liquid stock or more than 3 million records on exchange rates.

Since transaction records are made at various times and are not necessarily integer multiples of a time unit such as one day, the timing of trades requires particular consideration. It is important to distinguish the price data indexed by transaction counts from the data indexed by time of associated transactions. Empirical evidence suggests that the price dynamics in calendar time and in transaction time differ significantly. The comparison of both sampling scales provides insights into the trading activity of an asset and its liquidity.

1.3.3 Definition of Returns

Time series of asset prices display a growing tendency in the long run. Occasionally, however, price series may switch from upward to downward movements and vice-versa in the short or middle run. For this reason, prices of the same asset sampled at different periods of time may exhibit unequal means. Since this feature greatly complicates statistical inference, it needs to be eliminated. A simple approach consists in transforming the prices into returns, which empirically display more stationary behavior.

Let us consider a financial asset with price p_t at date t that produces no dividends. Its return over the period $(t,t+H)$ is defined as

$$r(t,t+H) = \frac{p_{t+H} - p_t}{p_t}. \tag{1.4}$$

The return depends on time t and the horizon H. Very often, statistical analysts investigate returns at a fixed unitary horizon:

$$r(t,t+1) = \frac{p_{t+1} - p_t}{p_t}, \tag{1.5}$$

which in general display more regular patterns than the initial series of prices.

In theoretical or econometric analysis, the above formula is often replaced by the following approximation: Let us suppose the unitary horizon and a series of low-value returns: we obtain

$$\begin{aligned}\tilde{r}(t,t+1) &= \log p_{t+1} - \log p_t \\ &= \log\left(\frac{p_{t+1}}{p_t}\right) \\ &= \log\left(1 + \frac{p_{t+1} - p_t}{p_t}\right) \\ &\simeq \frac{p_{t+1} - p_t}{p_t} = r(t,t+1).\end{aligned}$$

The returns defined in (1.5) are used by banks, various financial institutions, and investors in financial markets. The differences of price logarithms conventionally represent the returns examined by researchers. However, it is important to note that

$$\begin{aligned}\tilde{r}(t,t+1) &= \log\left(1 + \frac{p_{t+1} - p_t}{p_t}\right) \\ &\simeq r(t,t+1) - \frac{r(t,t+1)^2}{2},\end{aligned}$$

when we consider the expansion at order two. Therefore, the approximation $\tilde{r}\ (t,t+1)$ undervalues the true return and may induce a significant bias due to replacing the theoretical definition of returns in (1.5) by the approximation.

1.3.4 Historical and Dynamic Analysis

The distributional properties of returns provide valuable insights on their future values. The analysis can be carried over in two frameworks. The *static* (*historical*) approach consists of computing marginal moments such as the marginal mean and variance from a sample of past returns and using these statistics as indicators of future patterns. The *dynamic* approach concerns the conditional distribution and conditional moments, such as the conditional mean and variance. These are assumed to vary in time, so that at each date t, new estimates need to be computed conditional on past observations. The conditioning is necessary whenever there are reasons to believe that the present returns, to some extent, are determined by the past ones. By the same argument, future returns depend on present and past returns as well, and their values can be used for forecasting.

Historical Approach

The historical approach explores the marginal distribution of returns. For instance, let us consider the series of returns on a single asset $y_t = r\ (t,t+1)$. The expected return is evaluated from the data on past returns by

$$Ey_t \simeq \frac{1}{T}\sum_{t=1}^{T} y_t = \bar{y}_T,$$

whereas the variance of the return is approximated by

$$Vy_t \simeq \frac{1}{T}\sum_{t=1}^{T} (y_t - \bar{y}_T)^2.$$

The historical approach can be refined by applying rolling estimators. Implicitly, this procedure assumes that marginal distributions of returns vary in time. It is implemented by introducing a window of a fixed length K and approximating the expected return at t by the *rolling average*:

$$Ey_t \simeq \frac{1}{K}\sum_{k=0}^{K-1} y_{t-k} = \frac{1}{K}(y_t + y_{t-1} + \ldots + y_{t-K+1}).$$

On the transition from t to $t+1$, the approximation of the expected return is updated by adding a new observation y_{t+1} and deleting the oldest one y_{t-K+1}.

Conditional Distribution

The analysis of the marginal distributions of returns is adequate for processes with a history that provides no information on their current values. In general, the expected values and variances of returns are partly predictable from the past. This property is called *temporal dependence* and requires a dynamic approach, which consists of updating the conditional moments in time by conditioning them on past observations. Very often, the analysis is limited to the first- and second-order conditional moments:

$$E(y_t \mid \underline{y_{t-1}}) \qquad \text{and} \qquad V(y_t \mid \underline{y_{t-1}}),$$

where $\underline{y_{t-1}} = (y_{t-1}, y_{t-2}, \ldots)$ denotes the information available at date $t-1$. Although the conditional moments are more difficult to approximate, in practice they yield more accurate forecasts.

Horizon and Observation Frequency

The conditional distribution may be used for predicting future returns at various horizons and sampling frequencies. While the predictions of future returns may not always be improved by conditioning on the past, the conditional expectations often yield better outcomes than the historical expectations. For illustration, we discuss below the prediction accuracy in computing the conditional variance of prices, called the *price volatility*.

Let us first assume that prices are observed at integer valued dates. The price volatilities at date t can be computed at one, two, or more units of time ahead:

$$V(p_{t+1} \mid p_t, p_{t-1}, p_{t-2}, \ldots) \qquad \text{at horizon 1,}$$
$$V(p_{t+2} \mid p_t, p_{t-1}, p_{t-2}, \ldots) \qquad \text{at horizon 2,}$$
$$V(p_{t+H} \mid p_t, p_{t-1}, p_{t-2}, \ldots) \qquad \text{at horizon } H.$$

This approach allows examination of the dependence of volatility on the forecast horizon (the so-called term structure of volatilities).

If prices are observed every two units of time and t is even, the volatility at horizon 2 is

$$V(p_{t+2} \mid p_t, p_{t-2}, p_{t-4}, \ldots).$$

It differs from the previously given volatility at horizon 2 in terms of the content of the conditioning set, for which observations at odd dates are omitted.

The above discussion suggests that price volatility is a complex notion comprised of the effects of time, horizon, and sampling frequency.

1.3.5 Nonlinearity

The complexity of financial time series has motivated research on statistical methods that allow accommodation of nonlinear dynamics. The nonlinear patterns result from the specificity of financial products and the complexity of strategies followed by investors. We give below some insights on the nature of nonlinearities encountered in theory and/or documented by empirical research.

Nonlinearity of the Variable to Be Predicted

Let us provide two examples of the nonlinearity of the variable to be predicted. First, market risk is related to the volatility of returns, commonly approximated by squared returns. Therefore, the variable to predict is a power function of the asset price. Second, there exist derivative assets with definitions that involve nonlinear transformations of the prices of underlying assets. For instance, the pricing formula of a European call is based on an expectation of $(S_T - K)^+$, which is a nonlinear transform of the stock price.

Nonlinearity of the Relationships between Prices

Even though prices of a derivative and of an underlying asset do not generally satisfy a deterministic relationship, they likely are randomly and nonlinearly related. For instance, the price of a European call $C_t(T,K)$ and the price S_t satisfy nonlinear inequality constraints due to the requirement of the AAO.

Nonlinearity with Respect to Parameters

Empirical evidence suggests that both the marginal and the conditional return distributions feature departures from normality. Essentially, research has documented the asymmetry of distributions and fat tails, implying a high probability of observing extreme returns. For this reason, standard analysis based on linear regression models, which involves the first two moments only, may be insufficient or even misleading in many financial applications.

Nonlinearity of the Dynamics

The observed dynamics of returns feature several nonlinear patterns. By looking at a trajectory of returns sampled daily or at a higher frequency, one can easily observe time-varying dispersion of returns around the mean or, equivalently, their time-varying variance (*volatility*). The first observation of this type was made by Mandelbrot in the early 1950s, who empirically found that large returns (positive or negative) have a tendency to be followed by large returns and that small returns have a tendency to be followed by small ones of either sign. This phenomenon is known as *volatility clustering* and points out not only the variation, but also the persistence of volatility. During the last twenty years, estimation and prediction of volatility dynamics have been given considerable attention and have resulted in a large body of literature on models with conditional heteroscedasticity. Technically, future squared returns are represented as functions of past squared returns, and nonlinearity arises from the presence of power functions.

In more recent developments, temporal dependence in volatility has been associated with *regime switching*, which means that episodes of high or low returns are explained by movements of a latent variable that admit a finite number of discrete states.

Nonlinearity of the Financial Strategies

The myopic or intertemporal optimizations of investors for dynamic portfolio management, hedging, and risk control are nonlinear with respect to the expected future evolution of prices. Then, at equilibrium, the behavior of investors induces nonlinear effects on future prices.

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