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## PROBLEM 1

### A Military Question: Catapult Warfare

Our opening problem has a military flavor. Imagine an invading army faced with a huge, VERY tall defensive wall. (Think of The Wall in HBO's *Game of Thrones*, protected by the Night Watch.) To breach the wall, the invaders decide to attack by launching massive projectiles, to hit as high up the face of the wall as possible. (For a particularly nasty, *really disgusting* attack, one not all that uncommon in ancient times, think of a catapult flinging wooden barrels full of fresh cow dung and/or the dead bodies of animals and soldiers up onto—even better over—the wall.) Given that the launching device (catapult, cannon, whatever) gives each projectile a “muzzle” speed of  $V$ , the launcher is distance  $D$  from the base of the wall, and  $g$  is the acceleration of gravity, you are to calculate the launch angle  $\theta$  that maximizes the height  $h$  of the projectile's impact point on the wall (see Figure P1.1). Indeed, what *is* this maximum height? Additionally, what's the flight time, from launch to impact, of the projectile when  $h$  is maximized? In all your calculations, ignore the effects of air resistance. *Note:* This problem can be done with nothing but algebra, a touch of trigonometry/geometry, and the quadratic equation. No calculus is required (other than knowing that distance is the integral of speed). No derivatives are required.

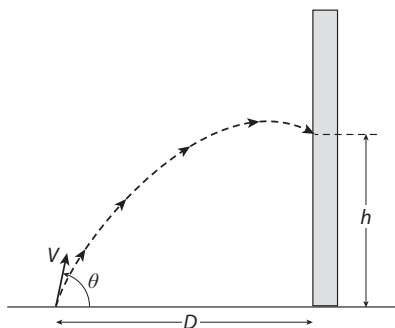


Figure P1.1. What is  $\theta$  to maximize  $h$ , given  $V$  and  $D$ ?

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