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# Dimensional analysis

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*a not so secret weapon*

## An apparently innocuous but surprisingly powerful remark

The secret weapon of the fly by night physicist is dimensional analysis.

Many small American towns boast a wooden sign on the road leading into the hamlet, stating some basic numerical facts about the place, for example,<sup>1</sup> “Welcome to Dum Dum Town, home of the Angry Ducks,<sup>2</sup> founded 1869, population 12, elevation 233 feet, total 2114.”

You laugh. Good, it means that you understand one of the foundational principles of physics: We can add various quantities together only if they are expressed in the same unit. The sign for Dum Dum Town should have given the population as 24 feet.<sup>3</sup> Furthermore, meaningful numerical values should be convention independent, which 1869 is not. It follows that, in physics, we should not refer temperature to Fahrenheit’s armpits.<sup>4</sup>

Even if various quantities are in the same unit, they still may not be added together. To underline this point, allow me to mention that my family and I were once at a bus stop on a desert road in Israel near the Dead Sea. A high-tech board displayed the destinations of various buses and their estimated times of arrival thus: Jerusalem 35 minutes, Beersheba 14 minutes, Eilat 29 minutes. My son Max, 6 at the time and having recently mastered addition, announced that the sum was 78. (What was meaningful about the sum, I pointed out to Max, was that it decreased by a multiple of 3 every time they updated the board. This prediction<sup>5</sup> was shortly verified observationally.)

We are allowed to add, or subtract, various quantities together only if they are expressed in the same unit. Thus, the left hand and right hand sides of an equation in physics must have the same physical dimension.<sup>6</sup> The reader

surely knows that this apparently innocuous statement is surprisingly powerful in physics, but people outside physics are often amazed.

## The harmonic pendulum

We begin with a canonical example from the early days of physics. The story is that one day in church, Galileo watched the various hanging incense burners swinging back and forth and silently measured their periods against his pulse.

So, consider a pendulum of length  $l$  subject to the acceleration of gravity  $g$ . What is its period for small oscillations?

## Convention used throughout this book and a word of caution regarding the paucity of letters

To do physics, we need units for mass, length, and time. Three and only three dimensions, which we denote by  $M$ ,  $L$ , and  $T$ , respectively. More about this in chapter IV.1. Why only three? Nobody knows. Could there be more? Very unlikely, at least until the day we discover that we need more.

No, Patrick, degree is not a fundamental unit of physics. Temperature is an energy, and should be measured in units appropriate for energy. Again, more in chapter IV.1.

For the time being, all I need to say is that I adhere to the convention of using square brackets  $[X]$  to denote the dimension of a physical quantity  $X$ . For example, recalling that kinetic energy is  $E = \frac{1}{2}mv^2$ , we see that energy has dimension  $[E] = M(L/T)^2 = ML^2/T^2$ . A table of dimensions is given in the front of this book.

Acceleration is time rate change of velocity: thus  $[g] = (L/T)/T = L/T^2$ . Indeed, when we first started studying physics, it was drummed into us that numerically  $g = 32$  feet per second squared, or 9.8 meters per second squared, depending on where you were.

Henceforth, I will use the three letters,  $M$ ,  $L$ , and  $T$ , generically to denote dimensions of mass, length, and time, respectively. In many cases (and one will come up presently), I will have to use one single letter to denote more than one physical quantity. Please do not be confused; the alphabet contains only so many letters.

Also, occasionally, it turns out to be more convenient to use energy  $E$  rather than mass  $M$ , as we will see.

## Period of a pendulum

So, what is the period  $T$  of a pendulum? Well, we are to form an expression with the dimension of time out of  $l$  and  $g$ , with dimension  $[l] = L$  and  $[g] = L/T^2$ ,

respectively. To get rid of  $L$ , we see that the ratio  $l/g$  is the only possibility, with dimension  $[l/g] = L/(L/T^2) = T^2$ . (You were forewarned:  $T$  here stands for two different concepts.) Thus, the period has to be

$$T \sim \sqrt{l/g} \quad (1)$$

Remarkably, we have determined the dependence of  $T$  on  $l$ , seemingly with no effort. For example, if we double the length of a pendulum, we know that the period  $T$  should increase by a factor of  $\sqrt{2} \simeq 1.4$ . This square root dependence of the period on the length is an experimentally verifiable prediction.

Lord Rayleigh was allegedly the first to systematically exploit dimensional analysis, which he called the “law of similitude,” in the late 19th century. I am surprised that it occurred so late, but surely the requirement that in physics dimensions must match was known to the founding fathers, almost by definition a bunch of plenty smart guys.

## A detailed look at a one line calculation

The layperson is often astonished that physicists could predict the behavior of pendulums without doing a detailed calculation, but, as you probably know, our reasoning hides a lot of important, even profound, physics that took centuries to understand.

Since this is our first example, we will examine it in detail, attempting to beat it to death, so to speak. The rest of this chapter consists of a series of comments.

Occasionally but not too often,<sup>7</sup> a student worries unduly about all sorts of possible complications. What about air resistance? What about the stretchiness of the string, if the pendulum consists of a weight attached to a string? The list could go on.

Over time, several characters have wandered into my three previous textbooks, the most beloved appearing to be Confusio,<sup>8</sup> judging from comments by students and others. Worrywort here is not confused, but merely worries too much. The self-evident answer to put Worrywort at ease is that the spirit of fly by night physics is to treat the simplest possible case first. If Worrywort persists, then how about the changing mass of the pendulum as the incense burns? How about the rotation of the earth? Perhaps the best answer is that idealization is an essential part of theoretical physics.

## Circular frequency over everyday frequency

Results obtained by dimensional analysis, such as (1), could easily be off by factors of 10 or more in one direction or the other. Indeed, the exact result for the pendulum is well known to be (see below)  $T = 2\pi \sqrt{l/g}$ .

Often, as in this case, factors of  $2\pi$  and such could be fixed, or at least guessed at, by a linear combination of physics sense and experience with similar

problems. In everyday usage, frequency, that is, the inverse of the period,  $f = 1/T$ , is defined as the number of repetitions per unit time. (For example, a hertz is 1 cycle per second.) But in physics, when we are faced with a situation periodic in time, after we set up the equation of motion, solve the differential equation, and so forth, we typically have solutions involving  $\sin \omega t$ ,  $\cos \omega t$ , or for the more sophisticated, the complex exponentials  $e^{\pm i\omega t}$ , with the so-called circular frequency  $\omega$ .

Evidently, these cyclic functions repeat themselves after a time duration  $T = 2\pi/\omega$ . But from experience, in the differential equation we solve, differentiation with respect to time would bring down factors of  $\omega$ . So our dimensional analysis should apply, not to  $T$ , but to  $\omega \sim \sqrt{g/l}$ . Thus, we expect  $T = 2\pi/\omega \simeq 2\pi\sqrt{l/g}$ .

Take a pendulum 1 meter long. Then  $T \simeq 6\sqrt{1/10}$  sec  $\simeq 2$  sec. Without the  $2\pi$ , we would have gotten a period of  $\sim \frac{1}{3}$  sec. You do not have to be a keen experimentalist to realize that 2 sec is more reasonable.

Moral of the story: always  $\omega$ , not  $f$ .

Indeed, in the rest of this book, I will drop the word “circular” and simply refer to  $\omega$  as frequency. Here is an instructive exercise for the curious undergrads in my class. Glance at a textbook on an advanced subject, for example, quantum field theory. See if you can find  $\omega$  or  $f$  more often.

The goal of this book is not to derive results that would agree with experimental data to  $n$  significant figures. But still, if you could obtain a better prediction, for free so to speak, by simply using a better variable, then why not?

## The dull function hypothesis

As is also well known, there is actually a dimensionless parameter in the problem, namely, the initial angle  $\theta_0$  of the swinging pendulum, and so dimensional analysis, strictly speaking, only tells us that  $T \simeq 2\pi f(\theta_0)\sqrt{l/g}$ . The unspoken assumption is that  $f(0)$  is a number of order 1.

Sometimes students ask: How do we know that  $f(0)$  does not vanish? A valid question. But then you have to give a reason why  $f(0)$  might vanish.

The unspoken assumption, again based on sense and experience, is that the majority of the functions appearing in physics are fairly boring functions that do not have especially wild properties. Of course, there are some exceptions.<sup>9</sup> But then, fairly obvious reasons usually would present themselves. Otherwise, we will end up with a major unsolved problem in physics.

In this particular example, it is fairly easy to see that the period will become independent of  $\theta_0$  in the small angle limit. Equating potential energy to kinetic energy, we have  $mg l \theta_0^2 \simeq \frac{1}{2} m v^2$ , thus giving  $v \simeq \sqrt{g l} \theta_0 \propto \theta_0$ . The pendulum swings slower and slower as  $\theta_0 \rightarrow 0$ , but by geometry, the distance traversed  $\simeq l \theta_0$  also  $\rightarrow 0$ . Dividing distance by velocity, we get  $T \propto \theta_0 / \theta_0 = 1$ , and we see explicitly that  $\theta_0$  disappears.

We will come back to the dull function hypothesis in chapter I.4.

## Exact calculation, yes, but only after a back of the envelope calculation

Here is another quote:\*

Erst kommt das Denken, dann das Integral. (Roughly, “First think, then do the integral.”) [Rudolf Peierls to the young Hans Bethe]

For the pendulum, the exact calculation is part of freshman physics and so is hardly hard.

A tactic that an advanced undergrad should have learned is that it is better to use energy conservation

$$E = \frac{1}{2}m(\dot{\theta})^2 + mgl(1 - \cos \theta) \simeq \frac{1}{2}m(\dot{\theta})^2 + \frac{1}{2}mgl\theta^2 \quad (2)$$

rather than the equation of motion. Effectively, we have already integrated the equation of motion once to turn the second derivative in time into the square of a first derivative. This is an important theme we will come back to often, for example, in chapter VII.3.

Plugging  $\theta = \theta_0 \cos \omega t$  into (2) and dividing through by  $ml^2\theta_0^2$ , we see that the expression on the far right, containing a  $\sin^2 \omega t$  and a  $\cos^2 \omega t$ , could be constant only if  $\omega^2 = g/l$ , thus giving  $T = 2\pi\sqrt{l/g}$ .

By the way, we could also readily solve the problem for  $\theta_0$  not necessarily small; it merely involves an unappetizing and unilluminating integral. The left half of (2) allows us to solve for  $\dot{\theta}$ : it is proportional to the square root of some stuff plus  $\cos \theta$ , that is,  $\dot{\theta} = (\text{stuff})\sqrt{\cos \theta - \cos \theta_0}$ . (Without having to do any tedious algebra, we know the additive constant inside the square root must be  $\cos \theta_0$ , since  $\dot{\theta}$  vanishes when  $\theta$  reaches  $\theta_0$ .) Thus,

$$T = \int dt = \int \frac{d\theta}{d\theta/dt} = 4(\text{stuff}) \int_0^{\theta_0} \frac{d\theta}{\sqrt{\cos \theta - \cos \theta_0}} \quad (3)$$

Nothing much doing with this integral alluded to above, but for small  $\theta_0$ , it becomes

$$\int_0^{\theta_0} \frac{d\theta}{\sqrt{\theta_0^2 - \theta^2}} = \int_0^1 \frac{\theta_0 du}{\theta_0 \sqrt{1 - u^2}} \quad (4)$$

After scaling by  $\theta = u\theta_0$ , we see with our very own eyes that  $\theta_0$  drops out of the problem.

Scaling is an important tactic that we will come back to again and again. One way of saying this is that we reject measuring angles following some loco Babylonian<sup>10</sup> idea of dividing a right angle into 90 equal parts. Instead, we measure the angle  $\theta$  more sensibly in units of  $\theta_0$ .

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\*We will literally apply this remark in a math interlude.

## Drive your calculus teacher mad

Of course, we could also use Newton's equation of motion\*  $F = ma$ :

$$ml \frac{d^2\theta}{dt^2} = -mg\theta \quad (5)$$

which amounts to differentiating (2) with respect to  $t$ .

Sure, you and I know how to solve differential equations like this. The fly by night physicist, however, prefers the "drive your calculus teacher mad" approach. Write  $\frac{d^2\theta}{dt^2}$  out as  $\frac{d}{dt} \frac{d\theta}{dt}$  and cancel the  $ds$ :

$$\frac{d}{dt} \frac{d\theta}{dt} \sim \frac{\theta}{t^2} \quad (6)$$

Note that in this respect, Leibniz<sup>11</sup> had better notation than Newton. Then (5) becomes, after some more canceling,  $ml \frac{\theta}{t^2} \sim mg\theta$ , that is,  $t^2 \sim l/g$ , the correct answer.

Your calculus teacher screams. But this sort of fly by night maneuver is basically correct,<sup>12</sup> and in essence amounts to dimensional analysis.

When we discuss the expanding universe in chapter VII.3, we will use this approach to great advantage.

## The profundity behind Einstein gravity

Imagine yourself in Galileo's shoes. Observation of the pendulum led you to  $T \propto \sqrt{l}$ . Historically, Galileo also knew that gravity produces a universal acceleration  $g$  (recall the legend of the cannonballs dropped from the leaning tower), and he even had a reasonably good value for  $g$ , obtained by, more sensibly, rolling balls down planes inclined at various angles. But still, to construct a theory that would produce  $T \propto \sqrt{l}$  would have been supremely difficult and would have to wait for the genius of Newton. Familiarity might breed contempt, but (5) contains an awful lot of conceptual profundities. We might be in a similar situation regarding some of the unsolved problems in fundamental physics, such as the cosmological constant puzzle.<sup>13</sup>

We now arrive at a profundity, one of the greatest in physics.

To Newton, mass measures the amount of stuff.<sup>14</sup> He quite naturally assumed that the mass  $m$  appearing in his law of gravity and the mass  $m$  appearing in his law of motion are one and the same.<sup>15</sup>

But a hair-splitting lawyer, or a habitual reader of mysteries, would surely have detected a hidden assumption here: Are the blonde<sup>16</sup> seen kissing the butler and the blonde caught leaving the house on the night of the murder really the same blonde? Are those two masses really the same mass?

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\*Which, curiously enough, is spoken as  $F = ma$  but written as  $ma = F$ . Talk about my confusion when first studying physics!

Did you ever notice that the two  $m$ s appearing in (2) have different conceptual origins?

## A couch potato's obligation to gravity versus his reluctance to move

To distinguish between the masses that appear in Newton's law of gravity and in Newton's law of motion, physicists called them the gravitational mass  $m_G$  and the inertial mass  $m_I$ , respectively. The former measures a couch potato's obligation to listen to gravity, the latter his reluctance to get up and move. Conceptually, they are quite distinct and could, by logic alone, not be equal.

Unlike the faculty in some other university departments, those in the physics department do not accept proofs by authority, not even a single-named giant in a likely apocryphal story about dropping stuff off a leaning tower. And thus the Hungarian Baron Loránd Eötvös, instead of doing whatever barons did in the 19th century, devoted much of his life performing ever more precise experiments establishing the equality of the gravitational mass and the inertial mass. In our days, a series of experiments, known collectively as Eötvös experiments, have established the equality of the gravitational mass and the inertial mass  $m_I = m_G$  to a fantastic degree of accuracy.<sup>17</sup>

So, in principle, we could only say that the pendulum's period  $T = 2\pi\sqrt{m_I l / m_G g}$ .

It turns out that this amazing equality of the gravitational mass and the inertial mass furnishes the key<sup>18</sup> to Einstein's theory of gravity. See appendix Eg.

### Exercises

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(1) A physics major planning to become a high school physics teacher (a noble profession!) told me that in her high school, she was taught to remember the "Big 4." I asked her what those are. They turned out to be, for example, the formula for the distance  $\Delta x$  covered in time  $t$  by an object with constant acceleration  $a$  and initial velocity  $v_0$ :  $\Delta x = v_0 t + \frac{1}{2} a t^2$ . In fact, these kinematic formulas follow essentially from the definitions of various concepts, such as acceleration and dimensional analysis. Show this and argue for the factor of  $\frac{1}{2}$ .

(2) Look through your high school physics textbook and pick out some results that follow essentially from dimensional analysis.

(3) To ace exams on elementary physics, always keep dimensional analysis in your pocket. For instance, suppose you were confused about whether work done is force times distance traversed or force times time elapsed. What to do?

(4) When a spring with Hooke's constant  $k$  is stretched by  $\Delta l$ , it exerts a force  $F = k \Delta l$ . Find the oscillation frequency of a mass  $m$  attached to this spring.

## 10 Chapter I.1

(5) Consider the 1-dimensional motion of a particle in the potential  $V(x) = g|x|^\alpha$ . Let  $t$  and  $d$  denote the characteristic time and distance, respectively, for this motion. Show by dimensional analysis that

$$t \propto d^{1-\frac{\alpha}{2}}$$

Apply this result to (a) the harmonic oscillator, (b) a particle in free fall on the surface of the earth, and (c) a planet falling radially toward a star.

## Notes

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<sup>1</sup> According to a photo, most likely photoshopped, circulating on the web several years ago.

<sup>2</sup> For those readers unfamiliar with American culture, the name of a long defunct football team.

<sup>3</sup> And 1869 light years converted into feet.

<sup>4</sup> To be fair, Daniel Gabriel Fahrenheit (1686–1736) was actually a great physicist for his time.

<sup>5</sup> And any deviation from this law could be explained, namely, that one of the buses was delayed.

<sup>6</sup> To me this statement is so self evident that I absolutely refuse to spend 30 or so pages explaining why this must be true and orating about things like Buckingham's theorem (which, by the way, I had never heard of until I started writing this book and looking at the literature).

<sup>7</sup> I put in these two paragraphs at the request of a critical reader. But in fact, by the time a student reaches the course I mentioned in the preface, he or she almost invariably understands that idealization is a necessary part of physics.

<sup>8</sup> He first burst onto the scene in *QFT Nut*.

<sup>9</sup> And they sometimes open up new fields of study.

<sup>10</sup> Babylonians were incredibly smart. See, for example, *GNut*, p. 214.

<sup>11</sup> I read somewhere that in fact he “envisioned”  $\frac{dy}{dx}$  as the division of one infinitesimal by another, *dy* by *dx*.

<sup>12</sup> The creators of LaTeX also thought so: the derivative is written as “slash frac.”

<sup>13</sup> For example, *GNut*, chapter X.7.

<sup>14</sup> Newtonian physics cannot entertain the existence of massless particles.

<sup>15</sup> The following paragraph is taken from my book, *On Gravity*, Princeton University Press, 2018.

<sup>16</sup> As to what type of blonde, see the classification in the scholarly study *Blonde Like Me* by Natalia Ilyin (Simon & Schuster, 2000).

<sup>17</sup> In particular, an ingenious effort, led by my former colleague Eric Adelberger at the University of Washington, is fondly referred to as the Eöt-Wash experiment. See <https://www.npl.washington.edu/eorwash/node/1>. Nerd humor in full force here!

<sup>18</sup> For more details, see, for example, *GNut*. For a brief discussion, see T. P. Cheng, *Einstein's Physics*, Oxford University Press, 2013.

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