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♣ CHAPTER 1 ♣

Pits, Pebbles, and Bones

Rolling to Discover Fate

Gaming is a principle inherent in human nature.
—Edmund Burke, British House of Commons Record, February 11, 1780

Imagine life in the last Ice Age. Those Neanderthals, with their orangutan jaws and beetle brows, burbling some mono-vowel language, sharpening spears in preparation for a hunt of hungry scimitar-toothed black tigers, reflexively gambling every day against the impending extinction of their race.[1] Ground tremors, as common as cloudy days, triggered by great weights of melting ice continually relaxing gargantuan pressures of the earth's indomitable crust; ordeals of menacing elements, snow and freezing temperatures; hunger, pain, and weakness from the bruises of long, fierce hunts; and most worrisome of all, the daily threats of nearby ravenous beasts stealthily looking for supper. Humans have been gambling ever since that unfathomable island of time, when herds of pachyderms and hump-shouldered mammoths freely wandered over the frozen lakes of the Neander Valley.

Our extinct fellow proto-humans looked brutally intimidating and menacing with their powerful muscles, fleshy fingers, and massive limbs, but they were innocently carrying fierce looks for passive

self-protection.[2] They cared for their young, who played with discarded dry bones by kicking, banging, and tossing them here and there in amusement or quite possibly in a shilly-shally appreciation of primordial sport. How natural it is for a child to create games by hurling things. We can readily imagine a Neanderthal child uttering, "My bone for your two that I can throw mine further," even in a primitive language of consonants mixed with *ah* as its only vowel.[3]

We can picture the adults, in their rare, brief leisure time, wagering on who can throw the farthest spear or on who can down the nearest rhino. They may have tossed bones in games the way kids now toss marbles, laugh when something is funny, or cry when injured. Neanderthals smiled to express joy, frowned to convey displeasure, embraced in camaraderie, and gambled every day with their own lives in decisions of whether or not to go out on a hunt or to wander far from their recognized comfortable safety zones where the fire was warm.

Risks are the gambles, the games, the balance of expectation and fate. And luck rarely comes without risking the possibility of loss, injury, trouble, vulnerability, ruin, or damage in a universe of opposing chances. We also know (from bone sample evidence) that our Neanderthal friends were subjected to a high rate of injury during their lifetimes, most likely from close-range hunting of fast and ferocious sabertooths, whose sword-like canines could effortlessly pierce and slash the skin of even the toughest males. And if the cats—those felidae with the courage to raze mighty mammoths—didn't occasionally slash those hardy men to death, then the bruises those cats inflicted surely disabled them. That was the true gamble—to eat or be eaten.

Gambling is about odds, the chances of things going one way or another. Will the team bring down the mammoth, or will a tusk lethally impale someone? So we humans are programmed to gamble. It's not only about the team. We take risks, leave our houses and explore the uncertain boundaries of secure and reliable neighborhoods, all as part of the animal nature of survival. The urge to take risks is just one of the hardwired universals of being human, along with smiles, frowns, cries, and laughs.

When we come closer to our own time—not much closer, still in the late Pleistocene era, 10,000 to 40,000 years ago—we find our early dark-skinned Cro-Magnon ancestors in the Danube Valley painting on the walls of caves with sticks of charcoal, carving moon calendars or in performance of some ritual event to applaud the supernatural owner of the wildlife they hunt or to thank the shamans.[4] They, too, performed risks and had to ask: *Is it safe to paint today? Should we leave home and take care not to be mauled by the nearest beast? Or should we stay protected by warm fire and eat the spoils of yesterday's kill?* It was all a gamble at a time when humans were skillfully tuning maneuvers of feet, arms, and wrists to influence and direct flights of their sharp weapons.

Their tools, those spears, arrows, flints, and fires, gave them hunting advantages their brawny Neanderthal neighbors never had: the opportunity to hunt from safe distances. And with them came prophesizing games and innocent gambling. Innocent, because players were not necessarily wagering their fine spears or furs nor—what should have been quite reasonable—staking their best pickings, but rather banking on the moods of randomness for providential guidance and help from the phantoms of predictability in forecasting decisions. A shaman might roll a pair of, say, sheep astragals (anklebones) to determine if the tribe should go out on a hunt the next day. Die-like objects such as filed and sanded astragals have been found in abundance at archaeological sites almost everywhere from central France to as far east as the Punjab. What they were used for is anyone's guess, but more likely than not they brought some form of entertainment or a means of communication with the gods.[5] Someone would ask a question, and, depending on how the astragals fell from the shaman's hand, there would be an answer. One answer was accepted when wide sides faced upward, another when the narrow. Surely these bones were biased; however, it did not take long for our clever ancestors to find a way of evening the odds by rasping the sides of an astragal and smoothening out six faces of a stone or piece of wood for fairer outcomes in inventing the die. Certainly, sticks and odd-shaped stones must have been used for playing against chance. Fruit pits, pebbles, shells, teeth, seeds, and acorns must have given hints of rolling to discover fate.

When we come to the more modern hunting and gathering societies of 10,000 years ago, we see that gambling advances with better impressions of randomness. We find dice being made from carefully carved pieces of wood, stone, and ivory. It's more than just the archaeological findings; now we have the literature, albeit through oral tradition, to back it up. Homer's *Iliad* tells us that gambling and chance have their roots in the beginning of time when Zeus, Poseidon, and Hades drew lots for shares in the universe.

Lot is the etymological root in the words *allotment* and *lottery*. It is also something that happens to a person when the *lot* has fallen—*it was his lot*. The *casting of lots* would have meant any decision-making procedure or mechanism, the flip of a coin, the roll of a die, the pick of a straw. The *lot* itself might have been an object such as a piece of wood, a pebble, a die, a coin, or a straw that could be used as one of the counters in determining answers to vital questions by the position in which it comes to rest after being tossed or picked. But if the lot is to be fair, it must be far more unbiased than an astragal, which surely does not have equal chances of falling on any of its four sides.

Drawing lots was thought to be the fair way to settle a choice that could not be established by reason. And since every lot banks on the whim of Fortune, or on the very misunderstood impulse of randomness, it might be said that every unreasoned choice is a gamble. Indeed, children of all ancient cultures must have muttered some variant of *eenie meenie mynie mo* among friends making a choice. Still, for the adults, it was more likely thought of as a means of communication with some supernatural spirit. Getting the short end of the stick might have been the random pick of the draw, but it could also have simply been the will of God. The Mishnah (the section of the Talmud connected to oral laws) says that *to draw lots* one must have an urn of tablets marked to describe a fate and that those tablets must be alike in size and shape so that any pick is as likely as any other. The Bible says that to atone for the sins of his house, Aaron was to cast lots to decide which of two goats would be sacrificed and which would be sent back into the wilderness.

In Exodus there is a vague description of *the breastplate of judgment*, a part of Aaron's priestly garment. Aaron was to wear it on entering a

holy place to seek God's judgment on difficult questions affecting the welfare of Israel.[6] The breastplate of judgment refers to a vestment of embroidered fabric set with twelve precious stones representing the twelve tribes of Israel and worn by any of the high priests seeking God's guidance on matters concerning the welfare of the community. According to instructions outlined in Exodus, the breastplate must contain the so-called Urim and Thummim, which literally translate as "the Lights and the Perfections." According to some modern commentators, these were two sacred lots, chance instruments such as coins or dice, used for the purpose of determining the will of God on questions of national importance.[7] It may have been that the priest would cast the lots but also understood that while the lot is cast, God manipulates the lot to determine the outcome—"The lot is cast into the lap; But the whole disposing thereof is of the Lord" (Prov. 16:33).

We take "the Lights and the Perfections" here to mean the perfect determination of the truth by means of unbiased casting of lots—the perfect throw of perfect dice for the fairest possible decision.

Fairness is, unfortunately, seldom a functioning human trait, but when it comes to decision-making, inequality is inherently recognizable. The child who must share a piece of cake after being given the opportunity of dividing and cutting it to permit others to choose pieces will try to divide and cut with fairest precision. Humans can recognize overt inequities. So it shouldn't have taken many rolls of astragals to upset early gamblers and cause them to think of a fairer, more random way to cast lots. Though the typical astragal is shaped somewhat like an elongated cube, it has only four sides to fall on; its two end faces are so uneven and knuckly it would be highly unlikely for it to remain standing on one face. Yet, astragals were used for centuries before real cubical dice took over, sometime long before the first millennium BCE, when cubes that could (more or less) fall fairly on one of six faces were introduced. Since then, dice variations have been used in every part of the world from America to Japan, from Sweden to Africa. Recent (2004) archaeological digs in the Bronze Age city of Shahr-I Shokhta (literally, the Burnt City) in southeast Iran unearthed a five-thousand-year-old backgammon set made of ebony with cubical dice.

Dice playing enters stories such as the ancient Indian Sanskrit epic poem *Mahabharata*, the fifth century BCE tale of Cyrus the Great, and the story of Isis and Osiris in which we learn of the Egyptian game of *tau* (akin to English draughts or checkers, which goes back to at least 1600 BCE). Older still is the Royal Game of Ur, going back to before 2500 BCE.[8] Two players would race their pieces from one end of the board to the other according to moves controlled by specific landings of a die, a prototype of backgammon. The die would have been either a four-sided stick or tetrahedron.

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Modern dice, numbered as ours with opposite sides summing to 7, have been found at archaeological sites in Thebes and elsewhere in Egypt.[9] And we have evidence that the Egyptians played a game called *atep*—a game still played almost everywhere in the world and one I recall playing as a kid when we had to either choose sides or choose who would go first in a game. We'd call out *odds* or *evens* and then extend either one or two fingers on the split second after calling out *1-2-3-shoot*.[10] For such a game there are no physical lots but rather mental choices (as if flipping two coins at once) to make fair decisions.

When we come to the Romans we find gambling rampant, though we also find evidence for the first laws against gambling to dampen uncontrolled behavior associated with gaming. It was a time steeped in sexual marathons and drinking sessions mixed with gambling by dice, cards, and quail fights. In nineteenth-century archaeological excavations of Rome, hundreds of gaming tables were found. The tables were typically designed as *tabula lusoria* (table of play) in the form of three horizontal lines, each containing twelve signs with words arranged to make a sentence with thirty-six letters. The taverns patronized by gamblers used such poetic forms in their signs to warn against fighting over games, no doubt swayed by the thirty-six (6×6) distinct possible results of throwing two dice.

LEVATE	LVDERE
NESCIS	DALVSO
RILOCV	RECEDE

These six terms with thirty-six letters are abbreviations of words that form a sort of haiku rune that unravels to this rough translation: *Rise! If you don't know the game make room for better players*!

FIGURE 1.1. Royal Game of Ur, southern Iraq, ca. 2600–2400 BC. From the British Museum Room 56: Mesopotamia. Copyright © The Trustees of the British Museum.

Such *tabula lusoria* would signify good or bad luck, warn of the skill needed to play well, or warn of the risks of gambling. Others were unguarded invitations to gamble.

We know from Plutarch that Marc Anthony was a consummate gambler; that Augustus was an ardent dice player; and that Nero played some variant of craps. Claudius had a special carriage designed for playing dice; he even wrote a book on dice playing. And Caligula, after losing all his money at an ancient variant of craps, ran into the street, confiscated money from two Roman guards, and returned to his game.[11]

Dice have been found all along trails used by the crusaders. Throughout the Middle Ages, from northern Europe to Brindisi at the heel of Italy, crusading armies played dice games at taverns along their way.

Late in the thirteenth century, Alfonso X, king of León and Castile, commissioned the writing of a book of games. Known as *Libro de*

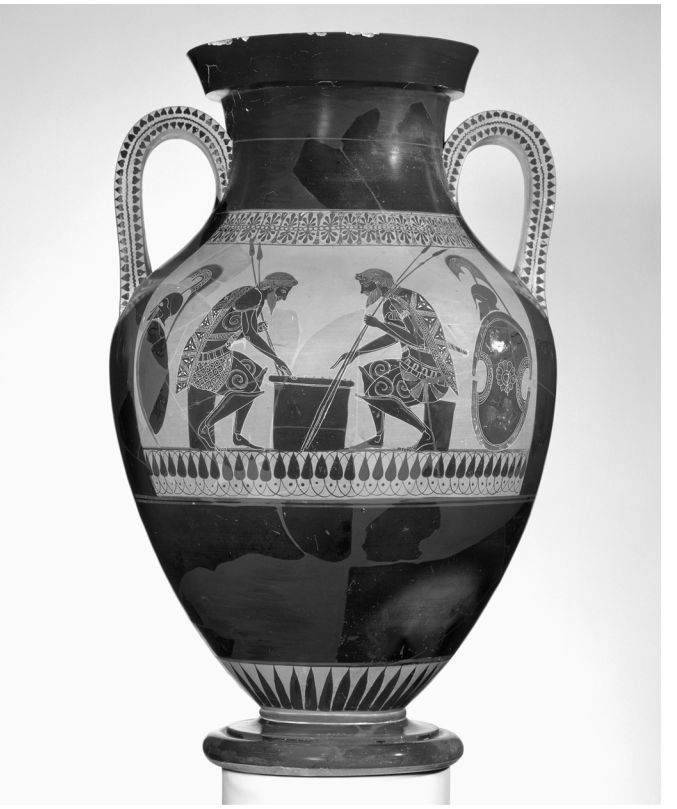

Figure 1.2. Achilles and Ajax playing dice (Attic bilingual amphora), ca. 525–520 BC. Reproduced courtesy of the Museum of Fine Arts, Boston.

Los Juegos (Book of games), it contains descriptions and illuminated illustrations of all sorts of games from chess to backgammon, including dice and tables.[12]

The story told in Alfonso's book of games is that there was once a king who would often consult his three wise men over the nature of things, and on one particular occasion the debate came to the question of gaming and of the advantages of luck and brains.[13] One wise man said that brains were more valuable than luck because thinking gives order to life and even if he lost, he would not be to blame because he used reason. Another said that luck was more valuable

FIGURE 1.3. Image from folio 75 verso of the *Libro de Los Juegos* by Alfonso X, depicting two ladies playing the game of *seis, dos, y* (six, deuce, and ace). Each player started with fifteen pieces. In this illumination one player has 8 on the sixth column, 4 on the second, and 3 on the ace point. The other player has 5 on each of the remaining three columns. The players are moving their men in one direction around the board (as in backgammon) to get to the opposite side of their starting positions. Note that three dice are being used. Reproduced courtesy of Bridgeman Art Library.

than brains because win or lose, his brains could not avoid his destiny. And the third said it would be better to have both—to use the brain to his reasoning advantage and to use luck to protect him from any potential harm.

Alfonso was on to something that was to become the core understanding of all professional gambling from cards to hedge funds. The balance of luck and reasoning could be interpreted through a rational measure of how favorable the outcome might be. Though Alfonso had no conception of risk management and certainly no perception of positive expectancy (the mathematical tool that modern professionals

use to quantify a future event), he did understand that the blends of luck and skill behind gambling games fall into a wide spectrum.

Alfonso's book of games tells us that dice are perfect cubes, made of wood, stone, bone, or, best of all, metal—as perfect as thirteenth-century craftsmen could manufacture—otherwise they would roll more often on one side than any other and would be a trick of luck. The spots are marked just as they would be on our modern dice with opposite sides summing to 7, but for some reason—possibly to acknowledge the holy trinity in hopes that they may have some influence—games were played with three dice.[14] The games were simple: in *mayores*, he who rolled highest won; in other games, he who rolled lowest won.

Alfonso's game book makes the point of saying that many games of the day were designed to resemble events and customs of the times, showing how kings during war would fight alongside their soldiers or how individual soldiers of other kings would be killed, captured, or expelled from the land.

> And also as in the time of peace they are to show their treasures and their riches and the noble and strange things that they have. And according to this they made games. Some with twelve squares (per side), others with ten, others with eight, others with six and others with four. And thus they continued descending down to just one square, which they divided into eight parts. And all this they did because of the great similarities according to the ancient knowledge, which the wise men used.[15]

There was still no notion of a mathematical measure of likelihood. Such an exotic concept would have required knowing something about permutations and combinations of objects, a subject that was almost entirely unknown. A permutation of n objects means all of the possible distinct arrangements of those n objects; so, for example, *ABC* is considered different than *ACB*. A combination of n objects means only that n objects have been selected; therefore, *ABC* is no different from *ACB*. These are two critical notions that are at the heart of gambling, for every random event entails several possible outcomes with or without regard to order.

FIGURE 1.4. The eight combinations of two symbols.

The Chinese were already engaged in some thoughts on permutations and combinations. The *I-Ching* (Book of Changes) entertains a symbol system to identify order in random events. It goes back to the third millennium BCE and may be the earliest work dealing with *combinatorics*, the branch of mathematics dealing with combinations and permutations of members of a group of objects and the mathematical relations that characterize their properties. We take the yin yang symbols to represent 0 and 1. If the solid bar (yang) represents 1 and the split bar (yin) represents 0, then these so-called trigrams are simply binary representations of the numbers 0,1,2,3, . . . Moreover, from just combinations of two symbols (the solid line and the broken line) taken three at a time, we get eight distinct objects.

Abstractly, these eight objects represent all the combinations that can be made by taking two things in groups of three. As an example of how that may play out in a gambling game, take the game of flipping three coins (heads yin, tails yang) and wagering exactly two heads will appear. The probability of getting exactly two heads is 3/8, since there are three groups with exactly two split bars.

As for the Greeks, aside from the few cases of combination counting that we learn through Plutarch, it seems that they never developed a systematic theory of combinatorics. Plutarch tells us that in the fourth century BCE, the Greek philosopher and mathematician Xenocrates computed the number of different combinations of syllables in sensible words of the Greek language as 1,002,000,000,000.[16] (Xenocrates' calculation must have been based on finding the number of possible syllables in the Greek language, surely a daunting lexicographical as well as mathematical exercise.) In the sixth century, however, the philosopher Boethius (who was recently elevated to sainthood by Pope Benedict XVI) figured out that the number of combinations of n things taken 2 at a time is simply

$$\frac{n(n-1)}{2}.$$[17]

This is a very easy calculation, if one looks at it as simply writing out the n things twice. Write out the two strings of n objects as

1, 2, 3, . . . n
1, 2, 3, . . . n.

Then notice that to pair two at a time would mean pairing each number of the first string with all the numbers of the second. That gives you $n(n-1)$ parings. Note that the $n-1$ occurs because you have to eliminate n pairings of a number with itself. But notice that you have counted twice, and so you must divide the result by 2 to get

$$\frac{n(n-1)}{2}.$$

One might say some of the essential mathematics of gambling were around as far back as the eighth century with the interest in Jewish mystical writings that calculated various ways in which the letters of the Hebrew alphabet can be arranged and came up with the correct colossal number.[18] The twelfth-century Spanish biblical commentator Rabbi Abraham be Meir ibn Ezra carried out some of the earliest of the impressive calculations of combinations. Ibn Ezra was able to calculate the number of combinations of 7 objects taken k at a time where k could be anything less than 7. His interest was the possible conjunctions of the seven known planets, which then included the sun and moon. The twelfth-century Indian mathematician Bhaskara extended Boethius's computations in his arithmetic textbook *Lilavati* (The beautiful), written for his child, Lilavati, giving rules for finding the number of ways to choose a group of r objects out of a group of n objects, and posing questions in illustrative mathematical narrative.[19] And surely, the fourteenth-century work of the mathematician and biblical commentator Levi ben Gerson, *Maasei Hoshev* (Art of calculation), should have been a significant contributor, as it correctly demonstrated the general formula for the number of combinations of n things taken r at a time, the principal tool in calculating odds.

However, this was a time when great ideas emerged in isolation, from biblical commentators who worked within the confines of a village, monks who never left their monasteries, and mathematicians who rarely circulated more than a single copy of their work. And so, alas, these contributions were unknown to the collaborative scientific communities of Europe and had to wait centuries before being rediscovered as if new.

They were rediscovered in the mid-thirteenth century. A manuscript of the Latin poem *De Vetula* was found in Ovid's tomb and immediately became a medieval best-seller. It was copied and distributed to libraries all across Europe, though very likely not written by Ovid himself. The poem itself is autobiographical, three books about a poet who changes his lifestyle because of a regrettable love affair. Leading a licentious life (described in detail), he has an affair and falls in love with a beautiful woman. When her husband dies twenty years later, he marries the woman and discovers that she is now old and that he was conned. Depressed, he turns his life to more lofty and moral pursuits of mathematics, philosophy, music, and, of course, religion. In the first book he gives a discourse on the laws of chance applied to gambling with three dice along with his reasons for avoiding dice games.[20]

Though the poem is a medieval morality verse, it does give evidence that some basic mathematical rules of permutations and combinations were known at the time of the discovery of the manuscript, at least as far back as early fourteenth-century France and quite possibly much earlier in India, since the knowledge likely came from Arabic and Indian sources. Regardless of its authenticity, the *De Vetula* contains the earliest known calculations involving serious probability through the observation that in the random throw of dice certain numbers have more ways of occurring than others—the smallest and greatest sums occur more rarely than those near the mode, the most frequent value, just as they do for a pair of dice. (See figure 2.1.)

By Henry VIII's time gambling was illegal in England and there were ordinances against gambling in many European countries, but at court almost everything seemed to be legal. Kings and queens could play as they wished as long as it was in the private apartments of royal residencies. It was customary to announce "His Majesty is

Figure 1.5. From an edition of Boccaccio's *De Casibus Virorum Illustrium* (Paris, 1467), MSS Hunter 371–72 (V.1.8–9), volume 1, folio 1r. Lady Fortune with the Wheel of Fortune. As the wheel turns some men may rise from poverty and hunger to greatness, while some great men may fall. Such scenes of the rise and fall of man were typical in the Middle Ages.

out" when the king entered a game; with that announcement, it was understood that court formality, ceremony, and etiquette could cease. When it was his pleasure to discontinue the game, it was announced "His Majesty is at home," whence playing would cease and the ceremony of the palace was returned to normal.

A parliamentary act passed under Henry VII forbade gambling at any time of year except during the twelve days of Christmas. During those twelve days the public was not only permitted to gamble but encouraged to do so in church.

> Whereby they thinke, throughout the yeare
> to have good luck in play,
> And not to lose: then straight at game
> till day-light they do strive,
> To make some pleasant proofe how well
> their hallowed pence will thrive.
> Three Masses every priest doth sing
> upon that solemne day,
> With offerings unto every one,
> that so the more may play.[21]

Like the Romans, Elizabethans were eager gamblers. Despite legal obstacles that continued up to the time of Elizabeth, we know from Shakespeare's plays that gambling popularity was widespread before and during the Renaissance. Gamblers flocked to the vibrant city of London, where festivities lasted through the year, a city where the individual could lose identity and escape into rhythms of fantasy.[22] Society and royalty made no attempt to conceal gaming. Sir Francis Drake, Thomas Digges, William Gilbert, and Ben Johnson frequently gambled at hazard (the popular seventeenth- and eighteenth-century forerunner of the dice game craps)—it was, after all, the social norm of gentlemen. Christopher Marlowe, Thomas Middleton, Sir Walter Raleigh, and the queen herself often played tables and hazard, and occasionally wagered in the popular blood sport of cockfighting.[23] Shakespeare saw gambling as an integral part of his world and—like all his other apt observations of human eccentricities—he skillfully used it for suitable metaphors.

Fate was directly linked to the mechanical movements in the sky. In *King Lear*, the bastard, Edmund, struggles with the connection between his bastardhood and movements in the sky. Even as late as 1606, when Shakespeare wrote his play about the mythical impetuous old king, the sky was thought of as a fixed (firm) canopy, studded with diamond-like stars, an impression that lingered through the centuries ever since Aristotle declared that the stars influence even birth and extinction.[24] People still believed in a mechanistic, deterministic universe where the notions of destiny, action, and reaction were indubitably linked. And what about *The Tragedy of Hamlet?* Elizabethans, and even Jacobins, would have no trouble believing that the murder of Hamlet's father was the cause of all that followed—Hamlet's madness, Ophelia's drowning, and, ultimately, the deaths of Gertrude, Laertes, and Hamlet—but the murder itself, *that* would have been initiated by the movements of the diamond-studded crystalline spheres nested, one in the next, with the earth at the center, each with a glowing jewel set in its transparency, all moving in perfect musical harmony. It is easy to understand how the impressions of fate seemed mechanical to a person who believes in a finite universe. It is hard to imagine the thoughts of an Elizabethan lying face up in a country field on a dry, warm, and moonless night, staring at the vast and wonderful Milky Way, yet he or she must have believed the universe finite and wondered about its size and how the mechanism of its laboring motion determined his or her luck.

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