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1 Introduction

1.1 Prehistory

The quest in physics has been historically dominated by unraveling the simplicity of physical law, moving more and more toward the elementary. Although this approach is not guaranteed to succeed indefinitely, it has been vindicated so far. The other organizing tendency of the human mind is toward unification: finding a unique framework for describing seemingly disparate phenomena.

The physics of the late nineteenth and twentieth centuries is a series of discoveries and unifications. Maxwell unified electricity and magnetism. Einstein developed the general theory of relativity, which unified the principle of relativity and gravity. The late 1940s witnessed a culmination of two decades' efforts on the unification of electromagnetism and quantum mechanics. In the 1960s and 1970s, the theories of weak and electromagnetic interactions were also unified. Moreover, around the same period, a wider conceptual unification also took place. Three of the four fundamental forces known were described by gauge theories. The fourth, gravity, is also based on local invariance, albeit of a different type, and so far it stands apart.¹ The combined theory, containing the quantum field theories (QFTs) of the electroweak and strong interactions together with the classical theory of gravity, formed the Standard Model (SM) of fundamental interactions. It is based on the gauge group $SU(3) \times SU(2) \times U(1)$. Its spin-1 gauge bosons mediate the strong and electroweak interactions. The matter particles are quarks and leptons of spin $\frac{1}{2}$ in three copies (known as generations and differing widely in mass), and a spin-0 particlethe Higgs boson-that is responsible for the spontaneous breaking of the electroweak gauge symmetry.

 $^{^1}$ Today we have some intriguing evidence that even gravity may be a strong-coupling facet of an extra underlying four-dimensional gauge theory.

2 | Chapter 1

The SM has been experimentally tested and has survived 45 years of accelerator experiments.² This highly successful theory, however, is not satisfactory:

- A classical theory—namely, gravity, as described by general relativity—must be added to the SM to agree with experimental data. This theory is not renormalizable at the quantum level. In other words, new input is needed to understand its high-energy behavior. This has been a challenge to the physics community since the 1930s, and (apart from string theory) very little has been learned on this subject since then.
- The three SM interactions are not completely unified. The gauge group is semisimple. Gravity seems even further from unification with the gauge theories. A related problem is that the SM contains many parameters that look a priori arbitrary.
- The model is unstable as we increase the energy (known as the hierarchy problem of mass scales), and the theory looses predictivity as one starts moving far from current accelerator energies and closer to the Planck scale. Gauge bosons are protected from destabilizing corrections because of gauge invariance. The fermions are equally protected due to chiral symmetries. The real culprit is the Higgs boson.

Several attempts have been made to address the above problems.

The first attempts focused on improving unification. They gave rise to the grand unified theories (GUTs for short). All interactions were collected in a simple group, SU(5)in the beginning, but also SO(10), E_6 , and others. The fermions of a given generation were organized in the (larger) representations of the GUT group. There were successes in this endeavor, including the prediction of $\sin^2 \theta_W$ and the prediction of light right-handed neutrinos in some GUTs. However, the theories required that the Higgs bosons break the GUT symmetry to the SM group, and the hierarchy problem took its toll by making it technically impossible to engineer a light electroweak Higgs.

The physics community realized that the focus must be on bypassing the hierarchy problem. An early idea attacked the problem at its root: it attempted to banish the Higgs boson as an elementary state and to replace it with extra fermionic degrees of freedom. It introduced a new gauge interaction (termed "technicolor"), which binds these fermions strongly and one of the techni-hadrons, should have the right properties to replace the elementary Higgs boson in its role as responsible for the electroweak symmetry breaking. The negative side of this line of thought is that it relied on the nonperturbative physics of the technicolor interaction. Realistic model building turned out to be difficult, and eventually this line of thought was mostly abandoned.

A competing idea relied on a new type of symmetry—supersymmetry—that connected bosons to fermions. This property turned out to be essential, since it could force the bad-mannered spin-0 bosons to behave as well as their spin- $\frac{1}{2}$ partners did. This works well, but supersymmetry stipulated that each SM fermion must have a spin-0 superpartner with equal mass. This being obviously false, supersymmetry must be spontaneously broken at

² With the exception of the neutrino sector, which was suspected to be incomplete and is currently the source of interesting discoveries.

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an energy scale not far away from today's accelerator energies. Further analysis indicated that the breaking of global supersymmetry produced superpartners whose masses were correlated with those of the already known particles, in conflict with experimental data.

To avoid such constraints, global supersymmetry needed to be promoted to a local symmetry. As supersymmetry transformations are in a sense the square root of translations, a theory of local supersymmetry must also incorporate gravity. This theory was first constructed in the late 1970s, and was further generalized to make model building possible. The flip side of this development was that the inclusion of gravity opened Pandora's box of nonrenormalizability of the theory. Hopes that (extended) supergravity might be renormalizable soon vanished.³

In parallel with the developments outlined above, a part of the community resurrected the old idea of Kaluza and Klein of unifying gravity with the other gauge interactions. If one starts from a higher-dimensional theory of gravity and compactifies the theory to four dimensions, the result is four-dimensional gravity plus extra gauge interactions. Although gravity in higher dimensions is more singular in the ultraviolet (UV), physicists hoped that, at least at the classical level, one would obtain a theory that is very close to the SM. Although progress was made, a stumbling block turned out to be obtaining a four-dimensional chiral spectrum of fermions as in the SM.

Although none of the directions described above resulted in a final and successful theory, the ingredients were very interesting ideas that many felt would form part of the successful theory.

1.2 The Case for String Theory

String theory has been the leading candidate over the past almost four decades for a theory that consistently unifies all fundamental forces of nature, including gravity. It gained popularity because it provides a theory that is UV finite.⁴

The basic characteristic of the theory is that its elementary constituents are extended strings rather than pointlike particles as in QFT. This makes the theory much more complicated than QFT, but at the same time it imparts some unique properties.

One of the key ingredients of string theory is that it provides a finite theory of quantum gravity, at least in perturbation theory. To appreciate the difficulties with the quantization of Einstein gravity, look at a single-graviton exchange between two particles (Figure 1.1a). The amplitude is then proportional to E^2/M_P^2 , where *E* is the energy of the process, and M_P is the Planck mass: $M_P \sim 10^{19}$ GeV. It is related to Newton's constant by

$$M_{\rm P}^2 = \frac{1}{16\pi \,G_{\rm N}}.\tag{1.2.1}$$

 3 But recent progress in on-shell loop corrections have made some people hope again that it might still be renormalizable.

⁴ Although no rigorous proof to all orders establishes that the theory is UV finite, there are several all-orders arguments as well as rigorous results at low-loop order. In closed-string theory, amplitudes must be carefully defined via analytic continuation, standard in S-matrix theory. When open strings are present, divergences occur. However, they are interpreted as infrared (IR) divergences (due to the exchange of massless states) in the dual closed-string channel. They are subtracted in the "Wilsonian" S-matrix elements.

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Figure 1.1. Gravitational interaction between two particles via graviton exchange.

Therefore, the gravitational interaction is irrelevant in the IR ($E \ll M_P$) but is strongly relevant in the UV. In particular, this implies that the two-graviton exchange diagram (Figure 1.1b) is proportional to the dimensionless ratio

$$\frac{E^2}{M_{\rm p}^4} \int_0^\Lambda d\tilde{E} \; \tilde{E} \sim \frac{\Lambda^2 E^2}{M_{\rm p}^4} \;, \tag{1.2.2}$$

where E is the outgoing particle energy, and \tilde{E} is the internal particle energy. This is strongly UV divergent. It is known that Einstein gravity coupled to matter is nonrenormalizable in perturbation theory. Supersymmetry makes the UV divergence softer, but the nonrenormalizability persists.

There are two ways out of this:

- It is possible that a nontrivial UV fixed point governs the high-energy behavior of quantum gravity. To date, no credible example of this possibility has been offered.
- It is possible that new physics emerges at $E \sim M_P$ (or even lower), and Einstein gravity is the IR limit of a more general theory, valid at and beyond the Planck scale. You could consider the analogous situation with the Fermi theory of weak interactions. There, one had a nonrenormalizable current–current interaction with similar problems, but today we know that this is the IR limit of the standard weak interaction mediated by the W^{\pm} and Z^0 gauge bosons. So far, no consistent field theory has been found that makes sense at energies beyond M_P and contains gravity. Good reviews of the ultraviolet problems of Einstein gravity can be found in [1, 2].

Strings provide a theory that induces new physics at the string scale M_s , which in perturbation theory (the string coupling g_s being weak) is much lower than the Planck scale M_P . It is still true, however, that string perturbation theory becomes uncontrollable when the energies approach the Planck scale.

There are two important ingredients responsible for why closed-string theory does not have UV divergences. One is the fact that string dynamics and interactions are inextricably linked to the geometry of two-dimensional surfaces. For example, for closed strings,

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Figure 1.2. Open (left) and closed (right) Riemann surfaces.

by decomposing the string Feynman diagrams, it is obvious that there is essentially a universal three-point interaction in the theory. This interaction is dictated by the twodimensional geometry of closed Riemann surfaces, as is obvious from Figure 1.2. The other related ingredient is the presence of the infinite tower of excitations with masses in multiples of the string scale. Their interactions are carefully tuned to become soft at distances larger than the string length ℓ_s but still longer than the Planck length ℓ_P .

For open strings, the situation is subtler. There, UV divergences are present, but are interpreted as IR closed-string divergences in the dual closed-string channel. This UV–IR open–closed-string duality is at the heart of many of the recent developments in the field.

Another key ingredient of string theory is that it unifies gravity with gauge interactions. It does this in several different ways. The simplest is via the traditional Kaluza-Klein (KK) approach. Superstring theory typically is defined in ten dimensions. Standard fourdimensional vacua can be obtained via compactification on a six-dimensional compact manifold. However, gauge symmetry can also arise from D-branes that sometimes are part of the vacuum (as in orientifolds). There is even gauge symmetry coming from a nongeometrical part of the theory, as happens in the heterotic string. The unified origin of gravity and gauge symmetry extends even further to other interactions. For example, the Yukawa interactions, crucial for giving mass to the SM particles, are also intimately related to the gauge interactions.

Unlike earlier KK approaches to unification, string theory is capable of providing, upon appropriate compactifications, chiral matter in four dimensions. This happens via a subtle interplay between anomaly-related interactions and the process of compactification.

Another characteristic ingredient of string theory is that the presence of spacetime fermions in the theory imply the appearance of spacetime supersymmetry at least at high energies. Consequently, high-energy supersymmetry is an important ingredient of the theory.

For several decades, researchers hoped that the theory would prove to be unique, although there are many possible vacua that could be stable. Further understanding of nonperturbative dualities in the 1990s has strengthened the belief in the uniqueness of this string theory structure. It has also pointed out other aspects of the overall theory, such as M-theory, that are new.

The advent of the AdS/CFT correspondence over the past 20 years and the radical input that it injected into an old suspected gauge-theory/string-theory correspondence has gradually changed the way we view string theory. Flux compactifications of string theory, of the

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type relevant for the AdS/CFT correspondence, had already indicated that the number of "vacua" of string theory is astronomical.

The AdS/CFT correspondence suggests that large numbers of vacua are dual to QFTs. Many researchers believe today that this implies that string theory is not a unique theory, but a theoretical framework similar to the framework of QFT. Moreover, the AdS/CFT correspondence further suggests that these two distinct-looking frameworks are in fact part of the same whole. Never in the past have string theory and QFT looked so close.

A big "hole" in string theory has been its perturbative (only) definition. With the advent of nonperturbative dualities, it was hoped that this shortcoming can be bypassed. Although the nonperturbative dualities have shed light in many obscure corners of string theory (obscured by strong-coupling physics), they never managed to bypass the Planck barrier. The Planck scale is always duality invariant, and any dual description is well defined for energies well below that Planck scale. We have no clue from string theory what happens near or above the Planck scale, as the relevant physics looks nonperturbative from any point of view.

In contrast, the AdS/CFT correspondence has given an unexpected hope to being able to define string theory nonperturbatively, in terms of dual UV-complete QFTs. This possibility extends earlier proposals for a nonperturbative definition of string theory via matrix models. All such definitions, however, although in principle nonperturbative, are valid on patches of the traditional string theory landscape, exactly as QFTs are connected only to those other QFTs emanating from them by relevant renormalization group (RG) flows. Such nonperturbative definitions hold the promise of elucidating gravitational physics beyond the Planck scale, although the relevant variables may not be the gravitational ones. This topic is being actively explored today.

This shift of paradigm has put a new light on finding observable physics in string theory or any of its siblings. Experimental confirmation still looks more complicated today than in the 1980s, and not all researchers agree on what is the best way this can be achieved.

Today string theory has a history of 50 years of active research, but the problems it has been called on to solve have varied widely during these 50 years. This reflects the evolution of our understanding of the theory. However, during all this period, it has been a source of many fundamental theoretical advances that have deepened considerably our knowledge of QFT and gravitational interactions.

It is fair to say that what was considered through the years as "string theory" has become an ever-expanding body of problems. Today, string theory has become broad enough to include a big part of QFT, gravitation, and conventional string theory/supergravity. We are in a remarkable era, where all tools in QFT and string theory come together to attack many varied problems that enter mathematics, condensed matter physics, quantum chromodynamics (QCD), and quantum information theory.

Judging from past experience, it is a bad idea to speculate on what string theory will be tomorrow, as almost certainly our guesses will be wrong. I can only say that many interesting problems remain unsolved, and the young and smart physics practitioners of today will push the barriers of knowledge further.

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1.3 A Stringy Historical Perspective

In the 1960s, physicists tried to make sense of a large amount of experimental data relevant to the strong interaction. There were lots of particles (or "resonances"), and the situation could best be described as chaotic. Some regularities were observed:

• Almost linear Regge behavior. It was noticed that a relatively large number of resonances could be nicely put on (almost) straight lines by plotting their mass—versus their spin *J*,

$$m^2 = \frac{J}{\alpha'} + \alpha_0, \tag{1.3.1}$$

with $\alpha' \sim 1 \text{ GeV}^{-2}$, and this relation was checked up to J = 11/2.

• *s*-*t* duality. If we consider a scattering amplitude of two hadrons \rightarrow two hadrons (1, 2 \rightarrow 3, 4), then it can be described by the Mandelstam invariants,

$$s = -(p_1 + p_2)^2, \ t = -(p_2 + p_3)^2, \ u = -(p_1 + p_3)^2,$$
 (1.3.2)

with $s + t + u = \sum_i m_i^2$. We are using a metric with signature (- + + +). Such an amplitude depends on the flavor quantum numbers of hadrons (for example, SU(3)). Consider the flavor part, which is cyclically symmetric in flavor space. For the full amplitude to be symmetric, it must also be cyclically symmetric in the momenta p_i . This symmetry amounts to the interchange $t \leftrightarrow s$. Therefore, the amplitude should satisfy A(s, t) = A(t, s). Consider a *t*-channel contribution due to the exchange of a spin-*J* particle of mass *M*. Then, at high energy,

$$A_J(s,t) \sim \frac{(-s)^J}{t-M^2}$$
 (1.3.3)

Therefore, this partial amplitude increases with *s*, and its behavior becomes worse for large values of *J*. If one sews amplitudes of this form together to make a loop amplitude, then there are uncontrollable UV divergences for J > 1. Any finite sum of amplitudes of the form (1.3.3) has this bad UV behavior. Moreover, such a finite sum has no *s*-channel poles. However, if one allows an infinite number of terms, then it is conceivable that the UV behavior might be different.

A proposal for such a dual amplitude was made by Veneziano [3]:

$$A(s,t) = \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s) - \alpha(t))},$$
(1.3.4)

where Γ is the standard Euler $\Gamma\text{-function,}$ and

$$\alpha(s) = \alpha(0) + \alpha' s \,. \tag{1.3.5}$$

By using the standard properties of the Γ -function, it can be checked that the amplitude (1.3.4) has an infinite number of *s*, *t*-channel poles:

$$A(s,t) = -\sum_{n=0}^{\infty} \frac{(\alpha(s)+1)\dots(\alpha(s)+n)}{n!} \frac{1}{\alpha(t)-n} \,.$$
(1.3.6)

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In this expansion, the $s \leftrightarrow t$ interchange symmetry of (1.3.4) is not manifest. The poles in (1.3.6) correspond to the exchange of an infinite number of particles of mass $M^2 = \frac{(n-\alpha(0))}{\alpha'}$ and high spins. It can also be checked that the high-energy behavior of the Veneziano amplitude is softer than any local QFT amplitude, and the presence of an infinite number of poles is crucial for this.

It was subsequently realized by Nambu, Goto, Nielsen, and Susskind that such amplitudes came out of theories of relativistic strings. However, string theories had several shortcomings in explaining the dynamics of strong interactions:

• All of them seemed to predict a particle with negative mass squared, the tachyon.

• Several of them seemed to contain a massless spin-2 particle that was impossible to get rid of.

• All of them seemed to require a spacetime dimension of 26 to keep from breaking Lorentz invariance at the quantum level.

• All of them contained only bosons.

At the same time, experimental data from SLAC showed that at even higher energies, hadrons have a pointlike structure; this opened the way for QCD as the correct theory to describe strong interactions.

However, some work continued in the context of "dual models", and in the mid-1970s several interesting breakthroughs were made.

• It was understood by Neveu, Schwarz, and Ramond how to include spacetime fermions in string theory.

• It was also understood by Gliozzi, Scherk, and Olive how to get rid of the omnipresent tachyon. In the process, the constructed theory had spacetime supersymmetry.

• Scherk and Schwarz, and independently Yoneya, proposed that closed-string theory, always having a massless spin-2 particle, naturally describes gravity and that the scale α' should be related to the Planck scale. Moreover, the theory can be defined in four dimensions using the KK idea, namely, considering the extra dimensions to be compact and small.

However, the big new impetus for string theory came in 1984. After a general analysis of gauge and gravitational anomalies [4], it was realized that anomaly-free theories in higher dimensions are very restricted. Green and Schwarz [5] showed that open superstrings in ten dimensions are anomaly free if the gauge group is O(32). The group $E_8 \times E_8$ is also anomaly free but could not appear in open-string theory. In [6] it was shown that another supersymmetric string exists in ten dimensions, a hybrid of the superstring and the bosonic string, which can realize the $E_8 \times E_8$ or O(32) gauge symmetries.

Since the early 1980s, the field of string theory has been continuously developing. There was a big rush on heterotic model building in the late 1980s, and the matrix model approach to two-dimensional string theory was developed in the early 1990s, followed by the study of stringy black holes. In the mid-1990s, nonperturbative dualities between

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different supersymmetric string theories were uncovered. This development gave rise to the aspiration that the theory is unique, and the name M-theory was coined. D-branes were discovered and studied. They turned out to be crucial for the construction of controllable models for the identification of black-hole microstates and the microscopic explanation of the black-hole entropy. Moreover, this led to the formulation of AdS/CFT correspondence and its generalizations.

String theory is a continuously evolving subject, and this book gives only a brief introduction to some of its best-understood topics.

1.4 Conventions

Unless otherwise stated, we use natural units in which $\hbar = c = 1$. The string length ℓ_s is kept explicitly throughout the book. It is related to the Regge slope α' , and the string (mass) scale M_s by

$$\ell_{\rm s} = \sqrt{\alpha'} = \frac{1}{M_{\rm s}}.$$

In the literature, most of the time $\alpha' = 2$ in closed-string theory, $\alpha' = 1/2$ in open-string theory, and sometimes $\alpha' = 1$ in CFT.

The fundamental string tension is $T = \frac{1}{2\pi \ell_s^2}$. We denote by T_p the D_p brane tension, by T_{M_2,M_5} the respective M₂ and M₅ brane tensions, and by \tilde{T}_5 the NS₅ brane tension.

We use X^{μ} for the spacetime coordinates of the string and x^{μ} for their zero modes. By convention, the left-moving part of the string is the holomorphic part, with conformal dimensions (Δ , 0). The right-moving part is the antiholomorphic part with dimensions ($0, \overline{\Delta}$). The right-moving part is taken as the nonsupersymmetric side of the heterotic string.

 $F_{L,R}$ is the worldsheet (left- or right-moving) fermion number. The operator that we use is $(-1)^{F_{L,R}}$. In the NS sector, it counts the number of fermion oscillators modulo 2. Its action is explained in Sections 4.12 on page 76 and 7.7.1 on page 182. It should be distinguished from the "spacetime fermion number" operators $F_{L,R}$. $F_L = 0$ in the left-moving NS sector and equals 1 in the left-moving Ramond sector. A similar definition holds for F_R . Note that in the heterotic string, F_L is indeed the spacetime fermion number. In type-II string the spacetime fermion number is $F_L + F_R$ modulo 2.

We are using the "mostly plus" convention for the signature of the spacetime metric. Our curvature conventions are such that the *n*-sphere S^n has positive scalar curvature. When there is no risk of confusion, we use for the element of the metric $\sqrt{-\det g} \leftrightarrow \sqrt{\det g}$.

Our conventions on the two-dimensional geometry are spelled out in Appendix A on page 724. Those on differential forms, the ϵ -density, the related *E*-tensor, and the Hodge dual are found in Appendix B on page 726.

We use a unified notation for extended supersymmetry in diverse dimensions. Generically, *n* supersymmetries in *d* dimensions are denoted by $\mathcal{N} = n_d$. In two, six, and ten dimensions, because of the existence of Majorana-Weyl (MW) spinors, we may define

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extended supersymmetries with *p* left-handed and *q* right-handed MW supercharges. In this case, we use the notation $\mathcal{N} = (p, q)_d$. However, sometimes, even in such dimensions, if the chirality of the supersymmetry is not important for our purposes, we might still use the $\mathcal{N} = n_d$ notation.

For symplectic groups, we use the notation Sp(2N) with Sp(2)~ SU(2). For the groups SO(2n), the subscript \pm on the spinor indicates its eigenvalue \pm 1 under the appropriate generalization of γ^5 .

We also call the heterotic string based on the Spin(32)/ \mathbb{Z}_2 lattice the "O(32) heterotic string," for simplicity.

1.5 A Brief Guide to the Literature

The guides to the associated literature, presented in this book, have been compiled with pedagogy as their main motivation as well as to provide some benchmark points in the literature, where the student can start searching. This book is intended as a textbook, and an appropriately chosen bibliography is a crucial complement. Review articles have been favored here, but also original papers, when they are deemed to have pedagogical value or if they are good starting points in a bibliographical search.

Most of the papers and reviews after 1991 have appeared in the electronic physics archives and are referred to as "[arXiv:...]." They are available from the central archive site https://arXiv.org/

and its mirrors worldwide.

There are several books and lecture notes on string theory. The first benchmark is Green, Schwarz, and Witten (alias GSW) [7]. It is a reference two-volume set. It summarizes the older literature on string theory and describes in detail string compactifications up to the mid-1980s. The best and most detailed exposition of the Green-Schwarz (GS) approach to the superstring is given here in detail. There is a balance between covariant and light-cone methods used in the quantization.

The second benchmark is J. Polchinski's two-volume set [8]. It focuses on the modern approach to string theory via covariant quantization and the use of CFT methods. It also contains a description of D-branes and their roles in nonperturbative string dualities.

Becker, Becker, and Schwarz [9] came out around the same time as the first edition of this book. It is an advanced and up-to-date book covering many important topics in string theory. Blumenhagen, Lüst, and Theisen [10] is the modern sequel of the very good older lectures [11]. It gives a clear and up-to-date introduction to modern string theory, including a detailed presentation of many compactifications that are interesting from the phenomenological point of view.

An introduction to string theory, with emphasis on phenomenological aspects and compactifications, can be found in Ibañez and Uranga, [12]. M. Dine [13] also bridges the gap between the SM, supersymmetry, and string theory, presenting all ex aequo. Freedman and van Proeyen's *Supergravity* [14] is very useful for understanding the structure of string effective theories.

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Several other books have different specializations and characteristics. C. Johnson [15] provides an in-depth look at D-branes and their effects on string theory, while at the same time providing an introduction to the basics. Ortin [16] provides a coherent and in-depth exposition of geometric aspects of the theory and its many interesting classical solutions.

R. Szabo [17] is a short introduction to string theory (120 pages in all) that covers the very basics. A recent book by V. Schomerus [18] also provides a quick introduction appropriate for a semester course in string theory.

Last but not least is the recent book of B. Zwiebach [19], which is written at a more introductory level and is addressed to advanced undergraduates. It contains several interesting topics in string theory at an accessible level.

Several books [20]–[23] have appeared over the past 10 years that expand on various aspects of the AdS/CFT correspondence and its applications. Natsuume [20] provides a pedestrian introduction to the holographic correspondence, with a list of different applications to QCD, heavy ions, and condensed-matter problems. [21] is written by a combination of experts from QCD and string theory; it treats the application of holographic techniques to QCD and the physics of heavy-ion collisions. Nastase's first book [22] gives a general introduction to the holographic correspondence with a broad spectrum of applications from theoretical topics, such as integrability, to practical topics, like bottom-up holographic theories and condensed-matter applications. His second book [23] focuses on condensed-matter applications and presents both the famous condensed-matter problems as well as the holographic frameworks that address them.

Ammon and Erdmenger [24] provides a broad introduction to string theory, gravity, and the AdS/CFT correspondence and then treats essentially all holographic applications, from integrability to flavor physics and QCD to condensed-matter applications. Finally, the book of Zaanen et al. [25] is written by a combination of experts from condensed matter and string theory and provides an excellent review of both sides, with a more in-depth treatment of condensed-matter problems using holography.

A recent book by Baumann and McAllister treats inflation in string theory [26]. It includes the basic cosmology of inflation, basic string theory, and string-theoretic models of inflation.

Several other books and good reviews have appeared over the years [27]–[35]. Some may be limited by their scope or date of appearance. There is, however, some merit in consulting them since they may have other advantages, such as an in-depth description of special subjects or a successful pedagogical description of certain topics.

Marolf's resource letter [36] is an excellent source of various articles on and reviews of string theory. It includes a wide spectrum of sources, from popular science to specialized reviews. The review article of Seiberg and Schwarz [37] provides a useful overview of the field, its achievements, and its goals.

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